ARITHMETIC

THEORETICAL AND PRACTICAL

RIVINGTONS

London	... ...	*Waterloo Place*
Oxford	... ...	*High Street*
Cambridge	... ...	*Trinity Street*

A School Edition of this Work may be had without the Appendix in small 8vo, price 3s. 6d.

ARITHMETIC

THEORETICAL AND PRACTICAL

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SECOND EDITION, REVISED AND ENLARGED

RIVINGTONS
London, Oxford, and Cambridge
1870

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

PREFACE TO THE SECOND EDITION.

THE special object of this Treatise is to furnish explanations of the fundamental principles of Arithmetic more full and clear than those usually given.

It should be held to be of the utmost importance that the reason of every step in every process employed should be made perfectly intelligible to the learner; if at least Arithmetic is to be looked on as an important branch of mental training, rather than a mere series of operations to be mechanically performed.

This book is consequently a protest against that still common process of teaching "sums," which may be called the "magical process:" "Follow the *rule* as laid down," says the master, "do not trouble yourself about the *reason*: but do this, do that, and—hey presto! the answer."

The considerable additions and alterations made in this Edition do not consist in the introduction of any new methods, so much as in the further development of those already laid down in the first Edition. The compendious method of Division is now used throughout the book, and is introduced in the process of finding the Greatest Common Measure; the contracted method of Multiplication and Division is likewise extended to compound quantities. The growing importance of the question of a decimal and an international coinage, and of a more uniform system of weights and measures has led to the introduction of a new Chapter on that subject: while the Chapters on duodecimals and on the extraction of the square and cube root have been recast. A very large addition has also been made in the Appendix to the collection of Examination Papers; and for permission to publish these I am indebted to the courtesy of the Masters of the several Colleges and Schools wherein the Papers were originally set.

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ERRATA.

p. 225, Ex. 34. *For* £16992 ,, 1*s*. ,, 9*d*., *read* £16992 ,, 3*s*. ,, 9*d*.

p. 235, Ex. 12. *For* Russian 5 per cents. at $91\frac{1}{8}$ *read* at $91\frac{1}{2}$.

ARITHMETIC.

CHAPTER I.

FIRST PRINCIPLES AND SCALES OF NOTATION.

§ 1. QUANTITY is the answer to the question *quantus*, how much? It is therefore that property of objects by means of which, when two of the same kind are compared together, one can be said to be greater or less than the other.

§ 2. Magnitude, which is often used as identical with Quantity, is really the answer to the question, *how great?* and may be used of everything which admits of the notion of greater or less, although in common language it is usually used with reference to the bulk of an object.

§ 3. Unit or Unity is the name given to that quantity which is to be reckoned as *one*, when other quantities of the same kind are to be measured.

§ 4. Number is the relation of a quantity to its unit; the notion of number being suggested by successive repetitions of the individual unit.

§ 5. When men first began to count, they would count numbers of some particular thing: so many men, so many horses, &c. Next they would observe that whatever result is obtained, as by adding one number of men to another number of men, the *same* result would be true if the *same* numbers of any particular kind of thing were used; if 15 men and 3 men more made 18 men, and 15 horses and 3 horses more made 18 horses, generally 15 and 3 would make 18, whatever kind of thing was reckoned: and the idea of number *abstracted* from any particular kind of thing would thus be realized.

Hence we define *concrete* numbers to be those considered as belonging to some determinate species; *abstract* numbers to be those taken without reference to any particular species.

Thus in 12 inches, and 12 pence, the 12 is a *concrete* number. But if we say 7 and 5 make 12, or 7 times 5 are 35, the numbers used are all *abstract*. And even if we say that a foot is 12 *times* as great as an inch, the number 12 is still abstract.

§ 6. We can now explain more fully tho term unit: it is not itself *one;* but it is the magnitude which shall be represented by one in calculation. If all lengths be referred to the standard of an inch, all weights to the standard of a pound, all periods of time to the standard of a second, the inch would be called the unit of length, the pound the unit of weight, the second the unit of time: that is to say, the unit would be a length, or a weight, or a time. The symbol which represents the abstract conception of singleness as distinguished from multitude is 1, which is the unit of abstract arithmetic: but all concrete quantities must have units of their own kind; and indeed anything may be unity for other things of its own kind; *i.e.* the unit is at first arbitrarily fixed on: the unit of length might be a foot, or a yard; the unit of weight might be an ounce, or a stone.

§ 7. Arithmetic (*ἀριθμητική*, scilicet *τέχνη*) is the art of numbering; and is usually taken to mean the science of expressing numbers by symbols, and of applying set rules to the different operations in which numbers are used.

§ 8. Notation is the art of expressing numbers by figures or symbols appropriated for that purpose.

§ 9. Numeration is generally applied to the converse process of expressing in words a number which is already expressed in symbols.

§ 10. *To explain what is meant by a scale of Notation.*

By a scale of Notation is meant a systematic arrangement for facilitating the computation of large numbers. Instead of giving independent names to the whole series of natural numbers beginning from unity, which would make an unlimited and most embarrassing nomenclature, it is arranged that a certain number of units arbitrarily fixed upon shall be grouped into a class; and that the same number of these classes shall be taken to form a class of the next higher order; and that the same number of these higher classes

shall be taken to form a class of a still higher order, and so on; advancing onwards from class to class as far as occasion may require. The number of units at first fixed upon to form a class is quite arbitrary: it might be five, it might be ten, it might be twelve, or any other number: but this being once fixed, the *same number* of each of the classes must be taken to form a class of the next higher denomination.

§ 11. *To explain the Decimal or Denary Scale of Notation.*

In the decimal scale the number ten is arbitrarily fixed upon as the basis of computation; and the cipher, which is the name given to nothing, and the first nine natural numbers are represented by the following symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. When the number ten is reached, this is considered as a new unit of a superior order; and the succeeding numbers are formed by successive combinations of the first nine natural numbers with ten, with two tens, with three tens, &c., until ten classes of ten each are gone through, when the last number in the last of these classes is called one hundred. This now becomes a unit of the next superior order, and another series of numbers is formed by combining one hundred with the numbers just enumerated; and when ten classes of hundreds are gone through, the last number is called not ten hundred, but one thousand. Neither tens of thousands nor hundreds of thousands have a separate name assigned to them[1]: but the same process of ascending by classes, ten of which form one of the next order, is continued until one thousand thousand is reached, and this is called a million. Proceeding onwards in the same way, a million million is called a billion; a million billion is called a trillion; a million trillion is called a quadrillion, and so on.

§ 12. *To explain the principle of Local Value.*

In order to represent numbers higher than nine, *i.e.* numbers which contain tens, hundreds, &c., the following device has been invented. No symbols besides those above enumerated are used, but it is agreed that each figure besides its individual shall have a *local* value, namely, a value depending

[1] Would not the assignment of distinct names to "tens of thousands" and "hundreds of thousands" facilitate the apprehension of high numbers, and render more obvious the law that a new unit is formed when ten of any class is reached?

upon the place it occupies; and that, while any figure standing simply by itself retains the individual value assigned to it, any figure standing to the *left* of another shall thereby be increased ten-fold. Thus when it is arranged that a figure standing in any particular place[1] shall represent so many *units*, the figure to the left of this will represent *tens*, or units of the second class: the figure to the left of this *ten* tens, or hundreds; the figure to the left of this, *ten* hundreds, or thousands, and so on; the local value of each figure continually increasing in a *ten-fold* degree as we advance one place further to the left. If in writing a number, any class, as units, tens, hundreds, &c. be wanting, the cipher[2] is used; which although without signification when standing by itself, serves when combined with other figures to fill up the vacant places, and so to give the significant figures their required local value.

[*Obs.* The peculiarity of *decimal* notation must not be confused with the *local* value assigned to the figures; the two things are perfectly distinct, and do not in any way depend upon each other. Indeed while many nations have used a decimal notation, very few traces of local value can be found in any system except the Hindoo-Arabic which we use. The decimal scale probably originated in the practice of counting on the fingers, whence the name *digits* for the symbols representing the first nine numbers. Had any nation counted only on *one* hand, such a system would have been the *quinary*, and six would have been the unit of the next superior order. Had they counted on fingers and toes the

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[1] In whole numbers the figure on the extreme right is said to be in the *units* place; the next figure to the left, in the *tens* place; the next figure to the left, in the *hundreds* place; the next, in the *thousands* place; and so on. But in writing decimal fractions (which, as will be shewn afterwards, afford the means of extending the decimal scale below unity) it is not the *right-hand* figure, but the figure to the *left of the decimal point* which stands in the place of units.

[2] The word cipher; ἡ τζίφρα, cifra, is from the Arabic term Tsaphara, "quod vacuum aut inane est," blank, or void. At the end or in the middle of any number the cipher is of use to keep the significant digits in their proper rank, when the units or the hundreds or any other denomination may be wanting, *e.g.* 60 means 6 tens followed by no units: 606 means 6 hundreds, with no tens, but 6 units. At the *beginning* of a number ciphers would be useless: if so placed they could only indicate the absence of any higher class; *e.g.* 096 means only 9 tens and 6 units; the cipher showing that there are no hundreds, which is equally intelligible if the cipher be omitted. The use of the word cipher led to the digits being all called ciphers, and so introduced the use of the verb to cipher.

system would have been *vicenary*. Some traces of both these systems are to be found, for instance, in our reckoning by *scores*[1]. The quinary, the denary, and the vicenary are the only *natural* systems; and it will be found that no other than these have ever prevailed in common use. The duodecimal scale, with 12 for the base, would present some peculiar advantages, as 12 is exactly divisible by 2, 3, 4, and 6; while 10 is so divisible by only 2 and 5: but in the infancy of any nation the method of reckoning by one of the natural systems seems to have been always first established, and not to have been afterwards disturbed by any more artificial arrangement.

When the practical method of numeration had been fixed, the numerical language to express it would be afterwards formed; and this would be succeeded by the invention of written symbols. The Hebrews, Phœnicians, and Syrians used the letters of their alphabets for numerical symbols; and the Greeks, who derived their alphabet from the Phœnicians, borrowed from the same source their system of numerical notation. From what source the Roman numeral symbols originated is a point which has given rise to much conjecture; one explanation, namely that the system was made up from signs used in reckoning by single units, will be noticed below. For the symbols which we now use no other origin has been suggested than that of arbitrary invention: the shape of several of the figures has been considerably modified in course of time; but the use of nine figures with zero, and the principle of local value[2] were introduced among the nations of Europe from the Arabs, first into Spain in the 12th century, and especially into Italy in the beginning of the 13th century. The Arabs derived them from India, where the Hindoos had used them from a period anterior to all written records, and attributed the invention of them to the Deity, "the invention of nine figures with the device of places to make them suffice for all numbers, being ascribed to the beneficent Creator of the universe[3]." The use of this method among the Hindoos can be traced up to the 5th century after

[1] The word *score* itself, *the long notch on the tally*, shews the method of counting which was most common among our forefathers.

[2] The Chinese possess a system of decimal Arithmetic not only of very great antiquity, but one in which a very close approximation is made to *local* value; they use however symbols for the superior units (hundreds, thousands, &c.) which in our system are expressed by position alone.

[3] Note 2 to page 4 of Colebroke's *Translation of Bháscara's Lilavati:* where it is stated that 'the place, where no figure belongs to it, is shewn by a blank; which to obviate mistake, is denoted by a dot or small circle.'

Christ; among the Arabs to the 9th century. It appears to have been communicated about 1136 by the Moors in Spain to the Spaniards, but at first to have been little used except in Astronomical works and calendars; its more general adoption was introduced into Italy by the writings of Leonardo Pisano in 1202; but Roman numerals still continued to be most commonly used throughout Europe for a long period subsequent to this; and indeed merchants' accounts were so kept until the middle of the 16th century.]

§ 13. It will be useful here to explain the methods of notation used by the Greeks and the Romans. The system of the Greeks will serve to illustrate the manner of representing numbers by the letters of an alphabet; while the peculiarities of the Roman numerals, still commonly adopted among ourselves, as in inscriptions, &c., ought to be well understood.

The Greeks then, in order to denote numbers, used the 24 letters of their alphabet, with three additional signs, which signs, as ordinary letters, had become obsolete at an early period: these were the Βαῦ or Digamma, originally the 6th letter of the alphabet, which under the form ϛ (called τὸ ἐπίσημον Βαῦ) denoted the number 6; the guttural Κόππα, which originally followed πῖ in the alphabet, written Ϙ or ϟ, called τὸ ἐπίσημον κόππα, and as a numerical sign denoting 90; and the arbitrary symbol Σαμπῖ (compounded from the old letter Σάν from the Hebrew *Zain*, and πῖ) written ϡ, and denoting 900. Their numbers therefore were represented as follows:

α′, β′, γ′, δ′, ϵ′, ϛ′, ζ′, η′, θ′,
1, 2, 3, 4, 5, 6, 7, 8, 9,

ι′, ια′, ιβ′...κ′, κα′, κβ′...λ′, μ′, ν′, ξ′, ο′, π′, Ϙ or ϟ,
10, 11, 12...20, 21, 22...30, 40, 50, 60, 70, 80, 90,

ρ′, σ′, τ′, υ′, φ′, χ′, ψ′, ω′, ϡ,
100, 200, 300, 400, 500, 600, 700, 800, 900,

͵α, ͵β, ͵γ, ͵δ, ͵ϵ, ͵ϛ, ͵ζ, ͵η, ͵θ,
1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000,

͵ρ or Μυ. for Μυριάς stood for 10,000

The word "air" (α′ = 1, ι′ = 10, ρ′ = 100) will help the memory to retain the first letters of the lines of units, tens, and hundreds.

Besides this notation there was an older method of expressing numbers (a method found on ancient inscriptions, &c.) by means of the initial letters of Ἴος for εἷς, Πέντε, Δέκα,

Ηεκατόν, Χίλιοι, and Μύριοι. In this system I = 1, II = 2, III = 3, IIII = 4, Π = 5, ΠΙ = 6, ΠΙΙΙΙ = 9, Δ = 10, ΔΙ = 11, ΔΔ = 20, ΔΔΔ = 30, Η = 100, ΗΗ = 200, Χ = 1000, ΧΧ = 2000, Μ = 10000. Also abbreviated combinations of Π with other letters were used; thus ΠΔ = *πεντάκις δέκα* = 50; ΠΗ = *πεντάκις ἑκατόν* = 500; ΠΧ = *πεντάκις χίλιοι* = 5000. Also by writing Μ beneath any letter its value was increased ten thousand fold: Thus $\underset{\mathrm{M}}{\gamma}$ was 30000; $\underset{\mathrm{M}}{\kappa\beta}$ was 220000. In writing fractions, either γ', $\iota\beta'$ alone meant $\frac{1}{3}$, $\frac{1}{12}$; or else the denominator was written *above* the numerator, like an index in algebra, as $\kappa\varsigma^{\mu\theta'}$ for $\frac{26}{49}$.

§ 14. Various conjectures have been made concerning the origin of the Roman numerals, and among others the following hypothesis has been put forward: Suppose that a person who counted on his fingers wrote a stroke for each successive unit up to ten; and when he had advanced as far as ten strokes, that he drew a cross line through them to denote that he had come to the end of his handful: his marks would be

I, II, III, ... ~~IIIIIIIIII~~.

If now he shortened his mark for ten into a single unit with a cross line drawn through it, he would have X for ten: for one hundred he might adopt the unit with two cross lines, as ; for one thousand he would require a unit with *three* cross lines, or four strokes, which might be written M, or ω, or even ↀ; next, if he *halved* these symbols he would have half X or V for five; half or L for fifty; half ↀ or D or IƆ for five hundred. Whether this hypothesis be correct or no, at any rate the Romans represented numbers by combinations of these symbols; they had a certain principle of local value as far as this, namely that a smaller symbol standing *before* a larger one, in numbers less than one hundred, was to be *subtracted*, but standing *after* it was to be *added*. Their notation therefore was as follows:

1.	I.	8.	VIII or IIX.
2.	II.	9.	VIIII or IX.
3.	III.	10.	X.
4.	IIII or IV.	11.	XI.
5.	V.	12.	XII.
6.	VI.	13.	XIII or XIIV.
7.	VII.	14.	XIIII or XIV.

15.	XV.
16.	XVI.
17.	XVII.
18.	XVIII or XIIX.
19.	XVIIII or XIX.
20.	XX.
30.	XXX.
40.	XXXX or XL.
50.	L.
60.	LX.
70.	LXX.
80.	LXXX or XXC.
90.	LXXXX or XC.
100.	C.
200.	CC.
300.	CCC.
400.	CCCC.
500.	D or IƆ.
600.	DC or IƆC.
700.	DCC or IƆCC.
800.	DCCC or IƆCCC.
900.	DCCCC or IƆCCCC.
1000.	M or CIƆ or ∞ or $\bar{I}$.
2000.	IIM or CIƆCIƆ.
5000.	IƆƆ or $\overline{V}$.
10000.	CCIƆƆ.
50000.	IƆƆƆƆ.

[*Obs.* We have stated that the reversed C (Ɔ), called apostrophus, with a perpendicular line preceding it, as IƆ, or drawn together as D, signifies 500. But in every multiplication with ten a fresh apostrophus is added, as IƆƆ = 5000, IƆƆƆ = 50000, &c.; and when a number is to be doubled, C is repeated as many times *before* the horizontal line as Ɔ stands *behind* it: thus if IƆ, or five hundred, is to be doubled, CIƆ = 1000; if IƆ, or five thousand, is to be doubled, CCIƆƆ = 10000, and so on.]

[*Obs.* The exercises which follow each chapter are intended both for an examination in the principles which have been laid down, as well as for practice in the various rules explained: and the amount of advantage derived from this book will mainly depend upon the fidelity with which these exercises are worked out. The great object to be kept in view is that elementary principles should be thoroughly mastered; and that all examples should be worked out from a knowledge of the reasons of the process adopted, and not by the help of a question of a similar sort, which happens to be worked at length in the book. Let the learner try to acquire habits of rapidity in his calculations as well as accuracy: too much time is generally wasted in *counting up* in addition, in using *too many words* in multiplication, &c.: whereas these processes ought to be done instantaneously, and without effort. The habit of making short calculations *in the head*, instead of writing down every figure is as much to be commended as it is generally neglected: it teaches rapidity, (generally the most rapid reckoner is also the most accurate,) and gives confidence as well: whilst it also proves whether first principles and the reasons of the processes are really understood; it being easy for the teacher to put the

questions in varied forms, so as to test this point especially. Above all, when a question is proposed of a novel kind, or of a familiar kind perhaps, but in a new shape,—inverted, or otherwise disguised,—let the learner, instead of giving up the attempt discouraged, fall back upon that excellent gift—common sense, and endeavour honestly to reason out the difficulty; and let him be assured that there are no mysteries in the science which may not be unravelled by a little careful thought, common sense, and perseverance.]

Exercise I.

1. Write down the numerical symbols for—

(1) Nineteen thousand and six.

(2) Sixteen hundred thousand, four hundred and two.

(3) Eight million, three hundred and eight thousand, seven hundred and ninety-one.

(4) One hundred and sixty-six million, four hundred and two thousand, and nine.

(5) A thousand million.

(6) Two billion, three hundred thousand million, four hundred and five thousand, six hundred and seven.

2. Write down in words the numbers expressed symbolically by—

(1) 123456789. (2) 9009009009.

(3) 777000777. (4) 896787542134.

(5) 4563218764529. (6) 378658459372156.

3. Explain what is meant by the unit of *length*, the unit of *weight*, the unit of *time*, &c.; and point out the difference between *concrete* and *abstract* numbers.

4. Explain what is meant by *a scale of notation:* in the *quinary* scale how would the number *seven* be represented? if only *seven* digits were used how would the number *thirteen* be represented?

5. Explain the *decimal* or *denary* scale of notation; and shew how with only nine symbols and the cipher we are able to represent any numbers however large.

Does the principle of *local value* depend in any way on that of *decimal* notation? or could the one principle be employed without the other?

6. Point out the conveniences of our method of notation, comparing it with any other method you are acquainted with.

What is the use of the cipher? may it be placed with equal propriety at the beginning, at the end, or in the middle of a number?

CHAPTER II.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

Obs. The three following Tables, viz. the Addition Table, the Subtraction Table, and the Multiplication Table should be thoroughly committed to memory[1]. In the first Table the *sum* of any number in the upper horizontal column and of any number in the left-hand vertical column will be found in the square formed by the intersection of the two columns in which the numbers stand; in the second, the *excess* of any number in the upper horizontal column over any number in the left-hand vertical column will be found in the square formed by the intersection of these two columns; and similarly in the third, the *product* of any two numbers, one in the upper horizontal column, the other in the left-hand vertical column, will be found in the square formed by the intersection of the two columns in which the respective numbers stand.

[1] When commencing the study of Arithmetic, after the fundamental principles have been thoroughly explained and understood, the learner should so commit to memory the tables here given, as to say at once, without mental effort, and as it were mechanically, the result of any simple Addition, Subtraction, or Multiplication. Very awkward habits are often formed by beginners; for instance, in numeration children count up through units, tens, hundreds, &c. instead of being taught to remember that the fourth figure is in the place of thousands, the seventh in the place of millions, the thirteenth in the place of billions, &c. In Addition, perhaps, they are allowed to count on their fingers, or by strokes upon the slate. These habits should be checked at the first; and the method of *ready reckoning* insisted on, that is, the method of performing these operations almost mechanically.

The Addition Table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

The Subtraction Table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
3			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4				0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5					0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
6						0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7							0	1	2	3	4	5	6	7	8	9	10	11	12	13
8								0	1	2	3	4	5	6	7	8	9	10	11	12
9									0	1	2	3	4	5	6	7	8	9	10	11
10										0	1	2	3	4	5	6	7	8	9	10
11											0	1	2	3	4	5	6	7	8	9
12												0	1	2	3	4	5	6	7	8
13													0	1	2	3	4	5	6	7
14														0	1	2	3	4	5	6
15															0	1	2	3	4	5
16																0	1	2	3	4
17																	0	1	2	3
18																		0	1	2
19																			0	1
20																				0

THE MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

[*Obs.* We must here explain that + for *plus*, − for *minus*, × for *into*, and ÷ for *by*, are the signs of *addition*, *subtraction*, *multiplication*, and *division* respectively; and that the sign =, or *equal to*, means that the quantities between which it stands are equal to one another.]

§ 15. ADDITION (from *addo*, *I give to*) is the putting together two or more quantities into one; this result, which is as large as all the original quantities together, is called their *sum*.

§ 16. Add together 1863 and 6789, and explain the process.

Axiom. The sum of two numbers is equal to the sums of their respective parts collected together.

Now 1863 = 1 thousand + 8 hundreds + 6 tens + 3 units,
and 6789 = 6 thousands + 7 hundreds + 8 tens + 9 units,

and as the sum of these two numbers is equal to the sums of their respective parts, that sum is

7 thousands + 15 hundreds + 14 tens + 12 units.

But here we may observe that 12 units make up 1 ten and 2 units; writing 2 in the place of units and carrying 1 to the place of tens, we have 15 tens; but 15 tens are equal to 1 hundred and 5 tens; writing 5 in the place of tens, and carrying 1 to the place of hundreds, we obtain 16 hundreds: but 16 hundreds are equal to 1 thousand and 6 hundreds; writing 6 in the place of hundreds, and carrying 1 to the place of thousands, we have 8 thousands. Hence the entire sum is 8 thousands, 6 hundreds, 5 tens, and 2 units; or is 8652.

This may be exhibited in another form thus:

1863 = 1 thousand + 8 hundreds + 6 tens + 3 units,
6789 = 6 thousands + 7 hundreds + 8 tens + 9 units,

The sum is

7 thousands + 15 hundreds + 14 tens + 12 units,
i.e. 7 thousands + 15 hundreds + 14 tens + 1 ten and 2 units,
i.e. 7 thousands + 15 hundreds + 15 tens + 2 units,
i.e. 7 thousands + 15 hundreds + 1 hundred and 5 tens + 2 units,
i.e. 7 thousands + 16 hundreds + 5 tens + 2 units,
i.e. 7 thousands + 1 thousand and 6 hundreds + 5 tens + 2 units,
i.e. 8 thousands + 6 hundreds + 5 tens + 2 units,
i.e. 8652.

Having observed the principle upon which the process depends, it will be sufficient in practice to use the following shortened form:

$$\begin{array}{r} 1863 \\ 6789 \\ \hline 8652 \end{array}$$

Instead however of performing the process by saying, "9 and "3 are 12, put down 2 and carry 1; 8 and 1 are 9, 9 and 6 are "15, put down 5 and carry 1; 7 and 1 are 8, 8 and 8 are 16, "put down 6 and carry 1; 6 and 1 are 7, 7 and 1 are 8," using as few words as possible, say only "9 and 3, twelve; 9 and 6, "fifteen; 8 and 8, sixteen; 7 and 1, eight."

§ 17. Add together £9 „ 19*s.* „ 11*d.* and £8 „ 18*s.* „ 8*d.* and explain the process.

	£	*s.*	*d.*
	9 „	19 „	11
	8 „	18 „	8
adding like denominations we obtain as the sum	17 „	37 „	19

But 19 pence are 1 shilling and 7 pence; writing 7 in the place of pence and carrying 1 to the place of shillings, we have 38 shillings; but 38 shillings are £1 and 18 shillings; writing 18 in the place of shillings, and carrying 1 to the place of pounds, we have £18. Hence the sum is £18 „ 18*s.* „ 7*d.*

§ 18. SUBTRACTION (from *subtraho, I withdraw*) is the removal of a less quantity from a greater; the quantity to be diminished (*minuendum*) is called the *minuend,* the quantity to be withdrawn (*subtrahendum*) is called the *subtrahend,* and the quantity which remains is called the *difference.*

§ 19. The subtraction of simple numbers, in accordance with the table given above, is effected by the memory; but in high numbers, especially where some of the figures in the subtrahend are greater than the corresponding figures in the minuend, a process must be adopted, the principle of which depends upon the two following axioms:

(1) The difference of two numbers is equal to the differences of their respective parts taken together.

(2) The value of the minuend is not altered by separating the various denominations of which it is composed, viz. tens,

hundreds, &c., into several parts, and reckoning one ten as 10 units, one hundred as 10 tens, &c.

Ex. Subtract 7495 from 9263.

If we take units from units, tens from tens, hundreds from hundreds, &c., the differences of these several parts taken together will make up the difference of the given quantities. Now, writing the subtrahend beneath the minuend,

$$\begin{array}{r} 9263 \\ \underline{7495} \end{array}$$

if we endeavour to take 5 units from 3 units, the 5 being the larger number cannot be taken away from the 3; therefore separate the 6 tens in the minuend into 5 tens and 1 ten, and add the 1 ten as 10 units to the 3 in the place of units; this will make 13 units in the place of units and leave 5 tens in the place of tens; take the 5 units in the subtrahend from the 13 units now in the minuend, and write in the remainder 8.

We have now 5 tens in the minuend, from which to take 9 tens in the subtrahend; as this is impossible separate the 2 hundreds in the upper line into 1 hundred and 10 tens; leave 1 in the place of hundreds and add 10 tens to the 5 in the place of tens, making 15 tens in the minuend; take 9 tens from 15 tens, and in the remainder write 6 in the place of tens.

There is now 1 hundred in the minuend, from which to take 4 hundreds in the subtrahend; this likewise being impossible, separate the 9 thousands in the minuend into 8 thousands and ten hundreds; leave 8 in the place of thousands, and add the 10 hundreds to the 1 in the place of hundreds, making 11 hundreds in the minuend. From 11 hundreds take 4 hundreds, and in the remainder in the place of hundreds write 7.

Lastly, we have 8 thousands in the minuend, from which to take 7 thousands in the subtrahend; and this being possible, in the remainder in the place of thousands write 1.

The minuend in its imaginary altered form would stand thus:

8 thousands + 11 hundreds + 15 tens + 13 units.

From which we can take

7 thousands + 4 hundreds + 9 tens + 5 units,

leaving as a remainder

1 thousand + 7 hundreds + 6 tens + 8 units.

§ 20. In the process adopted in practice the figures in the minuend are not actually altered; and perhaps we might more simply explain the practical process as follows:

$$\begin{array}{r} 9263 \\ \underline{7495} \end{array}$$

To subtract 5 from 3 is impossible; so separate 1 ten from the 6 tens, and adding it to the 3 units, say 5 from 13 leaves 8. Now we are supposed to have separated 1 ten from the 6 tens, but as the figure really remains 6, we still have to take 1 from it; also we have to take from it the 9 in the lower line; so instead of taking away first 1, and then 9 more, take away 10 at once; but 10 from 6 being impossible, separate 1 from the place of hundreds, and adding it as 10 tens to the 6 tens, say 10 from 16 leaves 6. As we have not really taken 1 from the 2 hundreds, we have still to take 1 from it, also we have to take the 4 in the lower line; instead of taking first 1 and then 4, take away 5 at once; but 5 from 2 being impossible, separate 1 from the place of thousands and add it as 10 hundreds to the 2 hundred, and say 5 from 12 leaves 7. As we have not really diminished the figure 9 in the place of thousands, we have still 1 to take from it, and likewise we have to take away the 7 in the lower line; so, taking away 8 at once from the 9, we have 1 left in the place of thousands, and the entire difference is 1768.

§ 21. Since the minuend *diminished* by the subtrahend equals the remainder, it follows that the remainder *increased* by the subtrahend equals the minuend.

Hence we may test the accuracy of any subtraction by *adding together* the remainder and the subtrahend; if their sum be equal to the minuend, the subtraction may be supposed to have been correctly performed.

From this consideration we may deduce a method of obtaining the correct result of any subtraction by asking ourselves what must be *added* to the subtrahend to make it equal to the minuend; thus:

$$\begin{array}{r} 9263 \\ \underline{7495} \\ 1768 \end{array}$$

5 *and eight* are 13; write down 8; and whenever the number resulting from the addition is ten or more than ten, (as here 13) carry 1 to the next figure in the lower line; 10 *and six*

C

are 16; write down 6, and carry 1 to the next 4; 5 *and seven* are 12; write down 2, and carry 1 to the next 7; 8 *and one* are 9; write down 1.

As this method will be referred to again, when we come to a compendious method of division, which will be mentioned below, we give another example of it, observing that we only write down the figures representing the words printed in italics, and if the sum be ten or more than ten, carry one to the next figure, without saying "put down so and so and carry one."

Ex. 2. Find the difference between 4567 and 3498.

$$\begin{array}{r} 4567 \\ \underline{3498} \\ 1069 \end{array}$$

8 *and nine* 17; 10 *and six* 16; 5 *and nought* 5; 3 *and one* 4.

Ex. 3. Subtract £5 „ 18*s.* „ 11*d.* from £10 „ 7*s.* „ 2*d.*

£	*s.*	*d.*
10 „	7 „	2
5 „	18 „	11
4 „	8 „	3

We cannot take 11*d.* from 2*d.*; therefore separate the 7*s.* in the minuend into 6*s.* and 12*d.*, and add the 12*d.* to the 2*d.*, making 14*d.*; from 14*d.* take 11*d.*, and write 3*d.* in the remainder. Now from the 7*s.* (as the figure remains unaltered), we have still to take away 1*s.*, besides the 18*s.* in the subtrahend, *i.e.* we have to take away 19*s.*; but as we cannot take 19 from 7, separate the £10 in the minuend into £9 and 20*s.*, and add the 20*s.* to the 7*s.*, making 27*s.*; take 19*s.* from 27*s.*, and write 8*s.* in the remainder. From the £10 remaining unaltered we have still to take £1, besides the £5 in the subtrahend; *i.e.* we have to take away altogether £6; subtract £6 from £10, and write £4 in the remainder; whence the entire *difference* is £4. 8*s.* 3*d.*

[The above examples give the reason for the method of what is commonly called *borrowing* and *carrying;* but as these terms by no means explain the operation, the principle of the process employed in subtraction is often not understood. The term *carrying,* which is proper enough in addition, is hardly correct in subtraction, as it would be difficult to say from what figure anything is *carried;* while the term *borrowing* needs explanation, and means, as will have been

seen, the separation of the different denominations of the minuend into several parts.]

§ 22. MULTIPLICATION (from *multiplex*, manifold) is a shortened method of performing addition; when one of two given numbers is to be taken as many times as there are units in the other.

The quantity which is to be multiplied (*multiplicandum*, that which is to be taken manifold times) is called the *multiplicand;* the quantity by which it is to be multiplied is called the *multiplier;* and the result, the *product.*

§ 23. The fundamental principles upon which the process of multiplication depends are these:

(1) If we separate any multiplicand into any number of parts, and multiply each part severally by any number and add the results, the *whole* multiplicand is thus multiplied: *e.g.* 15, which may be separated into 8 and 7, is multiplied by 9, if 8 and 7 be each multiplied by 9 and the results added together.

(2) If the multiplier be separated into any number of parts and the multiplicand be multiplied severally by each of these parts and the results added together, this is equivalent to multiplication by the *whole* multiplier: *e.g.* if it be required to multiply 17 by 12, and we multiply 17 by 4 and 17 by 8 and add these results, we have then taken 17 exactly $4 + 8$ times, or 12 times.

From these principles we may deduce the following, viz.

(3) Any number is multiplied by 10 by annexing *one* cipher; by 100 by annexing *two* ciphers; by 1000 by annexing *three* ciphers, &c., *e.g.* $58 \times 10 = 580$; for by annexing the cipher the 8 *units* have become 8 *tens*, and the 5 *tens* have become 5 *hundreds; i.e. the several parts* of the multiplicand have each received a tenfold increase, and therefore the whole number has been multiplied by 10.

§ 24. We can now proceed to explain the process of multiplying any number by a single figure; and then, of multiplying any number by any other number.

(*a*) Multiply 6789 by 5; and explain the process.

$6789 = 6$ thousands $+ 7$ hundreds $+ 8$ tens $+ 9$ units,

we must therefore multiply each of these parts by 5, and add together the results:

Now 9 units multiplied by 5 will give 45 units.
8 tens . . . . . 40 tens.
7 hundreds . . . . 35 hundreds.
6 thousands . . . . 30 thousands.

writing these results in the ordinary way, and adding them together we have

45 units	=	45
40 tens	=	400
35 hundreds	=	3500
30 thousands	=	30000

and *sum* of these, which is the required *product*, is 33945

To shorten this form in practice it is sufficient to write the multiplier under the multiplicand, and to multiply each denomination, units, tens, hundreds, &c., severally by 5, *carrying* whenever it is necessary to the next highest denomination: *e.g.*

$$\begin{array}{r} 6789 \\ 5 \\ \hline 33945 \end{array}$$

Five times nine, 45; write down the 5 and carry 4 to the place of tens; five times eight 40, and 4 are 44; write down the 4 and carry 4 to the place of hundreds; five times seven 35, and 4 are 39; write down 9 and carry 3 to the place of thousands; five times six 30, and 3 are 33; write down 3 in the place of thousands, and 3 in the place of ten thousands. In performing the operation, however, use as few words as possible in practice.

(β) Multiply 6789 by 2345, and explain the process.

The multiplier 2345 may be separated into $2000 + 300 + 40 + 5$; if then we multiply the multiplicand by each of these parts and add the results, we shall obtain the product required:

$$\begin{array}{lrcr} \text{Now} & 6789 \times 5 & = & 33945 \\ & 6789 \times 40 & = & 271560 \\ & 6789 \times 300 & = & 2036700 \\ & 6789 \times 2000 & = & 13578000 \\ \hline \end{array}$$

and the sum of all these is 15920205, which is the *product* required.

If the ordinary method of performing this operation be compared with the detailed process here given, it will be observed that by arranging the figures in the second line of multiplication one place to the left of those in the first, those in the third one place to the left of those in the second, and so on, we retain the figures in each line in their proper places

without the addition of the ciphers at the end of each line, the abbreviated form standing in practice as follows:

$$\begin{array}{r} 6789 \\ \underline{2345} \\ 33945 \\ 27156 \\ 20367 \\ \underline{13578} \\ 15920205 \end{array}$$

§ 25. (1) *The product of two numbers is the same if the multiplicand and multiplier be reversed: e.g. 5 times 27 = 27 times 5.*

For 5 times 27 means that 27 is to be taken 5 times. Now if we had 5 groups each containing 27 things, that would be 27 taken 5 times; and if we took one out of each of these 5 groups, and arranged them when so taken in a group by themselves, we should have a group of 5; and this process might be 27 times repeated before the original 5 groups would be all exhausted, and then we should have 27 new groups, each containing 5 things, or we should have 5 taken 27 times. And since there are in each case the same number of things taken, *i.e.* since the *product* is in each case the same, we see that

5 times 27 = 27 times 5.

(2) The product of two numbers is said to be a *multiple* of both multiplier and multiplicand.

For since $5 \times 3 = 3 \times 5 = 15$, the product 15 contains exactly 3 fives or 5 threes; and 15 is called a *multiple* of 5 and 3; for 15 contains both 5 and 3 an exact number of times. Hence

Def. A *multiple* of two numbers is a number which contains each of the two numbers an exact number of times.

A *common multiple* of several numbers is a number which contains each of the several numbers an exact number of times.

The *least common multiple* of several numbers is the least number which contains each of the several numbers an exact number of times.

(3) *The multiplication of one number by a second, and of that product again by a third number, is equivalent to one multiplication by the product of these two multipliers; e.g.* the multiplication of 15 by 3 and of that result by 4, will be the same as the multiplication of 15 at once by 12.

For $15 \times 3 = 45$, whence we may say that forty-five *contains* 15 exactly 3 times; therefore 4 forty-fives will contain 15 four times as often, or 4 times 3 times, or 12 times.

The same thing may be exhibited thus:

$$15 \times 3 \times 4 = 45 \times 4 = 180$$

$$15 \times 3 \times 4 = 15 \times 12 = 180.$$

(4) The result of the multiplication of one number by a second, and of that product again by a third number is called the *continued product* of the three numbers: *e.g.*

$$2 \times 3 \times 5 = 6 \times 5 = 30,$$

where 30 is the *continued product* of 2, 3, and 5.

Also, since $\quad 2 \times 3 \times 5 = 2 \times 15 = 30,$

we see that 30 contains each of the numbers, 2, 3, 5, 6, and 15 an exact number of times; therefore 30 is a *common multiple* of 2, 3, 5, 6, and 15. Likewise as no number less than 30 will contain *all* the numbers 2, 3, 5, 6, and 15, an exact number of times, 30 is the *least common multiple* of these numbers.

The simplest process for verifying the correctness of multiplication is perhaps the following:

Add together all the digits of the multiplicand; then add together the digits of the resulting number; and so on, until by such successive additions a single figure is arrived at. Next add together the digits of the multiplier; and add the digits of the resulting number; and so on, until a single figure is arrived at. Multiply together the single figures thus obtained from the multiplicand and multiplier; and reduce the result by successive additions to a single figure. Call this *the first resulting figure.* Then add together, in a similar manner, the digits of the product of the original numbers; and by successive additions reduce this also to a single figure. Call this *the second resulting figure.* When the first resulting figure tallies with the second resulting figure, the work may be presumed to be correct. For example, let the multiplicand be 78596, let the multiplier be 769, and let it be assumed that the product is 60440324. To test the accuracy of this, say,

$$7 + 8 + 5 + 9 + 6 = 35; \quad 3 + 5 = 8;$$

$$7 + 6 + 9 = 22; \quad 2 + 2 = 4;$$

$$8 \times 4 = 32; \quad 3 + 2 = 5;$$

this is the first resulting figure.

Then $6 + 4 + 4 + 3 + 2 + 4 = 23$; $2 + 3 = 5$;
this is the second resulting figure,—and as these correspond, the work is presumably correct.

This method, like the ordinary process of "casting out the nines," (upon the principle of which, indeed, this is based,) is not an infallible test of accuracy; but it forms a very useful practical check in long calculations.

§ 26. We have seen that we can *add* together concrete quantities as pounds, &c., and that we can *subtract* such quantities one from the other; we cannot however *multiply* them together; to attempt to multiply together *pounds* and *pounds* is to attempt an impossibility. It is a common error nevertheless to suppose that £5 multiplied by £2 gives as a result £10; but from the definition of multiplication, which requires that one quantity should be taken as *many times* as there are units in the other, it will be seen that to take £5 "*two pounds times*" is mere nonsense; we can take £5 *two times*, and the result will be £10; that is to say, we can multiply any concrete quantity by any abstract number; but no concrete quantity can be multiplied by another concrete quantity, whether of its own, or of any other denomination: that is to say, in multiplication the *multiplier* must *always* be an abstract number; the *multiplicand* may be either abstract or concrete; but if the multiplicand be concrete, the *product* must also be concrete. There is a seeming exception to this rule, where feet multiplied into feet give square feet, but this will be explained below, in the rule of cross multiplication. Hence so-called compound multiplication consists of the multiplication of concrete quantities by abstract numbers, *i.e.* of the repetition of concrete quantities a certain number of times.

Ex. 1. Required to multiply £17. 18*s*. 9*d*. by 8, and to explain the process.

Multiplying the several denominations by 8 we obtain

£	*s*.	*d*.
17 „	18 „	9
		8
136 „	144 „	72

Here we observe that 72 pence make 6 shillings, writing 0 in the place of pence, and carrying 6 to the place of shillings we have 150 shillings; but 150 shillings make £7 and 10 shillings;

writing 10 in the place of shillings and carrying 7 to the pounds, we have 143 pounds; hence the product in its simplest form is £143. 10*s*. 0*d*.

Ex. 2. Multiply £23. 7*s*. 11*d*. by 63.

Since 9 times 7 is 63, if we here multiply the given quantity by 9 and by 7 successively, *i.e.* multiply £23. 7*s*. 11*d*. by 9, and then multiply that result by 7, we shall by that means effect the multiplication by 63: hence,

£	*s*.	*d*.
23 „	7 „	11
		9
210 „	11 „	3
		7
£1473 „	18 „	9

Ex. 3. Multiply £105. 10*s*. 2*d*. by 39.

Since $$6 \times 6 + 3 = 39,$$

if we multiply the given sum of money by 6, and then that result by 6, and to that continued product add 3 times the original sum of money, we shall obtain the product of £105. 10*s*. 2*d*. and 39.

£	*s*.	*d*.	
105 „	10 „	2 × 3	
		6	
633 „	1 „	0	= product by 6
		6	
3798 „	6 „	0	= product by 36
316 „	10 „	6	= product of top line by 3
£4114 „	16 „	6	= product by 39.

The multiplication of large sums of money is rendered more easy by certain processes which are explained under the Rule of "Practice."

§ 27. DIVISION (from *divido, I part asunder*) is generally defined to be a shortened form of performing subtraction; when we require to know how often one number called the *divisor* may be subtracted from another called the *dividend;* the number which expresses *the number of times* the subtraction may be repeated, is called the *Quotient*, (from *quoties, how many times?*)

We may however define division more generally as *the converse of multiplication.* We have seen that there is only one form of multiplication; we can only require that any quantity should be taken a certain number of times; and although the multiplicand may be either abstract or concrete, yet the multiplier must of necessity be abstract. But when we come to the converse operation, we shall see that there are two different forms in which it may be required to be done; we may, for instance, either wish to know how many times 6 can be subtracted from 24; or we may wish to separate 24 into 6 equal parts: similarly, in concrete quantities, we may either wish to know how many sums of £6 each there are in £24, or we may wish to divide £24 into 6 equal parts. Now in either of these cases we know the *product:* but when we ask how many times 6 can be subtracted from 24, or how often £6 can be subtracted from £24, we have the *multiplicand* given, and are to find the *multiplier;* when we wish to divide the number 24, or £24, into 6 equal parts, we have the *multiplier* given to find the *multiplicand.* If therefore we define division as a *method of performing a series of subtractions,* we only include the former of these processes; whereas by defining it as *the converse of multiplication,* both are included.

The explanation of the process, however, need only refer to *one* of these cases, as we know (cf. § 25. 1) that it makes no difference in the product if we transpose the multiplier and multiplicand, *i.e.* that $6 \times 4 = 24$, as well as $4 \times 6 = 24$. Hence the simplest process of division would be that of successive subtractions: *e.g.* to divide 24 by 6, subtract 6 from 24, and then subtract 6 from the remainder, and then 6 from that remainder; and so on, until the remainder is either nought, or less than 6; then the *number of subtractions* which have been made will be *quotient.* Thus, the quotient of 24 divided by 6 is 4, because 4 such subtractions could be made; the quotient of 25 divided by 6 is 4, with a remainder 1.

In order to avoid the labour of repeated subtractions, we may lay down the following principle, *viz.:* that if we separate any dividend into any number of parts, and find how often the divisor may be subtracted from each of these parts, (or how often the divisor is *contained,* as it is called, in each of them,) we shall, by adding these results, obtain the correct quotient of the whole dividend divided by that divisor; because it is

evident that the *whole* dividend will contain the divisor as many times as its *several parts together* contain it.

Ex. 1. Divide 3213 by 9, and explain the process.

3213 may be separated into $3200+13$; now 3200 contains 9 more than 300 times: subtract 9×300, or 2700, from 3200, and there remains 500; so that the entire dividend now remaining is $500+13$; but 513 contains 9 more than 50 times; subtract 9×50, or 450, from 513, and there remains 63; but 63 contains 9 exactly 7 times; subtract 9×7 from 63, and the remainder is 0. We have therefore made in all $300+50+7$ subtractions, or the quotient is 357. The form commonly adopted depends upon the reasoning above stated, although it is shortened in practice by the omission of ciphers.

The process of division may be shown to be the exact converse of that of multiplication by the following illustrations. First *multiply* 123 by 8; separate 123 into $100+20+3$, and multiply each part by 8; they become $800+160+24$, and these added together amount to 984. Now *divide* 984 by 8: from 984 subtract 8 *one hundred times; i.e.* take away 800, leaving 184 as a remainder. From this remainder subtract 8 *twenty times, i.e.* take away 160, leaving 24 as a remainder; from this remainder subtract 8 *three times, i.e.* take away 24, and there is no remainder. We may exhibit this in the following tabular form, where multiplicand, multiplier, and product, answer respectively to quotient, divisor, and dividend:

Multiplicand.		Multiplier.		Product.
$(100+20+3)$	multiplied by	8	is	984.

$$\begin{array}{lrl} \text{Divisor.} & \text{Dividend.} & \text{Quotient.} \\ 8) & 984 & (100+20+3 \\ & \underline{800} & \\ & 184 & \\ & \underline{160} & \\ & 24 & \\ & \underline{24} & \\ & 0 & \end{array}$$

Again $\quad 456\times78=400\times78+50\times78+6\times78$

$$=31200+3900+468$$

$$=35568.$$

Now let it be required to divide 35568 by 78.

	From 35568	
Subtract 78×400, or	31200	
	4368	= remainder after 400 subtractions of 78.
Subtract 78×50, or	3900	
	468	= remainder after 50 more subtractions of 78
Subtract 78×6, or	468	
	0	= remainder after 6 more subtractions of 78,

hence there have been 456 subtractions in all; or the quotient of 35568 by 78 is 456.

The process called "Long Division" may now be easily explained: let it be required to divide 550974 by 1472, and to explain the operation:

```
1472) 550974 (300 + 70 + 4
      441600
      109374
      103040
        6334
        5888
         446 remainder.
```

The dividend may be separated into 550 thousands, 9 hundreds, and 74; now 5509 will contain 1472 more than 3 but less than 4 times: therefore 550974 will contain 1472 more than 300 but less than 400 times. Subtract 300 times 1472 from the dividend, *i.e.* subtract 441600; and there remains 109374. Now 10937 contains 1472 more than 7 but less than 8 times; therefore 109374 contains 1472 more than 70 but less than 80 times: subtract 70 times 1472 from 109374, *i.e.* subtract 103040; and there remains 6334; lastly 6334 contains 1472 more than 4 but less than 5 times: subtract 4 times 1472, *i.e.* subtract 5888 from 6334, there now remains 446, which is less than the divisor; and consequently no further subtractions can be made. Hence the quotient is 374, and there is a remainder 446.

The form is shortened in practice by omitting the ciphers not only in the quotient but also in each successive subtrahend: care being taken to keep the figures in each of these lines in their proper places without them; also by not bringing down all the figures in the dividend every time, but only those which from time to time are required to take part in the process. The term "remainder" is sometimes misunderstood. It might be explained in the example just given that the dividend contains the divisor more than 374, but less than 375 times; so that, after 374 subtractions of the divisor, there

is left the number 446, which is not sufficiently large to contain the divisor once, and is therefore written as a remainder. To say that the correct quotient is $374\frac{446}{1472}$ would be to anticipate an acquaintance with fractions.

The correctness of any sum in division may be proved by multiplying together the *divisor* and *quotient* and adding to their product the *remainder:* if this produces the original *dividend*, the operation has been correctly performed.

§ 28. We have stated (in subtraction § 21) that the difference between two numbers may be obtained by writing down what must be *added* to the less to make it equal to the greater. By adopting this process in division, we may combine in one row of figures the two processes of multiplication and subtraction; that is to say, instead of writing down at length each product of the divisor multiplied by the several figures of the quotient and then subtracting, we may mentally multiply each figure of the divisor, and write down as we proceed only what must be *added* to it to make it equal to the corresponding figure in the minuend; *e.g.* in dividing 35861 by 763 the process will stand as follows:

```
763) 35861 (47
      5341
       000
```

Where the first step is to multiply 763 by 4 and at once subtract the result from 3586; these combined processes are effected thus: "4 times 3 are twelve; 12 *and four* are 16; "4 times 6 are 24 and 1 carried (because 16 is more than 10) "are 25; 25 *and three* are 28; 4 times 7 are 28 and 2 carried "are 30; 30 *and five* are 35." Only the figures printed in italics are written down; and the 1 from the upper line being brought down we say again, (this time using fewer figures in the process, and performing each step of the multiplication mentally,) "21 *and nought* are 21, carrying two; 44 *and nought* are 44, carrying four; 53 *and nought* are 53."

It is well to suppress as many figures as possible in this process, arriving at the results of multiplication mentally without saying "7 times 3 are 21," &c., and working rather as in the second line of the above example than as in the first, where for the purpose of explanation a process more full than otherwise necessary is adopted.

This compendious method of division is worthy of attention both for its clearness and for the saving of trouble which it effects; other examples are subjoined, the ordinary process being set side by side, that it may be seen how many figures are saved by the shorter form.

Ex. 1. Divide 1699029276 by 4567283.

Compendious Method.	*Ordinary Process.*
4567283) 1699029276 (372	4567283) 1699029276 (372
32884437	13701849
09134566	32884437
0000000	31970981
	9134566
	9134566
	....

N.B. In this and in the following example the figures *brought down* are printed in bold type.

Ex. 2. Divide 431135173 by 738245.

Compendious Method.	*Ordinary Process.*
738245) 431135173 (584	738245) 431135173 (584
6201267	3691225
2953073	6201267
000093	5905960
	2953073
	2952980
	93

N.B. The examples in division worked out in the remainder of this book will be generally exhibited in the compendious or shorter form.

§ 29. As we can multiply any quantity by 3 and then multiply that result by 5, instead of multiplying at once by 15, so instead of dividing at once by 15, it would be the same thing to divide first by 3 and then divide this result by 5; only observing that care must be taken in such a case to obtain the correct remainder: *e.g.* divide 6869 by 3 and 5 successively, instead of dividing at once by 15,

$$\begin{array}{r|l} 3 & 6869 \\ \hline 5 & 2289 \text{ rem. } 2 \\ \hline & 457 \text{ rem. } 4 \end{array}$$

Now the remainder 2 in the first quotient is 2 units of the upper line; that is, it is 2 ordinary units: but the remainder 4 in the second quotient consists of 4 units of the second line; and as each unit in the second line is *three times* as great as each unit in the upper line, the remainder 4 is equal to 3×4 units of the upper line, *i.e.* is equal to 12 ordinary units; hence the whole remainder is $2 + 12$; or is 14.

Ex. 2. Divide 24533279 by 432.

Since $6 \times 8 \times 9 = 432$, we may divide successively by these numbers,

$$
\begin{array}{r|l}
6 & \underline{24533279} \\
8 & \underline{\;\;4088879} \text{ rem. } 5 \\
9 & \underline{\;\;\;\;511109} \text{ rem. } 7 \\
 & \;\;\;\;\;\;56789 \text{ rem. } 8
\end{array}
$$

Here the first remainder consists of 5 units of the upper line, or of 5 ordinary units; but as each unit in the first quotient, *i.e.* in the second line, is 6 times each unit in the first line, the remainder, after the second quotient has been obtained, consists of 7 units, each of which is 6 *times as large* as the units in the first line; that remainder therefore it 6×7, or 42. Similarly the remainder after the third quotient consists of 8 units which are 8 *times* 6 *times* as large as the units in the first line; and this remainder therefore is 384. Hence the true remainder is $384 + 42 + 5$ or is 431. To find the true remainder, therefore, it is necessary to *multiply the remainders after every line by all the preceding divisors except their own, and add the results.* The quotient and remainder obtained in this case may be compared with those obtained by long division, using the compendious process:

```
432) 24533279 (56789
      2933
       3412
        3887
         4319
          431
```

§ 30. Although, as we have seen, we cannot *multiply* one concrete quantity by another, yet we can *divide* one concrete quantity by another of the same denomination. Observe, however, that £10 divided by £2 does not give £5 as a quotient, but the abstract number 5; and that just as £5 taken

twice gives £10, so £10 contains £2 exactly 5 times. Therefore if we divide any concrete quantity by any abstract number, the quotient is a concrete quantity; but if we divide any concrete quantity by another concrete quantity of the same denomination, the quotient is an abstract number.

Ex. 1. Divide £20 „ 4*s.* „ 10*d.* into 7 equal parts.

```
      £    s.   d.
  7) 20 „  4 „ 10
     ------------
     £2 „ 17 „ 10
```

Here the quotient obtained by dividing £20 by 7 is £2, with a remainder £6; bring £6 into shillings, and add the 120 shillings so obtained to the 4 shillings in the dividend; divide the 124 shillings by 7; the quotient is 17 shillings with a remainder 5 shillings; bring 5 shillings into pence, and add the 60 pence to the 10 pence in the dividend; divide the 70 pence so obtained by 7, and the quotient is 10 pence. Hence the entire quotient is £2 „ 17*s.* „ 10*d.*

Ex. 2. Divide £31 „ 8*s.* „ 8*d.* by £3 „ 18*s.* „ 7*d.* We must first bring both dividend and divisor to the *same denomination*, viz. to *pence*, before the division can be effected; therefore

```
£   s.   d.        £   s.  d.
3 „ 18 „ 7         31 „ 8 „ 8
20                  20
--                 ---
78                 628
 12                  12
---               ------
943              ) 7544 (8
                   7544
                   ....
```

Here the quotient is the abstract number 8, or the dividend contains the divisor exactly 8 *times.*

Ex. 3. Divide £374 „ 13*s.* „ 1*d.* between 61 men.

```
       £     s.  d.
  61) 374 „ 13 „ 1 (6
      008
       20
      ---
       173 (2
       051
        12
       ---
       613 (10
       003
```

Each man would receive £6 „ 2*s.* „ 10*d.*; and there would be 3*d.* remaining, which it would be impossible actually to divide among them. The share of the three pence belonging to each man could only be expressed as a fraction of a penny, namely, as $\frac{3}{81}$ *d.*

Ex. 4. Divide £125 „ 2*s.* „ 5*d.* by 56.

We may divide by factors in compound division also, if care be taken to apply the rule already given for finding the correct remainder.

$$\begin{array}{r|l} & \text{£} \quad s. \quad d. \\ 7) & 125 \,„\, 2 \,„\, 5 \\ \hline 8) & \;\;17 \;\; 17 \;\; 5 - 6 \\ \hline & \text{£}2 \;\; 4 \;\; 8 - 1. \end{array}$$

Hence $1 \times 7 + 6 = 13$, and there are 13 pence remaining.

Exercise II.

1. Add together the following numbers:

(1) 9999, 888, 77.

(2) 700653, 8949, 56735.

(3) 1568090, 7906, 150986, 289.

(4) 6895437, 17268, 5743986, 2571.

(5) 3678963, 2301530, 1010506, 1567375, 1441625.

(6) 47567395, 56783962, 78975783, 69894697, 85679876.

2. Subtract

(1) 2498 from 5673.

(2) 4782 from 8996.

(3) 487465 from 756051.

(4) 99999 from 100001.

(5) 10123589 from 98532101.

(6) 342589678 from 713241351.

3. From eight million, five hundred and thirty-two thousand, one hundred and five, subtract six million, seven hundred and sixty-four thousand, nine hundred and eighty-three; and then subtract from the remainder one million seven hundred and fifty-six thousand, one hundred and twenty-two.

4. Subtract sixty-nine million, seven hundred and eighty-four thousand, seven hundred and thirty-nine, from seventy-

two million, five hundred thousand, six hundred and ninety-one; and then from the remainder subtract two million, seven hundred thousand, nine hundred.

5. Multiply

(1) 56789 by 64 (2) 857943 by 978
(3) 2785693 by 5379 (4) 4579301 by 40067
(5) 764532 by 8459 (6) 66554433 by 227788.

6. Divide

(1) 455 by 13 (2) 1520370 by 3754
(3) 3632042225 by 7856931 (4) 173286295046 by 8534589
(5) 69224660505 by 7683945 (6) 4770011800 by 536725.

Exercise III.

1. Add together 4563 and 7948, and explain the reasons of the process.

2. What must be added to the sum of £5 „ 17*s.* „ 6*d.* and £7 „ 15*s.* „ 11*d.* in order to make the total equal to £20?

3. Subtract 7895 from 8234.

In what different ways could the operation be performed? Give reasons for the method you adopt; and prove the correctness of the answer.

4. Subtract £3562 „ 19*s.* „ 7*d.* from £4571 „ 13*s.* „ 2*d.* and explain the principle of the method commonly employed.

5. Multiply 2685 by 748.

Explain the meaning of the word multiplication; and state the principles upon which the process ordinarily adopted depends. In the above instance give a detailed explanation of the several steps by which the result is arrived at, and test its correctness.

6. Multiply £78 „ 4*s.* „ 9*d.* by 64.

7. Divide 1101948 by 2974; find the quotient and the remainder; explain the operation, and prove the result.

8. In an ordinary long division sum the divisor was 1472, the quotient was 374, and the remainder 446; what was the dividend?

9. How many times does £120 „ 9*s.* „ 2*d.* contain £17 „ 4*s.* „ 2*d.*?

10. What will remain after subtracting 4093 as often as possible from 143256?

11. If the multiplier be 2036 and the product 8764980, what is the multiplicand?

12. Show that a number is multiplied by 10 by the addition of a cipher to the right.

13. If the divisor be twice the quotient, and the quotient thrice the remainder, find the dividend when the remainder is 99.

14. Show that if the *sum* of 23 and 19 be added to the *difference* between 23 and 19 the result is *twice* 23; but if the *sum* of 23 and 19 be diminished by the *difference* between 23 and 19, the result is *twice* 19.

Assuming that what is here proved true for the particular numbers given is *generally* true for *all* numbers, enunciate in words the general proposition which may be deduced.

15. Multiply £6 „ 4*s.* „ 2*d.* by 24, by multiplying it first by 6 and then multiplying that result by 4.

16. In a similar way multiply £16 „ 15*s.* „ 4*d.* by 32.

17. Observing that $3 \times 5 \times 7 = 105$, employ short division to divide 4796292 by 105; find the true remainder, and explain the process by which it is obtained.

18. At a certain house of business the sums of money received and paid away daily throughout a week were as follows: Monday, receipts, £1073 „ 16*s.* „ 4*d.*, payments, £562 „ 18*s.* „ 9*d.*; Tues. receipts, £987 „ 15*s.* „ 3*d.*, payments, £739 „ 17*s.* „ 5*d.*; Wed. receipts, £854 „ 11*s.* „ 11*d.*, payments, £947 „ 16*s.* „ 11*d.*; Thurs. receipts, £9376 „ 19*s.* „ 2*d.*, payments, £1073 „ 15*s.* „ 3*d.*; Fri. receipts, £786 „ 17*s.* „ 6*d.*, payments, £693 „ 0*s.* „ 7*d.*; Sat. receipts, £1240 „ 0*s.* „ 10*d.*, payments, £892 „ 11*s.* „ 1*d.* What was the excess of the total receipts over the payments during the week?

19. What will be the cost of 1000 quarters of wheat at 53*s.* per quarter? And how many quarters may be bought for £4687 „ 17*s.* at the same price?

20. If it cost £14600000 annually to support a standing army of six hundred thousand men, what is the *average daily* cost of each man?

21. Bought 40 articles at 12*s*. 6*d*. each, and 60 more of the same kind for 15*s*. 6*d*. each; required the whole cost, and the *average price* of each article.

22. If 140 sheep cost £169 „ 3*s*. „ 4*d*., what is the price per score?

23. Employ the "compendious process of division" in dividing (1) 170765 by 4899. (2) 32016768 by 31024. (3) 202611423 by 457362.

24. Explain why it is impossible to multiply together £2 and £3. Of the three quantities £3, 10 yards, and £6, explain in how many ways it is possible to multiply the quotient of two of them by the third quantity.

CHAPTER III.

SOME PRACTICAL METHODS OF SHORTENING LABOUR IN THE FUNDAMENTAL RULES OF ARITHMETIC.

§ 31. (1) Since to multiply by 5 is to multiply by half ten, therefore to multiply any number by 5 add to it 0, which multiplies it by 10, and then divide by 2: Thus 456789 × 5 is the same as 4567890 ÷ 2; and we obtain the product of 456789 multiplied by 5 as follows:

$$\begin{array}{r} 2)\,\underline{4567890} \\ 2283945 \end{array}$$

(2) Similarly, since 100 ÷ 4 = 25, and 1000 ÷ 8 = 125, to multiply by 25 add two ciphers and divide by 4: to multiply by 125 add three ciphers and divide by 8.

e.g. the product 7854 × 25 may be obtained thus:

$$\begin{array}{r} 4)\,\underline{785400} \\ 196350 \end{array}$$

The product of 53267 × 125 may be obtained thus:

$$\begin{array}{r} 8)\,\underline{53267000} \\ 6658375 \end{array}$$

(3) Nine times any number is *one less* than ten times; therefore to any number add 0, and subtract the original number: the result will be the number multiplied by nine.

Thus: 45376 × 9 is the same as 453760 − 45376, which is 408384.

(4) The multiplication by 11 may be effected by adding a cipher, which multiplies by 10, and then adding the multiplicand; thus the product of 94872 × 11 is found as follows:

$$\begin{array}{r} 948720 \\ 94872 \\ \hline 1043592 \end{array}$$

(5) The multiplication by any number from 12 to 19 inclusive, may be effected in one line as follows: multiply by the figure of the multiplier in the units' place, and to the number to be carried add the figure of the multiplicand just multiplied; *e.g.* multiply 6378 by 19:

$$\begin{array}{r} 6378 \\ 19 \\ \hline 121182 \end{array}$$

The operation being performed as follows:

9 times 8 are 72; set down 2 and carry 7;

9 times 7 are 63, and 7 (carried) 70, and 8 (back figure) 78, set down 8, and carry 7;

9 times 3 are 27, and 7 (carried) 34, and 7 (back figure) 41, set down 1, and carry 4;

9 times 6 are 54, and 4 (carried) 58, and 3 (back figure) 61, set down 1, and carry 6;

6 (carried) and 6 (back figure) 12, set down 12.

The reason will appear obvious if we compare the usual form of the operation.

$$\begin{array}{r} 6378 \\ 19 \\ \hline 57402 \\ 6378 \\ \hline 121182 \end{array}$$

This *back figure system*, as it is sometimes called, may be extended to numbers between 20 and 30, and between 30 and

40, by adding to the number to be carried the *double* or the *treble* of the figure of the multiplicand just multiplied.

(6) When we can see that the multiplier may be separated into certain numbers, of which the largest is a multiple of the next below it, and that again a multiple of the next below it, and so on, we may perform an apparently long multiplication sum in a few lines; *e.g.* let it be required to multiply 234567891 by 118813212, using only three lines of multiplication.

$$\begin{aligned} 118813212 &= 118800000 + 13200 + 12 \\ &= 13200 \times 9000 + 13200 + 12 \\ &= 13200 \times 9000 + 12 \times 1100 + 12. \end{aligned}$$

Hence if we multiply the given number 234567891 by 12, then multiply that result by 1100, and then that result by 9000, we shall have multiplied it successively by 12, 13200, and by 118800000; and if we add together these three results, we shall obtain the product of 234567891 multiplied by 118813212, the operation will be as follows:

$$\begin{array}{r} 234567891 \\ 12 \\ \hline 2814814692 \\ 1100 \\ \hline 3096296161200 \\ 9000 \\ \hline 27866665450800000 \end{array}$$

Adding these three lines we have

$$\begin{array}{r} 27866665450800000 \\ 3096296161200 \\ 2814814692 \\ \hline 27869764561775892 \end{array}$$

(7) Since 4×25 is 100, and 8×125 is 1000, the division by 25 will be effected by multiplying the dividend by 4, and cutting off the last two figures from the product, in order to divide it by 100; and the division by 125 will be effected by multiplying the dividend by 8, and cutting off the last three figures from the product. In each case the figures *cut off*, when divided respectively by 4 or by 8, will be the *remainder*, and those *left* will be the *quotient*.

e.g. Divide 6934 by 25; and 78451 by 125.

$$\begin{array}{r} 6934 \\ 4 \\ \hline 277{,}36 \end{array}$$

therefore quotient is 277 with remainder 9.

$$\begin{array}{r} 78451 \\ 8 \\ \hline 627{,}608 \end{array}$$

therefore 627 is the quotient, with remainder 76.

To divide by 9, by 99, by 999, or by any number of nines, cut off from the right hand of the dividend as many figures as there are nines in the divisor; write the figures standing on the left of those cut off under the original dividend, and again cut off as many figures as there are nines in the divisor; repeat this as often as the number of figures in the dividend admits; add the results: the sum of the figures *cut off* is the *remainder*, the sum of the figures on the left of those cut off is the *quotient*.

If in the addition there be any number carried to the units' place of the figures forming the quotient, add the number carried likewise to the remainder (as in Ex. 2, where one is carried). If the sum of the figures cut off be all nines, add one to the quotient, and there is no remainder (as in Ex. 3).

Ex. 1. Divide 571117 by 99.

$$\begin{array}{r|l} 5711 & 17 \\ 57 & 11 \\ & 57 \\ \hline 5768 & 85 \end{array}$$

Hence quotient is 5768, with remainder 85.

Ex. 2. Divide 123456789 by 999.

$$\begin{array}{r|l} 123456 & 789 \\ 123 & 456 \\ & 123 \\ \hline 123580 & 368 \\ & 1 \\ \hline 123580 & 369 \end{array}$$

Here quotient is 123580, with remainder 369.

Ex. 3. Divide 7864643457 by 9999.

786464	3457
78	6464
	78
786542	9999
1	

786543 quotient, no remainder.

The reason of this is as follows; whatever number of hundreds any dividend contains, it contains an equal number of ninety-nines, together with an equal number of units. Thus, in Ex. 1, 571117 contains 5711 hundreds, with a remainder 17. It therefore contains 5711 ninety-nines, together with 5711 units, besides the remainder 17. Again, these 5711 units contain 57 hundreds, with a remainder 11; they therefore contain 57 ninety-nines together with 57 units, besides the remainder 11; consequently the original dividend contains 99 altogether 5711 times and 57 times, that is, 5768 times; while the remainders are $17+11+57$, which makes 85.

CHAPTER IV.

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

§ 32. In chap. 2, page 21, we used the term *multiple;* this we now proceed to explain more fully in connexion with the term *measure.*

Def. One number is a *Measure* of another, when it *measures, i.e. divides* that other an exact number of times.

Def. A *Common Measure* of several numbers is a number which divides *each* of the several numbers an exact number of times.

Def. The *Greatest Common Measure* (G.C.M.) of several numbers is the *greatest* number which divides each of the several numbers an exact number of times.

Def. One number is a *Multiple* of another when it *contains, i.e.* can be divided by, that other an exact number of times.

Def. A *Common Multiple* of several numbers is a number which contains *each* of the several numbers an exact number of times.

Def. The least common multiple (L. C. M.) of several numbers is the *least* number which contains each of the several numbers an exact number of times.

The terms Measure and Multiple are thus related; since 5 is a *measure* of 15, therefore 15 is a *multiple* of 5; here 5 is said to *measure* 15 by the units in the quotient 3.

By "an exact number of times" in the above definitions is meant that the division is effected *without any remainder.*

Def. Every number divisible by itself and unity alone is called a *prime* number.

Def. When two or more numbers have no common measure but unity, they are said to be *prime to each other.*

Def. A number which is divisible by other numbers besides itself and unity is called a *composite* number.

Hence a composite number is one composed of the product of 2 or more prime numbers: *e.g.* 6 is the product of 2 and 3; 12 the product of 2, 2 and 3.

Def. The *Factors* of a number are those which being *multiplied together* produce that number: *e.g.* 5 and 7 are the factors of 35. For although in strictness 5, 7 and 1 multiplied together give 35, yet it is usual to exclude unity when speaking of factors.

GREATEST COMMON MEASURE.

§ 33. To find the Greatest Common Measure of two quantities.

The Rule is as follows: *divide the greater number by the less, divide the less by the remainder, divide that remainder by the next remainder; and so on, until the remainder is* 0

or 1; *when the remainder is 0, the last remainder used as a divisor is the G. C. M. When the remainder is 1, the given quantities have no common measure except unity, i.e. they are prime to each other.*

The process will be best understood by an example; let it be required to find the G. C. M. of 816 and 561.

```
561) 816 (1
     561
     ---
     255) 561 (2
          510
          ---
           51) 255 (5
               255
               ---
               ...
```

or using the compendious form

```
561) 816 (1
     255) 561 (2
           51) 255 (5
               000
```

Hence 51 is the G. C. M. required.

Ex. 2. Find the G. C. M. of 22743 and 15827.

```
15827) 22743 (1
        6916) 15827 (2
               1995) 6916 (3
                      931) 1995 (2
                            133) 931 (7
                                 000
```

Hence 133 is the G. C. M. required.

Ex. 3. Find the G. C. M. of 273 and 935.

```
273) 935 (3
     116) 273 (2
           41) 116 (2
                34) 41 (1
                     7) 34 (4
                         6) 7 (1
                            1
```

Therefore the given numbers have no common measure except unity, or are prime to each other.

Since each remainder becomes in turn a divisor, the process may be neatly exhibited in a shortened form. After the first step in the division, in order to avoid writing the figures over again, we may divide the first divisor at the top by the remainder standing below, working backwards as it were, and placing the new quotient to the left of the first divisor: then after taking the next step forwards in the ordinary way, we may again work backwards, dividing by the next lower remainder. Exhibiting in this form the three examples just given, the process will stand in each case as under:

2	561	816	1
	51	255	5
		000	

561 is contained once in 816, with a remainder 255; 255 is contained in the 561, standing above it to the left, twice, with a remainder 51; 51 is contained five times in 255, with no remainder.

2	15827	22743	1
2	1995	6916	3
	133	931	7
		000	

2	273	935	3
1	41	116	2
1	7	34	4
	1	6	

§ 34. To find the G. C. M. of *three or more numbers.*

The Rule is as follows: *Find the G. C. M. of two of the given numbers; then the G. C. M. of the first G. C. M. obtained and of the next number; and so on; the last G. C. M. obtained is the G. C. M. of all the given numbers.*

Ex. 4. Find the G. C. M. of 2720, 5168, and 357.

2720) 5168 (1
2448) 2720 (1
272) 2448 (9
0000

i.e. 272 is the G. C. M. of 2720 and 5168.

Next find the G. C. M. of 272 and 357.

```
272) 357 (1
     85) 272 (3
         17) 85 (5
             00
```

therefore 17 is the G. C. M. of the three numbers 2720, 5168 and 357.

LEAST COMMON MULTIPLE.

§ 35. To find the least common multiple of two numbers.

If the two numbers be prime to each other, the least number which will contain them both is their *product;* but if they each contain several common factors, their product divided by the product of the common factor is the L. C. M.; hence the L. C. M. of two numbers is found by dividing their product by their G. C. M. It will therefore be sufficient in practice to *find the G. C. M. of the two numbers, to divide one of the given numbers by this G. C. M., and to multiply together the quotient and the other of the given numbers; this product is the L. C. M. of the two original numbers.*

Ex. Find the L. C. M. of 336 and 378.

We find that the G. C. M. of the two numbers is 42; and by dividing 336 by 42, we obtain a quotient 8: hence, as 8 multiplied by 378 gives 3024, we find the L. C. M. of 336 and 378 to be 3024, which number will contain 336 exactly 9 times, and 378 exactly 8 times.

§ 36. To find the L. C. M. of *three or more numbers.*

Let it be required to find the L. C. M. of 240, 378, and 1716. First find the L. C. M. of 240, and 378: this will be found, by the process given above, to be the product of the two numbers divided by their G. C. M. *i.e.* to be $(240 \times 378) \div 6$, or 40×378, or 1510. Next find the L. C. M. of 15120 and 1716: this will be found to be $(15120 \times 1716) \div 12$, or 1260×1716, or 2162160; which is the L. C. M. of the three given numbers.

§ 37. Hitherto we have only gone through the ordinary processes for finding the G. C. M. of two or of more numbers, and for finding the L. C. M. of two or of more numbers. When however we come to examine more closely the *reason* for the processes adopted, we shall see that in practice they may often be shortened with advantage.

The principle of finding the G. C. M. of two numbers depends upon the following axioms: (1) A measure of any number is also a measure of any *multiple* of that number. (2) A measure of each of two numbers is also a measure of their *sum* or *difference*.

Now if it be required to find the G. C. M. of 2652 and 19635, the operation at full length will be

```
2652) 19635 (7
      18564
      -----
       1071) 2652 (2
             2142
             ----
              510) 1071 (2
                   1020
                   ----
                     51) 510 (10
                         510
                         ---
                         ...
```

Here 18564 is a *multiple* of 2652, and 2142 is a *multiple* of 1071; and 1020 is a *multiple* of 510.

Therefore every number which is a common measure of 2652 and 19635 is a common measure of 18564 and 19635; and therefore is a measure of their difference 1071; and hence is a common measure of 2652 and 2142; and therefore is a measure of their difference 510: and hence is a common measure of 1071 and 1020; and therefore is a measure of their difference 51.

Also, 51 measures 510, and therefore measures 1020, therefore measures 1020 + 51, or 1071; therefore measures 2142, therefore measures 2142 + 510, or 2652; therefore measures 18564, therefore measures 18564 + 1071, or 19635.

Thus we have proved that every common measure of 2652 and 19635 measures 51; and likewise that 51 measures 2652 and 19635.

But 51 is the greatest measure of itself.

Therefore 51 is the greatest common measure of 2652 and 19635.

§ 38. In this proof it is to be observed that the process is not that of ordinary long division; the *quotients* are of no importance to the result, and in fact we are only finding the difference between a larger number used as a dividend, and a *multiple* of a smaller number, used as the corresponding divisor. This multiple therefore need not always (as in division) be less than the dividend; and it will be sufficient in practice to find the difference between the dividend and the *nearest* multiple of the divisor, whether that multiple be greater or less than the dividend. Attention to this will sometimes shorten labour in the operation: *e.g.* take Ex. 3, p. 41, and find the G. C. M. of 273 and 935; the operation may stand thus:

$$
\begin{array}{l}
273)\ 935\ (4 \\
\ \underline{1092} \\
157)\ 273\ (2 \\
\underline{314} \\
41)\ 157\ (4 \\
\underline{164} \\
7)\ 41\ (6 \\
\underline{42} \\
1
\end{array}
$$

Here in every case we have taken a multiple *greater* than the other number, and have found the difference by subtracting the *upper* from the *lower* number, instead of the *lower* from the *upper*, as in ordinary division; the process may sometimes be *shortened* by this method.

There is however no advantage in taking a multiple *greater* than the other number unless it be *nearer* to it than the next smaller multiple; but at any step in the process it is allowable to introduce a greater multiple if we see it to be the *nearest* to the other number; and subtracting the upper from the lower line, to proceed in the ordinary way.

Thus, in finding the G. C. M. of 2720 and 5168, (part of

Ex. 4, on page 42,) we may shorten the operation by taking the multiple of 2720 *nearest* to 5168:

$$\begin{array}{l} 2720)\,5168\,(2 \\ \quad\;\; \underline{5440} \\ \quad\quad 272)\,2720\,(10 \\ \quad\quad\quad\;\; \underline{2720} \\ \quad\quad\quad\;\; \ldots\ldots \end{array}$$

§ 39. In finding the L. C. M. of two or more numbers, if we can see at once that several of them can be divided by a number which is prime to the rest, we may divide all the numbers by this common factor, and find the L.C.M. of the given numbers by multiplying together the quotients, the numbers not divided, and the common divisor: *e.g.*

Find the L. C. M. of 12, 21 and 57.

Since 12, 21, and 57 are all divisible by 3, the L. C. M. required will be found if we divide each of these numbers by 3, and multiply together the *quotients, viz.:* 4, 7, and 19, and then multiply that result by 3.

For the L. C. M. of 12 and 21 is their *product* divided by their G. C. M.: *i.e.* is $(12 \times 21) \div 3$, which $= 4 \times 21$, or $= (4 \times 7) \times 3$. Hence 84 the L. C. M. of 12 and 21 may be obtained by dividing both 12 and 21 by the common factor 3, multiplying the quotients together, and multiplying that result by 3. Again the L. C. M. of 84 and 57, is $(84 \times 57) \div 3$, which from what has just been said $= (28 \times 19) \times 3$, and this $= (4 \times 7 \times 19) \times 3$.

Hence the L. C. M. of 12, 21 and 57, is obtained by dividing each of the numbers by 3, and multiplying these quotients together and then multiply that result by 3.

Besides this, in finding the L. C. M. of several numbers, if we can see that any one of them is contained in any other of them, whatever is a multiple of the *larger*, must also be a multiple of the *smaller* number; and therefore this latter need not be taken into account at all; hence in finding the L. C. M. of several numbers we may suppress all those which are divisors of any of the others. The operation therefore of finding the L. C. M. of several numbers whose common factors can be easily seen by inspection is frequently performed as in the following example:

Find the L. C. M. of 3, 8, 12, 16, and 22.

Here 3 may be suppressed altogether because it is a divisor of 12, and 8 because it is a divisor of 16; and we proceed with 12, 16 and 22, thus:

$$\begin{array}{r|l} 2) & 12,\ 16,\ 22 \\ \hline 2) & 6,\ \ 8,\ 11 \\ \hline & 3,\ \ 4,\ 11 \end{array}$$

Now as 3, 4, and 11 are all prime to one another, their L. C. M. is their continued product; therefore the L. C. M. of the given numbers 3, 8, 12, 16, and 22, is $3 \times 4 \times 11 \times 2 \times 2$, or is 528.

Instead of this process however, it is a simpler plan, when the given numbers are not large, to separate them into their respective factors, and then to take for the L. C. M. the number that is required to contain all those factors.

Thus, taking again the numbers in the example just given, it is obvious that the L. C. M. of 3 and 8 must contain the factors 3, 2, 2, 2: in order to include 12 it will require no other factor: to include 16 it must again contain 2: and in order to include 22 it must likewise contain 11.

Hence the L.C.M. will be

$$3 \times 2 \times 2 \times 2 \times 2 \times 11, \text{ or } 528.$$

[This method will be used afterwards, in reducing fractions to others having a common denominator. The principle on which the process depends is fully explained in § 40, page 49.]

It is likewise an improving exercise for the learner to find how often the G.C.M. is contained in the given numbers by tracing it backwards from the end to the beginning of the operation. Thus in the example in § 37, page 41, $510 = 51 \times 10$; $1071 = 51\ (2 \times 10 + 1)$; $2652 = 51\ (2 \times 21 + 10)$; $19635 = 51\ (7 \times 52 + 21) = 51 \times 385$. This is frequently the shortest method of finding these quotients.

§ 40. The common factors of many numbers may be found *by inspection* by being acquainted with the following *properties of numbers:* of which the *proofs* are not given here, because they are often long; the *results* however are very useful in practice, and, when carefully noted and accurately remembered, will often save the labour incurred by employing needless trial divisors.

Numbers are divisible by

2, when they are *even*.

3, when the *sum* of their digits is divisible by 3.

4, when their *two right-hand digits* are divisible by 4.

5, when they have 5 or 0 in the units' place.

6, when they are *even*, *and* the *sum* of their digits is divisible by 3.

8, when their *three* right-hand digits are divisible by 8.

9, when the *sum* of their digits is divisible by 9.

10, when they have 0 in the units' place.

11, when the *difference* between the sum of the digits in the odd places and the sum of the digits in the even places is either 0, or is divisible by 11.

12, when the two right-hand digits are divisible by 4, *and* the sum of the digits divisible by 3.

For the numbers 7 and 13 we may observe that since $1001 = 7 \times 11 \times 13$, it follows that if the number of thousands differs from the number of units by 0, or a multiple of 7 or 13, the number is divisible by 7 or 13.

[It should be explained that the numbers are to be read thus: units, tens, hundreds, of *units;* units, tens, hundreds of *thousands;* units, tens, hundreds of *millions;* &c. Thus in the number 31178, there are 178 *units* and 31 *thousands;* and $178 - 31 = 147 = 7 \times 21$; whence the number is divisible by 7. In the number 153517 there are 517 units and 153 thousands; and $517 - 153 = 364 = 7 \times 52 = 7 \times 13 \times 4$, whence the number is divisible by both 7 and 13.]

All *prime* numbers, except 2 and 5, have either 1, 3, 7, or 9 in the place of units; but it is *not* conversely true that all numbers having 1, 3, 7, or 9 in the place of units are prime.

We can now proceed to find the prime factors of any number.

[Obs. We must first explain that when a number is multiplied into *itself* any number of times, the product is called a *power* of the number; *above* the number and to the *right-hand* of it is written a small figure, which denotes the number of factors that produces the power; and this figure is called the *Index:* thus 2×2 is called 2 squared, or 2 raised to the second power, and is written 2^2; $2 \times 2 \times 2$ is called 2 cubed, or 2 raised to the third power, and is written 2^3; and so on.]

The method of decomposing or resolving any number into its prime factors is as follows: *Divide the given number successively, and as often as possible, by each of the prime numbers, 2, 3, 5, 7, &c., beginning with the lowest prime divisor that will measure the given number: when the last quotient is prime, this prime quotient and the several divisors which have been used are the prime factors required.*

Ex. 1. Decompose 4550 into its prime factors.

$$\begin{array}{r|r}
2 & 4550 \\ \hline
5 & 2275 \\ \hline
5 & 455 \\ \hline
7 & 91 \\ \hline
 & 13
\end{array}$$

And as the last quotient 13 is prime, the required prime factors are $2 \times 5 \times 5 \times 7 \times 13$, which may be written $2 \times 5^2 \times 7 \times 13$.

Ex. 2. Decompose 11088 into its prime factors.

$$\begin{array}{r|r}
2 & 11088 \\ \hline
2 & 5544 \\ \hline
2 & 2772 \\ \hline
2 & 1386 \\ \hline
3 & 693 \\ \hline
3 & 231 \\ \hline
7 & 77 \\ \hline
 & 11
\end{array}$$

Hence the prime factors required are $2^4 \times 3^2 \times 7 \times 11$.

As in this example we can see *by inspection* that 11088 is divisible by 8, since the two right-hand digits are divisible by 8, we might have divided *at once* by 8, or by 2^3; and then, after the next division by 2, we might have seen that the quotient 693 was divisible by 9, or by 3^2; and the operation in a shortened form might have stood thus:

$$\begin{array}{r|r}
2^3 & 11088 \\ \hline
2 & 1386 \\ \hline
3^2 & 693 \\ \hline
7 & 77 \\ \hline
 & 11
\end{array}$$

By decomposing numbers into their prime factors we may find either their G.C.M. or their L.C.M.

Take for instance, the numbers 1260, 10584, 12960; decomposed into their prime factors, they become respectively $2^2 \times 3^2 \times 5 \times 7$, $2^3 \times 3^3 \times 7^2$, $2^5 \times 3^4 \times 5$; and of these the only factors which are common to all, are 2^2 and 3^2; whence the greatest number which will measure them all, or the G.C.M., is $2^2 \times 3^2$, or is 4×9, or 36.

On the other hand, the least number which will contain all these prime factors must evidently contain the highest powers of each of them; that is, the L.C.M. must contain 2^5, 3^4, 5 and 7^2; and therefore the L.C.M. is the product $2^5 \times 3^4 \times 5 \times 7^2$, or is 635040.

From this we deduce the following rule for finding the G.C.M. or the L.C.M. of several numbers. *Decompose the given numbers into their prime factors: multiply together the* lowest *powers of those factors which are common to all; the product so formed will be the G. C. M. of the given numbers. Multiply together the* highest *powers of all the factors that occur; the product so formed will be the L. C. M. of the given numbers.*

Exercise IV.

I. Find the Greatest Common Measure of

1. 1729 and 5850.
2. 6409 and 7395.
3. 8645 and 12350.
4. 8398 and 29393.
5. 11050 and 35581.
6. 153517 and 7389501522.

II. Find the Least Common Multiple of

1. 792 and 936.
2. 1224 and 1656.
3. 1692 and 1708.
4. 11050 and 35581.

III. Find the G.C.M. of

1. 9139, 4403, 13949.
2. 6162534, 10190334, 19937610.

3. 7648, 13384, 63096.

4. 12562, 4568, 5139, 8565.

5. 4230, 141000, 95175, 3760, 27636. 6. 22578, 13144, 1113.

IV. Find the L. C. M. of

1. 3528, 25725, 23625, 432. 2. 316, 392, 553.

3. 1587, 575, 1035. 4. 1121, 413, 133.

5. 493, 68, 174, 153. 6. 14491, 16641, 3707.

V. Decompose into their prime factors

1. 1800. 2. 3528. 3. 40425. 4. 690690.

VI. Decompose into their prime factors, and thence find the G. C. M. and the L. C. M. of the numbers,

1. 17725554, 1054872, and 2406096.
2. 340362, 37818, and 7147602.

VII. Find the L. C. M. of 4, 12, 16, 20, and 36; also of 5, 7, 16, 28, 48, and 21.

VIII. Find the greatest number which will divide 398, and 442, leaving as remainders respectively 7 and 5.

IX. Required the least number which can be divided by 7, 12, 15, and 24, with a remainder 3 in every case.

X. Required the least number which when divided by 5, 8 and 9, gives in every case the remainder 2.

XI. Find the greatest number which will divide 6332, and 23999, leaving as remainders 5 and 2 respectively.

XII. Find the greatest number by which when 3863 and 4769 are divided, the respective remainders are 3 and 1.

XIII. Which of the numbers 16137, 41481, 47032, 6809517, 998216, are divisible by 8, 9, 11, or 12?

XIV. Test the numbers
91189, 71799, 51982, and 7389501522 for the factors 7 and 13.

CHAPTER V.

ENGLISH AND FOREIGN WEIGHTS AND MEASURES.

§ 41. As regards the different denominations of money, &c. we have hitherto only assumed that it is known that 4 farthings make a penny, that 12 pence make a shilling, and that 20 shillings make a pound. But it is a great inconvenience in our system of money, weights, and measures, that there is nothing uniform about it. The most convenient system is, no doubt, the decimal: and from time to time there have been attempts made to introduce such a system into England, and to adapt our coins, as well as our weights and measures, to those in general use on the Continent. But though the subject has been a good deal discussed, little progress has hitherto been made; the practical difficulties in carrying out alterations in long-established customs and habits have appeared so formidable, that they have been sufficient to deter the legislature from taking decisive action in the matter. We must therefore be content at present to use the following "Tables," in which the measures in common use are given. But inasmuch as the whole subject is attracting more and more attention, especially after the International Conference at Paris in 1867, (to which further allusion will be made below,) it appears only proper to subjoin to our own tables an account of the metric system of France; and then to give some account of the various schemes which have been proposed both for making our coinage decimal, and likewise for carrying into practical effect the project of a universal coinage for all nations.

The English tables in ordinary use are as follows:

Tables necessary to be accurately remembered.

AVOIRDUPOIS.

16 drams	make	1 ounce.
16 ounces	......	1 lb.
14 lbs.	......	1 stone.
28 lbs.	......	1 quarter.
4 qrs.	......	1 cwt.
20 cwt.	......	1 ton.

Hence 112 lbs. make 1 cwt.
8 stone ...... 1 cwt.
2240 lbs. ...... 1 ton.

LENGTH.

4 inches	make	1 hand.
12 inches	......	1 foot.
3 feet	......	1 yard.
6 feet	......	1 fathom.
5½ yards	......	1 rod, pole, or perch.
40 poles	......	1 furlong.
8 furs.	......	1 mile.
3 miles	......	1 league.

Hence
220 yards make 1 furlong.
1760 yards ...... 1 mile.

TROY.

For Gold, Jewels, &c.

24 grains	make	1 pennyweight.
20 dwts.	......	1 ounce.
12 oz.	......	1 lb.

7000 grains Troy, are equal to 1 lb. Avoirdupois; whence we can bring Troy measure into Avoirdupois, and vice versâ.

SURFACE.

144 sq. inches	make	1 sq. foot.
9 sq. feet	......	1 sq. yard.
30¼ sq. yds.	......	1 sq. rod, pole, or perch.
40 perches	......	1 rood.
4 roods	......	1 acre.
4840 sq. yards	......	1 acre.
640 acres	......	1 sq. mile.

The chain used for measuring land is 22 yds. or 66 feet, divided into 100 links, each of them 7·92 inches.

Hence

100 links	make	1 chain.
10 chains	......	1 furlong.
80 chains	......	1 mile.
10 sq. chains	......	1 acre.

APOTHECARIES.

20 grains	make	1 scruple.
3 scruples	......	1 dram.
8 drams	......	1 ounce.
12 ounces	......	1 pound.

The grain, ounce, and lb. are the same as in Troy weight.

CAPACITY.

2 pints	make	1 quart.
4 quarts	......	1 gallon.
2 gallons	......	1 peck.
4 pecks	......	1 bushel.
8 bushels	......	1 quarter.
5 quarters	......	1 load.

A barrel of *beer* contains 36 gals.
A hogshead of *beer* ...... 54 gals.
A hogshead of *wine*...... 63 gals.
A pipe of *wine* ...... 2 hhds.

CUBIC MEASURE.

1728 cubic inches make 1 cubic foot.

27 cubic feet make 1 cubic yard.

N.B.—A cubic foot of distilled water weighs 1000 ounces.

The difficulty of introducing any change in the established tables of weights and measures without stringent legal enactment, is shewn by the fact that prescriptions are still commonly dispensed according to the table of Apothecaries' measure given on the other side; while, nevertheless, the new British Pharmacopœia has published a new table of weights and measures, adopting the imperial *ounce* and *pound*, but *not* substituting a new medical grain for the Troy grain.

The new table stands as follows:

1 pound = 16 ounces = 7000 grains.
1 ounce = 437·5 grains.
1 grain.

Of Paper	24 sheets	make	1 quire.
	20 quires	......	1 ream.
	10 reams	......	1 bale.

---

12 dozen of any articles are called a gross.

Of Time	60 seconds	make	1 minute.
	60 minutes	......	1 hour.
	24 hours	......	1 day.
	7 days	......	1 week.
	12 calendar months	......	1 year.

A lunar month contains only 28 days; so that 13 lunar months make (within a day) the year.

In the solar year there are 365 days 5 hours 48′ 49·7″; in the civil year 365 days. So that the civil is shorter than the solar year by nearly one-fourth of a day.

To correct this, an intercalary day was added by Julius Cæsar to the month of February in every fourth year; hence Bissextile or Leap-year, having 366 days, the average Julian year contained 365·25 days.

But the true length of the solar year is only 365·2422414 days; and this error, increasing in the course of centuries, became at last so considerable that in 1582 Pope Gregory caused 10 days to be omitted; and directed that for the future 3 leap-years should be omitted in every 400 years; i. e. that all those years completing a century whose first two digits are not exactly divisible by 4, should not be leap years: thus 1700, 1800, 1900 not being reckoned as leap years, 2000 will be a leap year.

This Gregorian or New Style was adopted in England in 1752, when 11 days were omitted in order to correct the calendar.

The Standards which the Legislature prescribes for recovering the exact standard length or weight, should the existing standards be destroyed, are briefly as follows:

The Pendulum vibrating seconds of mean time in the latitude of London in a vacuum at the level of the sea, is to be divided into 39·1393 equal parts, called *inches;* and 36 of these inches are to form the standard *yard.*

A cubic inch of distilled water being weighed in air, at 62° of Fahrenheit, the barometer being at 30 inches, is to be taken as weighing 252·458 *grains:* and of such grains the Troy pound is to contain 5760, and the Avoirdupois pound 7000.

The capacity of the imperial gallon is determined by the space that would be filled by the weight of 10 pounds avoirdupois of distilled water, weighed in air, at 62° of Fahrenheit, the barometer being at 30 inches. And as that weight of distilled water is found to fill 277·274 cubic inches, that is the capacity of the standard gallon; while the bushel of eight gallons contained 2218·192 cubic inches.

These methods may be quite sufficient for restoring standards exact enough for all purposes of buying and selling: but they confessedly fail in perpetuating the precise exactness required by scientific purposes. So that it is now thought by many that the best way to maintain a measure is to construct, out of durable material, accurate copies of a fixed standard, and to preserve the copies carefully. And from the difficulty of obtaining a comparison of a measure of weight deduced from length with one already existing, where the nicest accuracy is required, it has been recommended that the standard of weight shall no longer be deduced from that of length, but

shall simply be a piece of metal, or other durable substance[1].

It would be impossible to exaggerate the inconsistencies of the English system, if that can be called a system which presents such uncouth and complicated tables, the innate absurdity of which is further aggravated by conflicting practice in various parts of the kingdom, notwithstanding repeated Acts of legislation. Why should a stone of meat in London be 8 lbs.? a stone of meat in the country 14 lbs.? while a stone of cheese is everywhere 16 lbs. Why should a sack of flour, which is usually 280 lbs., be in some parts of England 252 lbs.? What do we want with *two standard pounds*, the pound Troy and the pound Avoirdupois? and why do we allow besides these a table for Apothecaries, who buy wholesale by Avoirdupois, and retail by Troy, under names and measures of their own? Then there is an ordinary *Long* measure, and a separate *Cloth* measure, where the Flemish, English and French Ells are stated to be respectively three, five, and six quarters of a yard. Again, both the Scotch and the Irish mile differ from the English mile, and from one another, the Scotch mile containing 1984 yards, the Irish 2200. The English acre contains 4840 square yards, the Scotch acre 6184 square yards, the Irish acre 7840 square yards; while the Welsh acre is *double* of the English acre! Even in England there are local discrepancies, the Irish measure of 7 yards to the pole being still preserved in Lancashire, while in Cheshire 8 yards are reckoned to the perch, (rod, pole, and perch being three separate names for the same length). It is needless to prolong this list of erratic standards: a system so loose and unscientific is perfectly indefensible. It is to be hoped that the exaggerated idea of the difficulty of introducing alterations in practice will not be allowed to hinder much longer a reform so beneficial and so much called for.

§ 42. *The Metric System of Weights and Measures.*

The metric system of weights and measures was introduced into France in 1799, and has been largely adopted on the Continent. In England it was legalized by Act of Parliament in 1864, but no standards have been officially established; and at present the system is very little used in this country.

[1] See the *Report of the Commissioners appointed to consider the steps to be taken for the Restoration of the Standards of Weight and Measure*, 1841.

It is, however, greatly recommended by its simplicity and completeness. Each denomination increases or decreases in a tenfold ratio, and thus the system is essentially *decimal.*

For length, the unit adopted is the "Mètre," (whence the system is named metric). The Mètre is the ten-millionth part of the length of a line supposed to be drawn along the meridian from the pole to the equator of the earth.

From this unit of length the scale ascends and descends by powers of 10, the increasing decimal multiples having names distinguished by *Greek*, the decreasing decimal parts names distinguished by *Latin* prefixes. The Table stands as follows:

10,000 metres = 1000 dekametres = 100 hectometres = 10 kilometres = 1 myriametre (*myriam.*).
1,000 metres = 100 dekametres = 10 hectometres = 1 kilometre (*kilom.*).
100 metres = 10 dekametres = 1 hectometre (*hectom.*).
10 metres = 1 dekametre (*decam.*).
1 Metre.
·1 metre = 1 decimetre (*decim.*).
·01 metre = ·1 decimetre = 1 centimetre (*centim.*).
·001 metre = ·01 decimetre = ·1 centimetre = 1 millimetre (*millim.*).

For Surface the unit is the square metre, and the scale ascends and descends by powers of 10^2, or 100; and there would be the square dekametre, hectometre, &c.; and the square decimetre, centimetre, &c. But for measuring land it is found convenient to assign a distinctive name to a larger area than the square metre: consequently a square dekametre is taken as a fresh unit, and is called an "Are;" and from this are formed in precisely the same manner (by adding the Greek prefixes for the ascending and the Latin prefixes for the descending powers,) the terms dekare, hectare, &c.; and deciare, centiare, &c.

For Solidity the unit is the cubic metre: and this being called the "stere," the words dekastere, &c.; decistere, &c.; are similarly formed. (The dekastere is rather more than 13 cubic yards.)

For Capacity a fresh unit is again taken: and the cubic decimetre being called the "litre," from it are formed a similar series of terms, dekalitre, hectolitre, &c.; decilitre, centilitre, &c.

For Weight the unit is the "gramme," which is the weight

of a cubic centimetre of distilled water: and we have the ascending terms dekagram, hectogram, &c.; and the descending terms decigram, centigram, &c. The name given to 100,000 grams is a "quintal," while a "millier" is the name of a million grams.

The following tables of the metric values, and their British equivalents, are reprinted from the Act of Parliament already referred to (27 & 28 Vic. cap. 117):

MEASURES OF LENGTH.

Metric Denominations and Values.		Equivalents in British Denominations.			
	Metres.	Miles.	Yards.	Feet.	Inches.Decimals.
Myriametre	10,000	6	376	0	11·9
		or	10,936	0	11·9
Kilometre	1,000		1,093	1	10·79
Hectometre	100		109	1	1·079
Dekametre	10		10	2	9·7079
Metre	1		1	0	3·3708
Decimetre	$\frac{1}{10}$				3·9371
Centimetre	$\frac{1}{100}$				0·3937
Millimetre	$\frac{1}{1000}$				0.0394

(For an approximation sufficiently close, observe that 11 metres are nearly equivalent to 12 yards: or more nearly, that 32 metres are equivalent to 35 yards. It is better still to take the metre as equal to 39·37 inches.)

MEASURES OF SURFACE.

Metric Denominations and Values.		Equivalents in British Denominations.	
	Sq. Metres.	Acres.	Sq. Yards. Decimals.
Hectare, i.e. 100 Ares	10,000	2	2,280·3326
		or	11,960·3326
Dekare, i.e. 10 Ares	1,000		1,196·0333
Are	100		119·6033
Centiare, i.e. $\frac{1}{100}$ Are	1		1·1960

(The "are" is a little less than the fortieth part of an acre; and consequently the decare is rather less than a rood.)

MEASURES OF CAPACITY.

Metric Denominations and Values.	Cubic Metres.	Qrs.	Bush.	Pks.	Galls.	Qts.	Pts. Decimals.
Kilolitre, i.e. 1000 Litres	1	3	3	2	0	0	0·77
Hectolitre, i.e. 100 Litres	$\frac{1}{10}$		2	3	0	0	0·077
Dekalitre, i.e. 10 Litres	$\frac{1}{100}$			1	0	0	1·6077
Litre......................	$\frac{1}{1000}$						1·76077
Decilitre, i.e. $\frac{1}{10}$ Litre	$\frac{1}{10000}$						0·176077
Centilitre, i.e. $\frac{1}{100}$ Litre	$\frac{1}{100000}$						0·0176077

(Since a "litre" is rather more than $1\frac{3}{4}$ pints, the dekalitre is about equivalent to 2 gallons; or, more nearly, 50 litres to 11 gallons.)

WEIGHTS.

Metric Denominations and Values.	Grams.	Cwts.	Stones.	Pds.	Ozs.	Drams. Decimals.
Millier	1,000,000	19	5	6	9	15·04
Quintal	100,000	1	7	10	7	6·304
Myriagram	10,000		1	8	0	11·8304
Kilogram	1,000			2	3	4·3830
		(or 15432·3487 grains)				
Hectogram.........	100				3	8·4383
Dekagram	10					5·6438
Gram	1					0·56438
Decigram	$\frac{1}{10}$					0·056438
Centigram	$\frac{1}{100}$					0·0056438
Milligram	$\frac{1}{1000}$					0·00056438

(The "gram" being $15\frac{3}{8}$ grains Troy (very nearly), the kilogram is rather more than 2 lbs. 8 oz. Troy, or rather less than $2\frac{1}{4}$ lbs. Avoirdupois.)

N. B. The exercises on the Metric system will be given after the chapters on *Fractions* and *Decimals;* but the Tables are inserted here in order to exhibit side by side the English and French systems, although with some unavoidable anticipation concerning both fractions and decimals.

§ 43. To pave the way for the introduction into England of a uniform decimal system of weights and measures, it would be needful to commence with the coinage. The necessity of taking this step was warmly advocated a short time ago; it

was proposed to divide the pound into *ten* equal parts called florins, the florin into *ten* equal parts called cents, the cent into *ten* equal parts called mils. The coins used would have been

The Mil = 1000th part of a pound; a copper coin, somewhat *less* than a farthing (as a farthing is the 960th part of a pound). The double mil would be nearly a half-penny ($\frac{12}{25}d.$): and the 5-mil piece would be $1\frac{1}{5}d.$

The Cent = 100th part of a pound = 10 mils; a silver coin worth $2\frac{2}{5}$ pence, and therefore a little smaller than the present three-penny piece. The two-cent piece would be $4\frac{4}{5}d.$: the 5-cent piece the shilling.

The Florin = the 10th part of a pound = 10 cents = 100 mils; a silver coin worth 2*s.*

The half sovereign and sovereign as at present.

This system, while it would retain the present gold coinage as its basis, and would not therefore derange the accounts of the state, of bankers and of merchants, and would have the further advantage of retaining in circulation the silver coins of the shilling and crown, would nevertheless present the great disadvantage of abolishing the present copper coinage of the farthing, half-penny, and penny: as well as the silver coins representing 3*d.*, 4*d.* and 6*d.*: it would therefore disarrange the *poor man's* receipts and payments; and would cause confusion in all such cases as the *penny* postage, *penny* tolls, &c.; as well as in the cost of all those common necessaries, the price of which is calculated in pence. Besides which, any number of pence in the old coinage, with the exception of sixpence, could not be exactly represented in the new coinage.

Notwithstanding these drawbacks, the issue of the florin has done something towards introducing the system: but in order to popularize its adoption, surely the nomenclature should be altered. Florins, cents, and mils are not happily selected names. Let the coin representing two shillings be circulated under its own proper name; let the three-penny piece be withdrawn, and let a coin representing two-pence-halfpenny be issued under its proper name. When these two-shilling pieces and two-pence-halfpenny pieces are fairly established, let it be enacted that twelve-pence-halfpenny shall pass for the silver shilling. This would alter the value of the copper coinage 4 *per cent.* only: and this slight alteration

would be all that would be necessary; for 1000 farthings would then make a sovereign. Thus the pound would consist of *ten* two-shilling pieces; the two-shilling piece of *ten* two-pence-halfpenny pieces; and the twopence-halfpenny pieces of *ten* farthings.

Other systems of decimal coinage have been proposed; and the "International Association for obtaining a uniform Decimal System of Measures, Weights, and Coins" reported (1 March, 1865) that besides those who favoured the pound and mil scheme, there "were those who advocated the ten-penny scheme, viz. the maintenance of the penny as it is, and the issue of a silver coin of 10*d*., and another of gold of 100*d*.; whilst the third scheme contemplated taking the farthing as the unit, and multiplying that by 10, 100, and 1000; having thus a sovereign of 1000, instead of 960 farthings, or £1. 0*s*. 10*d*. Besides these schemes many other suggestions were made. One of these was to take the franc as a unit, introducing the French system as a whole. Another proposed to coin a dollar of 4*s*. 2*d*. or 50*d*., and make the unit of 100 halfpennies; the coin being thus nearly equivalent to the dollar of the United States, the five-franc piece of France, and the dollars circulating in China, India, and other countries."

Another scheme has been propounded by the present Dean of Ely, which deserves consideration from the few changes it would produce in the value of the existing coins. The Dean points out that the "pound and mil" scheme gives four coins of account, and only three of practical circulation; while of these he says that the florin is unfamiliar, and the cent new and as inconvenient as anything that can be devised; also that the ten-penny scheme, for the sake of keeping the penny fixed, compels the total destruction of our gold and silver coinage, and introduces two new coins of 10*d*. and 100*d*., which would bear the most tiresome and complicated relations to existing money. He then proposes to take as the unit of money one half the present pound, or ten shillings: by this no new coin is introduced, ten-shilling pieces being almost as common as sovereigns.

He proceeds to explain his scheme as follows:

"Suppose we take a ten-shilling piece as our first coin of "account. The second is the shilling, which is familiar enough. "The third will be the tenth part of a shilling, or 1·2 pence; "that is to say, it will be a coin worth one-fifth more or 20 per

"cent. more than our present penny. To substitute this coin "for the existing penny would involve little or no inconveni- "ence; the result would be that the bakers would make their "penny rolls one-fifth bigger than at present, boys would get "120 nuts for their penny instead of 100, and so on; and inas- "much as the copper coinage does not represent the value "indicated by it commercially, but only conventionally, the "existing copper coinage would serve under the new system "as well as under the old.

"But without saying anything more on this subject, let me "just explain how I should propose to introduce the change; "for after all this is the most important matter, and it is of no "use to shew that a system is good, without shewing that it "can be introduced without frightening people out of their wits.

"I divide the process of introducing a decimal system of "coinage into two operations, which I will call operation *A* "and operation *B*. These two operations may be made the "subject of one bill; or *A* may be enacted first and *B* after a "few years when people are familiar with *A*; or, lastly, *A* and "*B* may be enacted together, with the condition that *B* shall "not take effect till a certain number of years, say five, have "elapsed.

"*A*. Let it be enacted that after a certain date all public "accounts shall be kept in half-sovereigns instead of sove- "reigns. It may be presumed that bankers, and then shop- "keepers, would soon follow the example. The result would "be that one of the lines in the ordinary account-books would "become useless. Without knowing anything about decimals "people would find that after adding up the shillings, they "would not have to divide by 20 as now, but that the addition "was what is called in the books *simple*.

"There would be no inconvenience in comparing old ac- "counts with new, as it would be only necessary to multiply "or divide by 2. For example, if the national debt were "stated under the old system as 800,000,000, it would be "stated under the new as 1600,000,000, and so on.

"Nor would people be prevented from talking of pounds as "before; we know by experience how long guineas have re- "mained among us as a name after the coin has vanished, and if "so inconvenient a coin as the guinea can hold its place, there "need be no fear that the sovereign would lose its place as the

"principal coin of currency, after it had become obsolete as a "coin of account.

"*B.* Let it be enacted that from and after a certain date "the penny shall be one-tenth instead of one-twelfth of a shil-"ling. As soon as this is done the other line of the account-"book may be dispensed with, and compound addition of "money be forgotten. I have before explained that the result "of this will be to increase the value of the penny 20 per cent., "and any large holder of copper coinage would get a pre-"mium, but this is of no great consequence; the same coinage "would answer all purposes; and commodities would very "soon adjust themselves.

"It would be necessary to call in four-penny pieces, and "three-penny pieces would require to be marked 2½-pence in-"stead of 3. Bankers might reject half-pence as at present; we "should have three coins of account as now, and these three "would correspond very well as at present to the needs of the "rich, the middle class, and the poor.

"On the whole, the scheme now propounded

"(1) Involves no new-fangled or un-national coins;

"(2) Interferes with the convenience neither of rich nor poor;

"(3) Can be introduced without alarming people on the "subject of decimals[1]."

Having thus far explained the various schemes which have been proposed for a decimal coinage in England, we will go on to another branch of the same subject, namely, the project of an International coinage.

At the Paris Exhibition in 1867, an attempt was made to devise some feasible plan for introducing a Universal Coinage; and the French government invited delegates and commissioners of all the foreign States to meet and organize a Monetary Conference.

On the question of uniformity of weights and measures the Conference were well agreed: but the question of a universal coinage provoked considerable and animated discussion. As far as our present subject is concerned, the most important

[1] A full account of the above scheme may be found in an Article styled "Monetary Conventions and English Coinage," in the Number of *The Contemporary Review* for January, 1867.

resolutions arrived at by the Conference[1] were the 3rd, 4th, 5th, and 8th, to the effect that:

3. "It is desirable that each State should introduce among "its gold coins one piece at least of a value equal to that of "one of the pieces in use among other States interested, so "that there may be among all the systems a point of com- "mon contact. From that each nation will afterwards en- "deavour to assimilate gradually its system of coinage to that "which may be chosen as a uniform basis."

4. "The series of gold coins now in use in France, being "adopted by a great part of the population of Europe, is "recommended as a basis of the uniform system."

5. "Whereas, in consequence of accidental and happy "circumstances, the most important monetary units may be "adapted to the French gold piece of five francs, by means of "very small changes, this piece seems the most convenient to "serve as a basis of a monetary system, and the coins issued "upon such a basis may become, as soon as the convenience of "the nations will permit, multiples of this unit."

8. "It is extremely desirable that the system of decimal "numeration be universally adopted, and that the money of "all nations should be of the same fineness and of the same "form."

It will be observed that the Conference did not propound any specific plan of a universal coinage, but was content to aim simply at one point of accord in the different systems of coinage, in having one piece alike everywhere; and it fixed upon the five-franc piece in gold as on the whole the most convenient for the basis of the uniform system.

Now if these resolutions of this Conference are to be carried into effect in England, (and it should be borne in mind that already France has concluded a monetary convention with Italy, Belgium, and Switzerland,) the question of a decimal coinage is settled for us, and all we have to do is to assimilate our coinage to that of France. The British pound sterling having at present a value of 25·2215 francs, we should have to strike a coin of the exact value of 25 francs, or about $\frac{7}{8}$ths per cent. less than the present sovereign. This new coin of ours

[1] An account of this conference, with an able discussion of the merits of the plan it proposed, is given in a work on *Bullion and Foreign Exchanges*, by Ernest Seyd.

being the exact value of 25 francs would pass current in France, whilst the French 20 franc piece might be taken in England at the rate of 16 new shillings, for the shilling also would necessarily be affected by the change. The French Government has already taken the lead in this matter, having lately issued a new gold coin of the value of 25 francs, in deference to us and the Americans.

The necessity of having a new coin struck will be obvious if we consider that although upon each sovereign the difference would be only 22·15 centimes, or a little more than 2*d.*, yet upon large sums the case is very different; for upon £100,000 the difference would amount to £880 nearly.

Mr Seyd, from whose work we are now quoting, advocates a modification of the plan proposed by the Paris Conference, and suggests that, accepting the proposed piece of 5 francs as the basis of the system, it should be divided not into 5 parts (francs) but into 4 parts (shillings), which would give the following set of coins for Great Britain.

GOLD.					
Sovereign	=20	shillings,	or	25	Francs.
Half-sovereign	=10	...	,	12·50	...
Quarter-sovereign	= 5	...	,	6·25	...
SILVER.					
Dollar	= 4	...	,	5	...
Florin	= 2	...	,	2·50	...
Shilling	= 1	...	,	1·25	...
Half-shilling	= ½	...	,	0·625	cents.
Fifth-shilling	= ⅕	...	,	0·25	...

The Florin to be divided into 100 parts, each part about equal to the present farthing.

Whatever may be the ultimate issue of these various proposals, it is clear that England is too deeply interested in the question to stand aloof from the discussion: and although any change in the coinage of a country must entail some inconvenience, it would be better far for us to endure our share of that, than to see a question of such universal importance settled without our concurrence in the scheme.

Thus far for the project of a universal coinage[1], a project

[1] The Royal Commission appointed to consider the possibility of establishing an International Coinage report (see the *Times* of 7th

concerning which the amount of the trade of England, and the immense stake we have in the matter, gives us every right to be listened to. For our *coinage* there is something to be said; and it may be not impossible to arrive at a satisfactory solution of the problem of an international coinage without any violent disturbance of our existing system. But as regards *weights and measures*, our system is utterly indefensible, and the only course open to us would seem to be the early introduction of the Metric system, pure and simple, as it exists in the greater part of Europe and in different States of America.

Meanwhile, pending compulsory legislation on the subject, "the introduction of the system" (to quote from Professor Leone Levi's report of the Paris Conference) "should certainly "be encouraged in all schools, and demanded in all schools "receiving grants from the Privy Council; and an examination "in the same should also be required of teachers in the Normal "Schools, and candidates for Government certificates."

EXERCISE V.

I. Perform the following operations in Compound Addition:

	£.	s.	d.		£.	s.	d.
(1)	2781	19	11	(2)	456	17	11
	1462	4	9		239	14	9
	2468	3	7		754	16	11
	5782	14	11		892	15	9
	6341	12	2		621	12	6
	4122	15	8		432	9	11
	3456	10	11		679	7	4

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of October, 1868) *against* the proposal of the Paris Conference of reducing the value of the pound to that of 25 francs. They remark that while this alteration would disturb all existing obligations, it is after all only a partial measure; and they recommend that the general question in all its wider bearings should form the subject of an international conference.

	£.	s.	d.			£.	s.	d.
(3)	468	10	7		(4)	371	12	11
	279	18	$11\frac{1}{2}$			464	14	9
	345	9	9			953	9	$6\frac{1}{4}$
	989	6	$7\frac{1}{4}$			765	7	4
	67	4	$8\frac{3}{4}$			9	19	$11\frac{3}{4}$
	5287	19	2			2957	13	$7\frac{1}{2}$
	8	18	$6\frac{1}{2}$			32	5	5

	£.	s.	d.			£.	s.	d.
(5)	9999	17	$5\frac{1}{2}$		(6)	7517	8	$0\frac{1}{4}$
	0	2	$6\frac{1}{2}$			2482	11	$11\frac{3}{4}$
	7645	16	$4\frac{3}{4}$			9666	7	8
	2354	3	$7\frac{1}{4}$			6053	15	$8\frac{1}{2}$
	5978	2	9			3946	4	$3\frac{1}{2}$

II. (1) From £5761. „ 11*s*. „ $1\frac{1}{2}$*d*. subtract £2987. „ 19*s*. „ $11\frac{3}{4}$*d*.

(2) From £7003. „ 1*s*. „ $2\frac{1}{4}$*d*. subtract £4897. „ 11*s*. „ $9\frac{1}{2}$*d*.

(3) From £1101. „ 0*s*. „ $0\frac{3}{4}$*d*. subtract £783. „ 13*s*. „ 3*d*.

(4) From £72030. „ 9*s*. „ 7*d*. subtract £59913. „ 19*s*. „ $1\frac{1}{4}$*d*.

III. Multiply (1) £4098. „ 10*s*. „ $6\frac{3}{4}$*d*. by 3, by 5, by 7, and by 11: and add the products together.

(2) £6894. „ 12*s*. „ $10\frac{1}{2}$*d*. by 15.

(3) £3330. „ 10*s*. „ $4\frac{3}{4}$*d*. by 27.

(4) £6804. „ 18*s*. „ $8\frac{1}{2}$*d*. by 37.

(5) £33063. „ 17*s*. „ 4*d*. by 76.

(6) £9986. „ 18*s*. „ $4\frac{1}{4}$*d*. by 98.

(7) £1068. „ 10*s*. „ $7\frac{1}{4}$*d*. by $6\frac{1}{4}$.

(8) £863. „ 13*s*. „ $7\frac{1}{2}$*d*. by $6\frac{3}{4}$.

(9) £986. „ 12*s*. „ $0\frac{1}{2}$*d*. by $23\frac{5}{8}$.

(10) £48. „ 18*s*. „ $5\frac{1}{4}$*d*. by $30\frac{1}{5}$.

(11) £19. „ 10*s*. „ $8\frac{1}{2}$*d*. by 405.

(12) £18. „ 7*s*. „ $4\frac{3}{4}$*d*. by 965.

IV. Divide (1) £9801. „ 10*s*. „ $11\frac{1}{4}$*d*. by 3.
(2) £869. „ 17*s*. „ $8\frac{1}{2}$*d*. by 13.
(3) £786. „ 13*s*. „ $8\frac{1}{4}$*d*. by 246.
(4) £3004. „ 5*s*. „ $9\frac{1}{2}$*d*. by 379.
(5) £4024. „ 10*s*. „ 2*d*. by 401.
(6) £58098. „ 17*s*. „ $6\frac{3}{4}$*d*. by 1307.

V. Required the cost of

(1) 12 gallons at 6*s*. „ $10\frac{1}{2}$*d*. per gallon.
(2) 45 oxen at £14. „ 13*s*. „ 2*d*. each.
(3) 206 ounces of gold at £3. „ 11*s*. „ 4*d*. per ounce.
(4) $12\frac{1}{4}$ lbs. of butter at 1*s*. „ 4*d*. per lb.
(5) $34\frac{3}{5}$ lbs. of tea at 3*s*. „ 8*d*. per lb.
(6) 567 horses at £54. „ 12*s*. „ 9*d*. each.

VI. A man spends 1*s*. „ 9*d*. in beer every week: what does that amount to in a year?

VII. The price of 12 tons of coal was £15. „ 16*s*. „ 9*d*.: what was the price per ton?

VIII. If 20 cubic feet of timber cost £2. „ 15*s*., what was that per foot?

IX. The cargoes of 16 ships were valued at £998460: what was the average value of each cargo?

X. A gentleman spent £1000 a-year: what was his average daily expenditure?

XI. At a Railway station £5. „ 10*s*. was taken for 23 tickets: what was the average price per ticket?

XII. A person buys 64 lbs. of meat weekly: for 16 weeks the price is $8\frac{1}{2}$*d*. per lb.; for 26 weeks 8*d*.; and for 10 weeks $7\frac{1}{2}$*d*.: what is the amount of his butcher's bill, and the average price per lb. of meat during the year?

REDUCTION.

XIII. (1) Reduce £94. „ 10*s*. „ 6*d*. to pence.
(2) Reduce £100. „ 0*s*. „ $0\frac{1}{2}$*d*. to halfpence.
(3) Reduce £64. „ 2*s*. „ $2\frac{1}{2}$*d*. to farthings.
(4) Reduce 2460 pence to pounds, &c.

(5) Reduce 4490 farthings to pounds, &c.

(6) Reduce 5 guineas to farthings.

(7) How many three-penny pieces in £12. „ 0*s*. „ 6*d*.?

(8) How many four-penny pieces in £18. „ 1*s*. „ 8*d*.?

(9) How many pence in 96 crowns?

(10) How many fivepences in 16 half-crowns?

(11) In 4680 fourpences, how many fivepences?

(12) In 9990 half-pence, how many fourpences?

(13) In 1000 sixpences, how many farthings?

(14) In £100, how many fivepences?

(15) In 10 guineas, how many half-crowns?

(16) In 40 guineas, how many half-sovereigns?

(17) Bring 2806 half-pence to £. „ *s*. „ *d*.

(18) How many times could three farthings be subtracted from £20, before that sum would be exhausted?

(19) To how many persons could I give a crown out of 210 guineas?

(20) How many times could I pay away three halfpence if I had £24. „ 10*s*. „ $4\frac{1}{2}$*d*.?

(21) If I had 3640 guineas, how many florins ought I to receive for them?

(22) If I change 10080 pence for guineas, how many guineas should I receive?

(23) How often could tenpence be taken out of a purse containing £1. „ 10*s*. before the purse would be emptied?

(24) How often is $3\frac{3}{4}$*d*. contained in £1?

AVOIRDUPOIS.

XIV. (1) Add together

tons	cwt.	qrs.	lbs.	oz.
1 „	12 „	3 „	0 „	12
4 „	13 „	0 „	19 „	13
24 „	6 „	1 „	14 „	15
9 „	19 „	2 „	19 „	4

	tons	cwt.	qrs.	lbs.	oz.	drs.
(2) Add together	4	9	0	16	12	12
	3	19	3	27	4	9
	2	18	2	4	13	5
	1	9	1	13	6	8

	tons	cwt.	qrs.	lbs.	oz.
(3) From	4	0	0	3	4
Take		3	0	6	2

	tons	cwt.	qr.	lbs.	oz.	drs.
(4) From	9	0	1	0	4	0
Take		18	0	6	9	5

(5) Multiply 9 tons, 9 cwt. 1 qr. 10 lbs. 4 oz. by 24.

(6) Multiply 40 tons, 13 cwt. 3 qrs. 18 lbs. 9 oz. 9 drs. by 108.

(7) Divide 3 qrs. 14 lbs. 6 oz. 10 drs. by 6.

(8) Divide 8 tons, 9 cwt. 0 qrs. 9 lbs. 8 oz. by 12.

TROY WEIGHT.

	lbs.	oz.	dwt.	grs.
XV. (1) Add together	8	9	11	14
	2	0	19	22
		6	15	19
			6	8

	lbs.	oz.	dwt.	grs.
(2) Add together	11	10	9	14
	10	9	17	18
	6	3	11	10
		1	19	17

	lbs.	oz.	dwt.	grs.
(3) From	15	3	2	13
Subtract	14	8	11	17

	lbs.	oz.	dwt.	grs.
(4) From	10	0	0	1
Subtract	3	6	17	18

(5) Multiply 26 lbs. 10 oz. 18 dwt. 23 grs. by 30.

(6) Multiply 9 lbs. 3 oz. 17 dwt. 6 grs. by 43.

(7) Divide 9 lbs. 6 oz. 12 dwt. 12 grs. by 10.

(8) Divide 16 lbs. 11 oz. 16 dwt. 13 grs. by 30.

APOTHECARIES.

XVI. (1) Add

lbs.	oz.	drs.	scru.	grs.
6	6	6	0	9
5	8	6	2	10
4	9	3	1	14
2	4	7	2	10

(2) Add

lbs.	oz.	drs.	scru.	grs.
19	0	3	0	4
	9	7	1	19
6	0	6	2	14
23	6	5	1	12

(3) Subtract 3 lbs. 9 oz. 6 drs. 0 scru. 15 grs. from 4 lbs. 9 oz. 6 drs. 0 scru. 14 grs.

(4) Subtract 9 lbs. 8 oz. 6 drs. 2 scru. 19 grs. from 208 lbs. 4 oz. 4 drs. 1 scru. 18 grs.

(5) Multiply 14 lbs. 7 oz. 6 drs. 2 scru. by 15.

(6) Multiply 12 lbs. 3 oz. 6 drs. 2 scru. 6 grs. by 110.

(7) Divide 3 oz. 4 drs. 2 scru. 16 grs. by 8.

(8) Divide 9 lbs. 10 oz. 4 drs. by 36.

LONG MEASURE.

XVII. (1) Add

per.	yds.	ft.	in.
9	1	2	6
3	2	1	8
4	2	2	10
1	4	0	11

(2) Add

m.	f.	p.	yds.	ft.	in.
6	6	20	3	2	9
9	7	36	1	2	4
6	5	10	4	1	10
11	3	19	3	0	11

(3) Subtract 2 fur. 27 p. 4 yds. 2 ft. 8 in. from 7 fur. 19 p. 2 yds. 1 ft. 2 in.

(4) Subtract 38 mi. 6 fur. 23 p. 4 yds. 0 ft. 1 in from 200 mi. 6 fur. 23 p. 4 yds. 0 ft. 0 in.

(5) Multiply 6 fur. 3 p. 2 yds. 1 ft. by 12.

(6) Multiply 3 mi. 4 fur. 26 p. 4. yds. 2 ft. 10 in. by 27.

(7) Divide 4 yds. 2 ft. 6 in. by 6.

(8) Divide 6 fur. 36 p. $4\frac{1}{2}$ yds. by 9.

CHAPTER VI.

FRACTIONS.

§ 44. We proceed to give two definitions of a Fraction:

Def. 1. A Fraction is a quantity which represents a part or parts of an integer or whole. In its simplest form a Vulgar Fraction consists of two numbers, called the Numerator and Denominator; the Denominator shows into how many *equal* parts the whole is divided, and the Numerator shows how many of these equal parts are taken. The Numerator is usually placed over the Denominator with a line between them.

Obs. We suppose every integer to be divisible into any number of equal parts at pleasure.

Def. 2. A fraction is a simple manner of expressing the division of the numerator by the denominator[1].

Def. A proper fraction is one whose numerator is *less* than the denominator.

Def. An improper fraction is one whose numerator is *equal to*, or *greater than* its denominator[2].

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[1] The following definition of a Fraction has also been given: "Every sum which does not contain the unit of measurement an "exact number of times, but which can be measured by some part "of the unit an exact number of times, is a fraction."

Thus $\frac{3}{4}$ denotes a quantity which does not contain the unit of measurement so much as once; but which does contain a 4th part of that unit exactly 3 times.

This Definition however does not appear so simple as the definitions usually given; moreover it would not apply to complex fractions, nor to such fractions as $\frac{18s.\ 6d.}{£1}$.

[2] In an improper fraction the meaning may appear ambiguous: thus $\frac{5}{3}$ would appear to mean that the unit is to be divided into 3 equal parts, and 5 of those parts are to be taken. But in that case we must suppose as many units to be each divided into 3 equal parts as will give more than 5 of such parts, and then 5 of those parts to be taken.

Def. A compound fraction is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{3}{4}$: where $\frac{3}{4}$ is the quantity of which $\frac{2}{3}$ is to be taken.

Def. A complex fraction is one in which either the numerator, or denominator, or both, are fractions; as

$$\frac{2\frac{1}{2}}{3}, \quad \frac{3\frac{1}{4}}{4\frac{2}{3}}.$$

Def. A mixed number consists of a whole number and a fraction; as $5\frac{3}{7}$, which signifies 5 integers together with $\frac{3}{7}$ parts of an integer. Here the *plus* sign is understood, but not expressed.

Obs. Every whole number may be considered as a fraction whose denominator is 1: thus $7.=\frac{7}{1}$. For since the fraction expresses the division of the numerator by the denominator, $7 \div 1$ is equivalent to $\frac{7}{1}$.

§ 45. *To show that* $\frac{2}{3}$ *of* 1 *is* $\frac{1}{3}$ *of* 2.

$\frac{2}{3}$ of 1 is 2 third parts of unity.

Now 1 is 3 third parts of unity.

Therefore 2 is 6 third parts of unity.

Therefore $\frac{1}{3}$ of 2 is $\frac{1}{3}$ of 6 third parts of unity, or is 2 third parts of unity.

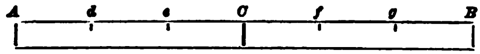
But $\frac{2}{3}$ of 1 is 2 third parts of unity.

Hence $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2.

Also since $\frac{1}{3}$ of 2 is $2 \div 3$, $\frac{2}{3}$ of 1 must $= 2 \div 3$.

Whence we see that one of the definitions given above involves the other.

Or, we may show that $\frac{2}{3} = \frac{1}{3} \times 2$ by the following illustration:



Let the length AB represent two yards, and divide each of the yards AC, CB into three equal parts Ad, de, eC; Cf, fg, gB.

Then, since these parts are all equal, Ad and de are together equal to Ad and Cf; but Ad and de, being each the third part of a yard, are together $\frac{2}{3}$ of one yard; and Ad and Cf, being each the third part of a separate yard, are together $\frac{1}{3}$ of 2 yards; therefore $\frac{2}{3}$ of 1 yard $= \frac{1}{3}$ of 2 yards; or, the same length is obtained whether we divide one yard into three equal parts and take two of them, or divide two yards both into three equal parts, and take one out of each of them.

§ 46. *To multiply any fraction by any integer multiply the numerator by it, or divide the denominator by it.*

If the numerator be doubled or trebled, while the denominator remains unaltered, the *number* of parts taken is doubled or trebled, but as the *magnitude* of the parts is unaltered, the value of the fraction is doubled or trebled. But if the denominator be divided by 2 or by 3, while the numerator remains unaltered, the *number* of parts taken is unaltered, but as the *magnitude* of the parts taken has been doubled or trebled, the value of the fraction is doubled or trebled.

Thus $$\frac{2}{11} \times 3 = \frac{6}{11}:$$

for in each of the fractions $\frac{2}{11}$ and $\frac{6}{11}$ the unit is divided into 11 equal parts; but *thrice* as many of these parts are taken in the latter case as in the former; hence the fraction $\frac{6}{11}$ represents the fraction $\frac{2}{11}$ taken 3 *times*, or multiplied by 3.

Again $$\frac{5}{12} \times 3 = \frac{5}{4};$$

for the unit is divided into 3 times as many equal parts in

$\frac{5}{12}$ as it is in $\frac{5}{4}$, and therefore each of the parts in $\frac{5}{4}$ is 3 times as great as each of the parts in $\frac{5}{12}$; but as *the same number of parts* are taken in both cases, the fraction $\frac{5}{4}$ must be 3 times as great as the fraction $\frac{5}{12}$.

§ 47. Conversely, *to divide a fraction by an integer, divide the numerator by it, or multiply the denominator by it.*

Thus $$\frac{4}{13} \div 2 = \frac{2}{13};$$

for in each of the fractions $\frac{4}{13}$ and $\frac{2}{13}$ the unit is divided into the same number of equal parts; but only *half* as many of those parts are taken in the latter case as in the former: hence the fraction $\frac{2}{13}$ represents *one-half* of the fraction $\frac{4}{13}$; or $\frac{4}{13} \div 2 = \frac{2}{13}$.

Again, $$\frac{5}{11} \div 2 = \frac{5}{22};$$

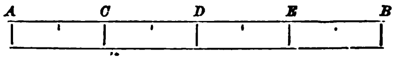
for the unit is divided into twice as many equal parts in $\frac{5}{22}$ as in $\frac{5}{11}$, and therefore each of the parts in $\frac{5}{22}$ is only half as great as each of the parts in $\frac{5}{11}$: but as *the same number of parts* are taken in both cases, the fraction $\frac{5}{22}$ must be only one-half of the fraction $\frac{5}{11}$.

§ 48. *To prove that the value of a fraction is not altered by multiplying both numerator and denominator by the same quantity.*

If any quantity be both multiplied and divided by the same number, its value is not altered. Now if the *numerator* of a fraction be multiplied by any number, the *fraction* is thereby multiplied by it (§ 46); and if the *denominator* of a fraction be multiplied by any number, the *fraction* is thereby

divided by it (§ 47); therefore by multiplying the numerator and denominator of a fraction by the same number, we both multiply and divide the fraction by the same number, and therefore do not alter its value.

Or, we may show that $\frac{3}{4}=\frac{6}{8}$ in the following manner:



Let the length AB represent one yard, and let it be divided into 4 equal parts AC, CD, DE, EB; then AE is $\frac{3}{4}$ of a yard; but if each of these fourth parts were divided into 2 equal parts, the whole yard AB would be divided into 8 equal parts, of which AE would contain 6; therefore AE is $\frac{6}{8}$ of a yard; hence $\frac{3}{4}$ and $\frac{6}{8}$ are the same thing; the line being divided into parts *twice as small* in the latter as in the former case, but *twice as many* of those smaller parts being taken.

Conversely, the value of a fraction is not altered by *dividing* both numerator and denominator by the same quantity.

49. We may hence reduce any fraction to lower terms by dividing both numerator and denominator by any common factor; and a fraction is said to be reduced to its *lowest terms* when its numerator and denominator, being both divided by their greatest common measure, are prime to one another. Thus, reduce to its lowest terms the fraction $\frac{3094}{4641}$.

First finding the G.C.M. of the numbers 3094 and 4641 (§ 33).

```
3094) 4641 (1
      1547) 3094 (2
            0000
```

Here 1547 is the G.C.M. required, and by dividing both numerator and denominator by this quantity the fraction $\frac{3094}{4641}$ is reduced to $\frac{2}{3}$; where 2 and 3 being prime to each other, the fraction is in its lowest terms.

If the given fraction be an improper one, first reduce it to a mixed number, and then reduce the remainder to its lowest terms: *e.g.* to bring $\frac{3471}{195}$ to its lowest terms.

$$\begin{array}{l} 195)\ 3471\ (17\frac{156}{195} \\ \quad\ \ 1521 \\ \quad\ \ \ 156 \end{array}$$

Here we have a remainder 156, and as we cannot carry the division further, we can only express the division of 156 by 195 by the fraction $\frac{156}{195}$; and the actual quotient is exhibited in the form $17\frac{156}{195}$; and finding the G.C.M. of 156 and 195, viz. 39, the given fraction in its lowest terms becomes $17\frac{4}{5}$.

It is not always necessary to find the G.C.M. of the numerator and denominator, if we can determine by inspection what factors are common to both: *e.g.* to reduce the fraction $\frac{14652}{15048}$ we may divide both numerator and denominator successively by the common factors 4, 9, and 11; thus,

$$\frac{14652}{15048}=\frac{3663}{3762}=\frac{407}{418}=\frac{37}{38}.$$

§ 50. *To bring fractions to others of the same value, having a common denominator.*

First, find the Least Common Multiple of all the denominators of the given fractions; divide this L.C.M. separately by the denominators of each of the given fractions, and by the respective quotients obtained by this division multiply both the numerators and denominators of the several fractions: this will not alter their value, but will reduce them to equivalent fractions having the least common denominator.

Ex. 1. Bring $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

Since 12 is the L.C.M. of 3 and 4, multiply both numerator and denominator of $\frac{2}{3}$ by 4, and both numerator and denominator of $\frac{3}{4}$ by 3;

then
$$\frac{3}{2}=\frac{2\times4}{3\times4}=\frac{8}{12},$$
$$\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}.$$

Hence $\frac{2}{3}$ and $\frac{3}{4}$ are equal respectively to $\frac{8}{12}$ and $\frac{9}{12}$, fractions which now have the same common denominator.

Ex. 2. Bring $\frac{4}{5}$, $\frac{5}{9}$, $\frac{19}{21}$, $\frac{17}{45}$ and $\frac{13}{63}$ to equivalent fractions, having the least common denominator.

By inspection (§ 39) the L. C. M. must contain the factors 5, 3, 3, 7.

Hence
$$\frac{4}{5}=\frac{4\times63}{5\times3\times3\times7}=\frac{252}{315},$$
$$\frac{5}{9}=\frac{5\times35}{9\times5\times7}=\frac{175}{315},$$
$$\frac{19}{21}=\frac{19\times15}{21\times5\times3}=\frac{285}{315},$$
$$\frac{17}{45}=\frac{17\times7}{45\times7}=\frac{119}{315},$$
$$\frac{13}{63}=\frac{13\times5}{63\times5}=\frac{65}{315}.$$

Hence $\frac{252}{315}$, $\frac{175}{315}$, $\frac{285}{315}$, $\frac{119}{315}$, $\frac{65}{315}$ are the fractions required.

§ 51. *To compare the value of different fractions, i.e. to find out which is the greatest and which the least.*

Bring the given fractions to others of the same value, having a common denominator; then the respective values of the fractions will depend upon their numerators, that fraction being greatest which has the greatest numerator.

Thus, to find which is the greatest, $\frac{6}{7}$ or $\frac{7}{8}$.
$$\frac{6}{7}=\frac{6\times8}{7\times8}=\frac{48}{56},$$
$$\frac{7}{8}=\frac{7\times7}{8\times7}=\frac{49}{56}.$$

And since $\frac{49}{56}$ is greater than $\frac{48}{56}$, it follows that $\frac{7}{8}$ is greater than $\frac{6}{7}$.

Ex. 2. Compare the values of $1\frac{3}{8}$ of $\frac{2}{3}$, $\dfrac{2\frac{1}{7}}{\frac{16}{7}}$, $\dfrac{4\frac{3}{5}}{4\frac{4}{5}}$, $7\frac{3}{4}$ of $\frac{1}{8}$.

These fractions when reduced become $\frac{11}{12}$, $\frac{15}{16}$, $\frac{23}{24}$, $\frac{31}{32}$, and the prime factors included in the L.C.M. being 2, 2, 3, 2, 2, 2, we have

$$\frac{11}{12}=\frac{11\times 8}{12\times 8}=\frac{88}{96},$$

$$\frac{15}{16}=\frac{15\times 6}{16\times 6}=\frac{90}{96},$$

$$\frac{23}{24}=\frac{23\times 4}{24\times 4}=\frac{92}{96},$$

$$\frac{31}{32}=\frac{31\times 3}{32\times 3}=\frac{93}{96}.$$

Hence the values of the given fractions are in the order of their equivalents as follows:

$$\frac{93}{96},\ \frac{92}{96}\ \ \frac{90}{96},\ \frac{88}{96}.$$

Ex. 3. Compare $\frac{6}{7}$, $\frac{8}{9}$, $\frac{12}{17}$, and $\frac{16}{19}$.

The comparison of fractions may sometimes be more concisely performed by transforming the given fractions to others having a common *numerator:* the fraction that has then the *least* denominator is the greatest: thus

$$\frac{6}{7}=\frac{96}{112},\ \frac{8}{9}=\frac{96}{108},$$

$$\frac{12}{17}=\frac{96}{136},\ \frac{16}{19}=\frac{96}{114}.$$

Hence $\frac{8}{9}$, $\frac{6}{7}$, $\frac{16}{19}$, $\frac{12}{17}$ are the fractions arranged in order of magnitude.

Ex. 4. Show that the fraction $\frac{5+6}{6+7}$ is greater than $\frac{5}{6}$ and less than $\frac{6}{7}$.

$$\frac{5+6}{6+7}=\frac{11}{13}.$$

Hence, comparing $\frac{5}{6}$ and $\frac{11}{13}$, we get

$$\frac{5}{6}=\frac{5\times 13}{6\times 13}=\frac{65}{6\times 13},$$

$$\frac{11}{13}=\frac{11\times 6}{13\times 6}=\frac{66}{6\times 13};$$

therefore $\frac{11}{13}$ is $>\frac{5}{6}$.

But, comparing $\frac{11}{13}$ and $\frac{6}{7}$, we have

$$\frac{11}{13}=\frac{11\times 7}{13\times 7}=\frac{77}{13\times 7},$$

$$\frac{6}{7}=\frac{6\times 13}{7\times 13}=\frac{78}{13\times 7};$$

therefore $\frac{11}{13}$ is $<\frac{6}{7}$.(1)

§ 52. *Proper fractions are increased and improper fractions are diminished by adding the same quantity to both numerator and denominator.*

We may observe in general, that the *higher* a number is, the *less*, relatively to another number, is its increase made by the addition of 1. Thus 2 is *double* of 1; but 3 is *not* double of 2; still less is 4 double of 3; or 100 double of 99. So that by adding an unit, or any number of units to each of two numbers, the increase to the smaller will be more in proportion than the increase to the larger. Hence, if a fraction be *proper*, *i.e.* if its numerator be less than its denominator, by adding the same quantity to both, the increase to the numerator will be more in proportion than the increase of the

(1) Of the signs > and < here used for "greater than" and "less than," it may help the memory to observe that the sign for "less than" is turned the same way as the letter L.

denominator, and the value of the fraction will be *increased;* while conversely, if the fraction be *improper*, the increase to the denominator will be less than the increase to the numerator, and the value of the fraction will be diminished.

Now let us take the proper fractions $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$, where each successive fraction is made by adding 1 to the numerator and denominator of the fraction preceding it. Reducing these to equivalent fractions having the same common denominator, they are respectively equal to $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{48}{60}, \frac{50}{60}$; and as of these fractions the first is the least and the last the greatest, we see that by adding the same quantity to the numerator and denominator of proper fractions, their value is continually increased.

But if the improper fractions $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$ be taken; these are respectively equal to $\frac{120}{60}, \frac{90}{60}, \frac{80}{60}, \frac{75}{60}, \frac{72}{60}$; and as of these fractions the first is the greatest and the last the least, we see that by adding the same quantity to both numerator and denominator of improper fractions, their value is continually diminished.

Whence we conclude that we cannot *add* the same quantity to the numerator and denominator of *any* fraction, without thereby altering its value.

Conversely, proper fractions are diminished and improper fractions increased by *subtracting* the same quantity from both numerator and denominator; whence we conclude that we cannot *subtract* the same quantity from the numerator and denominator of *any* fraction without thereby altering its value.

[*Obs.* It is of great importance to remember from this, in reducing fractions, that although we may *divide* both numerator and denominator by the same quantity, we may *not* take away the same quantity from both numerator and denominator by *subtraction.*]

CHAPTER VII.

THE ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF VULGAR FRACTIONS.

§ 53. The addition of two or more fractions is effected by finding some single fraction which shall express the sum of all the given fractions. It is however impossible to find such a fraction unless all the given fractions be first expressed with *a common denominator:* for, since the denominator of a fraction expresses the number of equal parts into which the unit is divided, it follows that in two fractions which have not a common denominator the unit is *not* divided into the same number of equal parts: therefore in endeavouring to add together two such fractions, for example $\frac{2}{3}$ and $\frac{3}{4}$, so as to express their sum by a single fraction, if we did not first bring them to equivalent fractions with a common denominator we should have to seek for a new denominator which would express that the unit was to be divided into three equal parts *and* four equal parts, while the new numerator must express that two of the three equal parts and three of the four equal parts were to be taken; but no single numbers could express this; and the process could only be represented symbolically thus, $\frac{2}{3}+\frac{3}{4}$: but if we reduce the fractions $\frac{2}{3}$ and $\frac{3}{4}$ to *others of the same value* having a common denominator (§ 50), they become $\frac{8}{12}$ and $\frac{9}{12}$ respectively; and the first fraction is made up of eight of the twelve equal parts into which the unit is now divided, while the second fraction is made up of nine of those parts; the sum of the two fractions must therefore contain eight and nine, or seventeen, of these twelfth parts; therefore $\frac{2}{3}+\frac{3}{4}=\frac{8}{12}+\frac{9}{12}=\frac{17}{12}$.

Hence the Rule for the Addition of Fractions: *Transform the fractions to be added to equivalent fractions having the least common denominator; take the sum of the*

numerators of the transformed fractions for the new numerator, and retain the common denominator as the new denominator.

§ 54. The addition of a whole number and a fraction is effected by writing the whole number as a fraction with 1 for a denominator, and then proceeding as in the ordinary addition of fractions, *e.g.* $7+\frac{5}{6}$ (which is commonly written $7\frac{5}{6}$, the sign of addition being omitted,) is equal to $\frac{7}{1}+\frac{5}{6}=\frac{42}{6}+\frac{5}{6}=\frac{47}{6}$: the shorter form usual in practice is to multiply the whole number by the denominator of the fraction, add the numerator of the fraction to it, write the sum as the numerator of the new fraction with the denominator of the original fraction.

§ 55. *Subtraction of Fractions.*

We may show, by reasoning similar to that used in the addition of fractions, that it is impossible to express by a single fraction the *difference* between two fractions, unless these be first reduced to equivalent fractions having a common denominator. We see then, that if it be required to subtract $\frac{2}{3}$ from $\frac{3}{4}$, the *difference* between $\frac{3}{4}$ and $\frac{2}{3}$ would be obtained by reducing these to equivalent fractions with a common denominator, *i.e.* to $\frac{9}{12}$ and $\frac{8}{12}$; and then the difference will be *one* of the twelve equal parts into which the unit is now in both cases divided.

Thus, $$\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}.$$

Hence the Rule for the Subtraction of Fractions: *Transform the given fractions to equivalent fractions having the least common denominator; take the difference of the numerators for the new numerator, and the common denominator for the new denominator.*

§ 56. *Multiplication of Fractions.*

We have defined multiplication to be an abbreviated method of performing addition; when one of two given quantities is to be taken as many times as there are units in the other.

Now applying this to fractions, to multiply one fraction by another, *e.g.* to multiply $\frac{2}{3}$ by $\frac{4}{5}$, will be to take $\frac{2}{3}$ as many times or parts of a time as there are units or parts of a unit in $\frac{4}{5}$; but as the proper fraction $\frac{4}{5}$ is less than one, this will be to take $\frac{2}{3}$ not so much as once, but four-fifths of once; *i.e.* to find the value of $\frac{4}{5}$ parts of $\frac{2}{3}$. But to take $\frac{4}{5}$ parts of $\frac{2}{3}$ is to divide $\frac{2}{3}$ into 5 equal parts, and to take 4 of those parts. Now the division of $\frac{2}{3}$ into 5 equal parts is effected by multiplying the denominator by 5 (§ 47), also taking 4 of these parts is effected by multiplying the numerator by 4 (§ 46).

Thus, $\frac{2}{3} \div 5 = \frac{2}{15}$,

and $\frac{2}{15} \times 4 = \frac{8}{15}$;

therefore, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Hence the rule for the Multiplication of Fractions, "*Multiply the numerators together for a new numerator,* "*and the denominators together for a new denominator.*"

§ 57. Since the value of a fraction is not altered by dividing both its numerator and denominator by the same quantity (§ 48), we may "cancel" in multiplying fractions together, *i.e.* may strike out of both numerator and denominator any common factor, before we multiply the numerators and denominators together: thus,

$$\frac{9}{10} \times \frac{15}{27} = \frac{135}{270};$$

and this, reduced to its lowest terms (§ 40) is $\frac{1}{2}$; but we might have obtained this result without multiplying 9 and 15, 10 and 27 respectively together, by observing that 9 and 27 are both measured by 9, 10 and 15 both measured by 5; and we may write

$$\frac{9}{10} \times \frac{15}{27} = \frac{9 \times 5 \times 3}{2 \times 5 \times 9 \times 3} = \frac{1}{2}.$$

Again, to find the *continued product* of any number of fractions, as of $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$ (1), instead of multiplying together all the numerators and all the denominators, we may write

$$\frac{1}{\cancel{3}} \times \frac{1}{\cancel{2}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{8}}{9} = \frac{1}{9}.$$

§ 58. When any number or fraction is multiplied by a proper fraction, it is taken so many parts of a time as there are parts of a unit in the proper fraction; this result is still called the product of the two quantities; but whereas in whole numbers the product is made by taking a number a certain number of times, and a whole number is therefore *increased* by being multiplied by any number larger than unity, so on the other hand by multiplying a whole number or a fraction by a proper fraction, the product is *less* than the original multiplicand; and the number or fraction, being only taken some part or parts of once, is *diminished* by being multiplied by a proper fraction: *e.g.* to multiply 2 by $\frac{1}{2}$ is to take 2 one-half of a time; *i.e.* the product is one-half of 2, *i.e.* is 1. Again, to multiply $\frac{1}{2}$ by $\frac{1}{2}$ is to take $\frac{1}{2}$ only one-half of a time; *i.e.* the product is one-half of $\frac{1}{2}$; *i.e.* is $\frac{1}{4}$.

§ 59. *Division of Fractions.*

We have defined division to be the converse of multiplication; where we require to know *how many times* one quantity called the divisor may be subtracted from another called the dividend; the quotient expresses the *number of times* that the subtraction can be performed.

Now to apply this to fractions: so long as the divisor is less than the dividend one fraction may fairly be said to be divided by another: but if the divisor be greater than the dividend, the division cannot be so performed as to give any whole number or improper fraction for a quotient; the result can only be expressed by a proper fraction. For instance

(1) This expression means that of $\frac{8}{9}$ we are to take $\frac{3}{4}$; of that result we are to take $\frac{1}{2}$; and of that result again we are to take $\frac{1}{3}$.

to divide $\frac{2}{3}$ by $\frac{4}{5}$ will be to enquire *how many times* $\frac{4}{5}$ can be subtracted from $\frac{2}{3}$; the quantity expressing *the number of times* the subtraction may be performed will be the *quotient.*

Reducing the given fractions to equivalent fractions having a common denominator, we have $\frac{2}{3}=\frac{10}{15}$, and $\frac{4}{5}=\frac{12}{15}$. Hence we see that we are enquiring how often a *larger* can be subtracted from a *smaller* quantity; and that as $\frac{12}{15}$ cannot be taken any whole number of times from $\frac{10}{15}$, so $\frac{4}{5}$ cannot be taken away any whole number of times from $\frac{2}{3}$: we must therefore enquire what fractional part or parts of a time $\frac{12}{15}$ can be taken from $\frac{10}{15}$; or in other words, what fractional part of $\frac{12}{15}$ is equal to $\frac{10}{15}$; for that is the part which can be taken from $\frac{10}{15}$ exactly, *i.e.* without leaving any remainder. This process is still called division, and the fraction expressing the required fractional part of the divisor is called the quotient.

Now if we divide $\frac{12}{15}$ into 12 equal parts and take 10 of them, we shall obtain ten twelfth parts of $\frac{12}{15}$; and $\frac{10}{12}$ of $\frac{12}{15}$ $=\frac{10}{15}$; therefore if $\frac{10}{12}$ of $\frac{12}{15}$ be taken from $\frac{10}{15}$ there will be no remainder, *i.e.* $\frac{10}{12}$ is the fractional part of $\frac{12}{15}$ which represents the *quotient:* and we may either say that we can subtract $\frac{10}{12}$ of $\frac{12}{15}$ from $\frac{10}{15}$ without leaving any remainder; or that $\frac{10}{12}$ of *unity* represents *the number of times* the required subtraction must be performed[1].

[1] The apparent absurdity of speaking of subtracting a quantity

It is here observable that we obtain the required fraction, viz.: $\frac{10}{12}$, by bringing the dividend and divisor to a common denominator, and then taking the numerator of the dividend for the numerator of the quotient and the numerator of the divisor for the denominator of the quotient. But this result might have been arrived at by reasoning thus:

$$\frac{2}{3} \div \frac{4}{5} \text{ is same as } \frac{10}{15} \div \frac{12}{15}.$$

Now $\frac{12}{15}$ will be contained in $\frac{10}{15}$ as often as 12 is contained in 10. But 12 is not contained in 10 any whole number of times; therefore we must write the result of such division as a fraction, viz., $\frac{10}{12}$; and the quotient of $\frac{2}{3} \div \frac{4}{5}$ is likewise the fraction $\frac{10}{12}$. Here the 10 in the numerator is obtained by multiplying 2, the numerator of the dividend, by 5, the denominator of the divisor; and 12 in the denominator is obtained by multiplying 3, the denominator of the dividend, by 4, the numerator of the divisor: or we see that

$$\frac{2}{3} \div \frac{4}{5} = \frac{2 \times 5}{3 \times 4} = \frac{2}{3} \times \frac{5}{4},$$

whence, without the trouble of bringing the fractions to a common denominator, we can deduce the practical rule for the division of fractions, viz., "*Invert the divisor, and proceed as in multiplication:*" the quotient obtained by this process will always give the number of times, or the fractional part of a time, that the divisor can be subtracted from the dividend without remainder.

The same result may also be arrived at from the following considerations: suppose it is required to divide $\frac{1}{2}$ by $\frac{1}{4}$; here the question asked is, "How often can $\frac{1}{4}$ be subtracted from $\frac{1}{2}$, so as to leave no remainder?" this is the same thing

only a part of a time arises from extending the application of the term 'division' to fractions at all.

as asking "How often must $\frac{1}{4}$ be added to itself to produce $\frac{1}{2}$?" the number of *additions* in the latter being the same as the number of *subtractions* in the former case.

Now $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$; *i.e. twice* $\frac{1}{4}=\frac{1}{2}$: therefore the number of times that $\frac{1}{4}$ can be subtracted from $\frac{1}{2}$ will also be *two*, but $\frac{1}{2}\times\frac{4}{1}=2$. Here again we obtain the required quotient by multiplying the dividend by the divisor inverted.

This illustration depends upon the self-evident consideration that the number of times the divisor must be added to itself to produce the dividend is the *same* as the number of times that the divisor can be subtracted from the dividend so as to leave no remainder.

§ 60. When any whole number or fraction is divided by a proper fraction, the number of times or the fractional part of a time that the divisor can be subtracted from the dividend is called the quotient. But whereas in the division of whole numbers the quotient is always *less* than the dividend, on the other hand in the division of fractions, whenever the divisor is a *proper* fraction, the quotient will be *greater* than the dividend: *e.g.* $2\div\frac{1}{4}=2\times\frac{4}{1}=8$; that is, $\frac{1}{4}$ may be subtracted exactly 8 times from 2. Again, $\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$, a fraction which is *greater* than the dividend $\frac{2}{3}$.

§ 61. Some examples in the above rules are now given, worked out at length to exhibit the processes employed.

Ex. 1. Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{15}$.

The denominators 4, 6, 8, 10, 15 are to be decomposed into their prime factors, and the *highest powers* of all the factors that occur are to be multiplied together to form the L.C.M.

Hence, by inspection, we say $2\times2\times3\times2\times5$ is the L.C.M.

(For $4=2\times2$; $6=2\times3$; $8=2\times2\times2$; $10=2\times5$; $15=3\times5$).

Then $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{9}{10}+\frac{11}{15}=\frac{90+100+105+108+88}{2\times2\times3\times2\times5}$

$$=\frac{491}{120}=4\frac{11}{120}.$$

Ex. 2. Add together $15\frac{5}{7}$, $6\frac{1}{4}$, $\frac{2\frac{3}{4}}{14}$.

Here $$\frac{2\frac{3}{4}}{14}=\frac{11}{4}\times\frac{1}{14}=\frac{11}{56},$$

and adding together the whole numbers separately, the expression becomes $15+6+\frac{5}{7}+\frac{1}{4}+\frac{11}{56}$.

Now $$\frac{5}{7}+\frac{1}{4}+\frac{11}{56}=\frac{40+14+11}{56}=\frac{65}{56}=1\frac{9}{56};$$

therefore $$15+6+1\frac{9}{56}=22\frac{9}{56}.$$

Ex. 3. Reduce the expression

$$\frac{\frac{1}{2}+\frac{1}{3}\text{ of }\frac{1}{4}+\frac{5}{6}}{\frac{1}{13}\text{ of }\left(1+5\frac{1}{2}\right)+\frac{5}{6}\text{ of }\frac{1}{23}\text{ of }\left(7-2\frac{2}{5}\right)-\frac{1}{3}}.$$

Observe that when two quantities, connected by the sign of multiplication, are combined with others by the sign + or −, these quantities must be first multiplied together, and the result then added to or subtracted from the other quantities; for instance, the numerator of the given fraction means that to $\frac{1}{2}$ is to be added the *product* of $\frac{1}{3}$ and $\frac{1}{4}$, and to this again $\frac{5}{6}$ is to be added; *not* that the sum of $\frac{1}{2}+\frac{1}{3}$ is to be multiplied by the sum of $\frac{1}{4}+\frac{5}{6}$; for had this been meant, the expression would have been written $\left(\frac{1}{2}+\frac{1}{3}\right)$ of $\left(\frac{1}{4}+\frac{5}{6}\right)$, brackets being used, as they are in the denominator of the given fraction.

We have therefore

$$\frac{\frac{1}{2}+\frac{1}{3}\text{ of }\frac{1}{4}+\frac{5}{6}}{\frac{1}{13}\text{ of}\left(1+5\frac{1}{2}\right)+\frac{5}{6}\text{ of }\frac{1}{23}\left(7-2\frac{2}{5}\right)-\frac{1}{3}}=\frac{\frac{1}{2}+\frac{1}{12}+\frac{5}{6}}{\frac{1}{13}\times 6\frac{1}{2}+\frac{5}{6}\times\frac{1}{23}\times 4\frac{3}{5}-\frac{1}{3}}$$

$$=\frac{\frac{6+1+10}{12}}{\frac{1}{13}\times\frac{13}{2}+\frac{5}{6}\times\frac{1}{23}\times\frac{23}{5}-\frac{1}{3}}$$

$$=\frac{\frac{17}{12}}{\frac{1}{2}+\frac{1}{6}-\frac{1}{3}}=\frac{\frac{17}{12}}{\frac{2}{6}}=\frac{17}{12}\times\frac{3}{1}=4\frac{1}{4}.$$

Ex. 4. Find the value of $\left\{1\frac{3}{8}+\frac{5}{4}\text{ of }\frac{7}{3\frac{4}{5}}-\frac{\frac{5}{6}}{2\frac{1}{2}}\right\}\div 1\frac{77}{228}$.

$$\text{The expression}=\left\{\frac{11}{8}+\frac{5}{4}\times\frac{7}{1}\times\frac{5}{19}-\frac{5}{6}\times\frac{2}{5}\right\}\div\frac{305}{228}$$

$$=\left\{\frac{11}{8}+\frac{175}{76}-\frac{1}{3}\right\}\times\frac{228}{305}$$

$$=\frac{627+1050-152}{456}\times\frac{228}{305}$$

$$=\frac{1525}{456}\times\frac{228}{305}$$

$$=\frac{5}{2}$$

$$=2\frac{1}{2}.$$

Exercise VI.

Addition of Fractions.

1. Add together $\frac{5}{8}$, $\frac{2}{5}$, $\frac{7}{12}$.
2. Add $\frac{5}{7}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{11}{21}$, $\frac{19}{42}$.
3. Required the sum of $\frac{7}{12}$, $\frac{3}{8}$, and $\frac{5}{14}$.

4. What is the sum of $\frac{1}{5}+\frac{5}{21}+\frac{3}{70}+\frac{7}{45}+\frac{5}{18}+\frac{2}{63}$?

5. Add together $7\frac{3}{10}$, $11\frac{7}{20}$, $3\frac{5}{7}$, $\frac{7}{8}$, and $\frac{17}{168}$.

6. Add together $\frac{5}{7}$ of $\frac{4}{15}$ of $3\frac{1}{2}$, $\frac{3}{5\frac{1}{2}}$, and $\frac{5\frac{3}{8}}{17\frac{1}{5}}$.

Subtraction of Fractions.

1. Subtract $\frac{5}{17}$ from $\frac{39}{51}$.

2. Subtract $2\frac{3}{7}$ from $5\frac{1}{18}$.

3. From $\frac{2}{3}$ of $5\frac{1}{7}$ of $1\frac{2}{5}$ take $\frac{3\frac{1}{2}}{\frac{3}{4}}$.

4. Find the difference between $\frac{13\frac{2}{3}}{16\frac{2}{5}}$ and

$$\frac{\frac{4}{7}\text{ of }\frac{3}{11}\text{ of }3\frac{1}{2}}{\frac{8}{9}\text{ of }\frac{3}{22}\text{ of }12}.$$

5. By how much does $\frac{1}{4}+\frac{3\frac{1}{7}}{5}+2$ exceed $\frac{22}{35}$ of $3\frac{1}{4}$ of $\frac{11}{26}$?

Multiplication of Fractions.

1. Multiply $\frac{4}{17}$ by 5, then by $\frac{5}{3}$, then by $\frac{\frac{5}{3}}{7}$, then by $\frac{5}{\frac{3}{7}}$.

2. Multiply $\frac{5\frac{3}{8}}{7\frac{1}{9}}$ by $\frac{1}{2\frac{41}{44}}$.

3. What fraction multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $3\frac{1}{2}$ gives $\frac{7}{9}$ as the result?

4. Required the product of $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$, and $\dfrac{2\frac{1}{3}}{3\frac{1}{2}}$.

5. What is the continued product of $\frac{3}{4}+\frac{4}{5}+\frac{7}{10}$, $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{10}$, $\dfrac{\frac{7}{12}+\frac{5}{9}}{\frac{7}{12}-\frac{5}{9}}$, and $\dfrac{1}{17\frac{2}{11}}$?

Division of Fractions.

1. Divide $\frac{3}{5}$ by 9, then by $\frac{7}{9}$, then by $\dfrac{\frac{7}{9}}{11}$, then by $\dfrac{\frac{7}{9}}{11}$.

2. Divide $3\frac{2}{7}$ of $13\frac{1}{8}$ by the sum of $13\frac{2}{7}+3\frac{1}{8}$.

3. Find the sum of the quotients of $\frac{8}{9}$ divided by $1\frac{1}{18}$, and $1\frac{1}{18}$ divided by $\frac{8}{9}$.

4. Compare the quotients of $\dfrac{5\frac{2}{3}}{3\frac{7}{9}}$ divided by $13\frac{1}{7}$ of $\frac{5}{8}$ of $\frac{4}{15}$ of $\frac{3}{5}$, and of $1\frac{1}{2}$ of $\dfrac{\frac{2}{3}}{5\frac{1}{2}}$ divided by $\frac{2}{3}$ of $\dfrac{5\frac{1}{2}}{1\frac{1}{2}}$ of $\frac{6}{143}$.

MISCELLANEOUS EXAMPLES.

1. Define a fraction; and bring to their lowest terms the fractions $\frac{969}{8721}$ and $\frac{65536}{5030912}$.

2. Add together $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{4}$ of $1\frac{1}{2}$; and find whether $\frac{2}{5}$ of $\frac{7}{9}$ is greater or less than $\frac{3}{8}$ of $\frac{5}{6}$.

3. Multiply the sum of $\frac{4}{7}$ of $\frac{1}{2}$, and $1\frac{1}{9}$ by $1\frac{3}{4}$ of the difference between $\frac{4}{11}$ and $\frac{1}{5}$.

4. Shew that $\frac{2+4+6}{3+5+7}$ lies between the greatest and the least of the fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$.

5. Divide $\frac{2+3}{4+5}$ by $\dfrac{4+3\frac{1}{2}}{5+5\frac{1}{2}}$.

6. What fraction multiplied into the sum of $\frac{3}{8}$, $1\frac{13}{24}$ and $\frac{17}{36}$ will make the product 3?

7. Explain the rule for the multiplication of fractions.

Multiply and divide $\frac{2}{3}+\frac{1}{5}$ by $1-\frac{1}{13}$: find which of these results is the greater, and express their difference in its lowest terms.

8. Reduce $\left(\dfrac{3\frac{1}{3}}{7}+\dfrac{2}{10\frac{1}{2}}-\frac{5}{18} \text{ of } \frac{4}{7}\right)\div\frac{4}{7}$.

9. Find the simple fraction equivalent to $\frac{1}{2}\cdot\dfrac{\frac{2}{3}-\frac{1}{2}}{2}\cdot\dfrac{\frac{3}{4}-\frac{2}{3}}{3}$.

10. Simplify $\dfrac{2\frac{4}{5}-1\frac{1}{2}+9\frac{1}{11}}{4\frac{1}{5}-2\frac{1}{4}+13\frac{7}{11}}$.

11. Obtain the value of three-sevenths of $\dfrac{\dfrac{\frac{1}{5}+\dfrac{3\frac{1}{5}}{7}}{2\frac{1}{3}}}{\dfrac{3}{4\frac{2}{7}}}$.

12. Find the value of $\dfrac{1}{2+\dfrac{2}{3+\dfrac{4}{5}}}$.

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13. Add together $\frac{1}{7}$, $\frac{2}{9}$, and $\frac{1}{3}$ of $\frac{2}{7}$; and explain the process.

14. Add together $3\frac{1}{2}$, $4\frac{1}{3}$, $5\frac{1}{4}$, $\frac{3}{4}$ of $\frac{7}{8}$, and $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{8}$.

15. Shew that the value of a fraction is not altered by multiplying both numerator and denominator by the same number. Is the value of a fraction altered by *adding* the same quantity to both numerator and denominator? Express the fractions $\frac{6}{45}$, $\frac{5}{7}$ and $\frac{9}{35}$ by corresponding fractions which have the same denominator, and find their sum.

16. Add together the fractions $\frac{10}{14}$, $\frac{2}{15}$, and $\frac{18}{70}$.

17. Reduce to their simplest forms $\frac{7}{8}-\frac{5}{6}$; and $\frac{6}{7}-\frac{4}{5}$.

18. Prove that the sum of the fractions $1\frac{1}{26}$ and $\dfrac{1}{1\frac{4}{9}}$ is equal to 5 times their difference.

19. Find the simple fraction which is equal to the difference of $\frac{1}{3}$ of $3\frac{7}{8}$ and $\frac{1}{4}$ of $5\frac{1}{4}$.

20. Reduce the expression $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}+\frac{1}{5}$ of $\frac{2}{3}$ of $1\frac{7}{8}+\frac{1}{4}$.

21. Determine, in its lowest terms, the continued product of $\frac{32}{51}$, $\frac{85}{112}$, $\frac{189}{207}$, and $\frac{23}{36}$.

22. Find the value of $\dfrac{1\frac{3}{4}-\frac{7}{6}\text{ of }\frac{18}{28}}{\frac{5}{6}\text{ of }\frac{12}{20}+5\frac{1}{2}}\div\frac{1}{6}$.

23. What is the exact value of

$$\left\{2\frac{3}{4}+\frac{5}{2} \text{ of } \frac{7}{3\frac{4}{5}}+\frac{\frac{4}{9}}{\frac{1}{3}}\right\} \div 4\frac{85}{228}?$$

24. Find the difference between $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\dfrac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}$ and

$$\frac{\frac{1}{4}-\frac{1}{6}}{\frac{1}{4}+\frac{1}{6}}-\frac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}.$$

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25. Reduce to their lowest terms the fractions $\frac{494}{221}$ and $\frac{1573}{689}$.

26. Reduce $\frac{2}{3}$ of $\frac{5}{7}+\frac{3}{5}$ of $\frac{4}{9}$ to a simple fraction.

27. Find the sum of $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{5}{6}$: and divide the result by $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{21}{9}$.

28. Explain why it is necessary before adding fractions to bring them to equivalent fractions having a common denominator.

Add together $\frac{4}{9}+\frac{3}{2}+\frac{5}{3}+\frac{7}{18}$.

29. What number added to the sum of $\frac{5}{8}$, $1\frac{17}{24}$ and $\frac{13}{36}$ will make the sum total equal to 3?

30. Add together $\frac{2}{5}$, $\frac{35}{80}$, $\frac{14}{100}$, $\frac{3}{140}$, and $\frac{3}{2800}$.

31. Reduce to simpler forms $\dfrac{17\frac{1}{3}}{73}$ and $\dfrac{7\frac{3}{7}}{40\frac{5}{9}}$; and find the quotient of the latter divided by the former.

32. Explain the rule for the division of fractions; divide the sum of $\frac{6}{7}$, $\frac{5}{3}$, $\frac{9}{14}$, $\frac{11}{6}$ by the difference between $\frac{4}{5}$ and $\frac{7}{9}$.

33. Add together $2\frac{1}{3000}$, $1\frac{5}{1600}$, $5\frac{1}{6000}$, and $2\frac{7}{8000}$.

34. Reduce $\dfrac{\left(\frac{1}{2}+1\frac{3}{7}+\frac{5}{6}\right)\times\left(\frac{4}{15}-\frac{3}{20}\right)}{\frac{11}{18}\text{ of }1\frac{14}{15}}$.

35. Find the simple fraction equivalent to

$$\frac{1}{2}\cdot\frac{\frac{1}{2}-\frac{1}{3}}{2}\cdot\frac{\frac{1}{2}-\frac{1}{4}}{3}\cdot\frac{\frac{1}{2}-\frac{1}{5}}{4}\cdot\frac{\frac{1}{2}-\frac{1}{6}}{5}.$$

36. Find the value of $\dfrac{\frac{3}{5}\text{ of }\frac{2}{9}-\frac{4}{7}\text{ of }\frac{1}{8}}{\frac{4}{15}\text{ of }\frac{1}{28}+\frac{3}{10}\text{ of }\frac{7}{9}}$.

§ 62. *To find the value of a given fraction of any concrete quantity.*

It is only here necessary to multiply the given fraction by that number which in whole numbers would reduce the denomination in which the fraction stands to the next lowest denomination: *e.g.* find the value of $\frac{2}{3}$ of £1.

$$\frac{2}{3}\text{ of £1 is }\frac{2}{3}\text{ of }20s.;$$

therefore $$\frac{2}{3}\times 20=\frac{40}{3}=13\tfrac{1}{3}s.,$$

and $$\frac{1}{3}\text{ of 1 shilling is }\frac{1}{3}\text{ of 12 pence;}$$

therefore $$\frac{1}{3}\times 12=4d.;$$

therefore 13*s.* „ 4*d.* is the required value.

§ 63. *To reduce a given quantity or a fraction of a given quantity to the fraction of another given quantity.*

In order to render the division of one concrete quantity by another concrete quantity possible, it is necessary that both

should be in the *same denomination*: Therefore bring the proposed quantities into the *same* (not necessarily the *lowest*) denomination; and then divide the quantity that is to be reduced *by* that to a fraction of which it is to be brought; *e.g.* Reduce 16*s.* 5*d.* to the fraction of £1.

Here 16*s.* 5*d.* = 197 pence, and £1 = 240 pence. Therefore the 197 pence in 16*s.* 5*d.* are to be divided by the 240 pence in £1: or $\frac{197}{240}$ is the required fraction of £1. The reason for this is as follows: Since £1 contains 240 pence, and 16*s.* 5*d.* contains 197 pence, if the pound be divided into 240 equal parts and 197 of them be taken, these 197 parts will be represented by 16*s.* 5*d.*; but the fraction $£\frac{197}{240}$ represents that the pound has been divided into 240 equal parts and 197 of them taken; therefore

$$16s.\ 5d. = £\frac{197}{240}.$$

Ex. 1. Find the value of $£\frac{3}{7}$.

$$\frac{3}{7} \text{ of } 20s. = \frac{60}{7}s. = 8\frac{4}{7}s.;$$

$$\frac{4}{7} \text{ of } 12d. = \frac{48}{7}d. = 6\frac{6}{7}d.;$$

therefore $$£\frac{3}{7} = 8s.\ 6\frac{6}{7}d.$$

Now to reverse this, *i.e.* to bring 8*s.* $6\frac{6}{7}d.$ to fraction of £1,

$$\text{say } 6\frac{6}{7}d. = \frac{48}{7} \text{ of } \frac{1}{12}s. = \frac{4}{7}s.;$$

$$8\frac{4}{7}s. = \frac{60}{7} \text{ of } £\frac{1}{20} = £\frac{3}{7}.$$

Ex. 2. What is the difference between $\frac{5}{12}$ of £1 and $\frac{5}{14}$ of a guinea?

$$\frac{5}{12} \text{ of } £1 = \frac{5}{\cancel{12}_{3}} \times \cancel{20}^{5} = \frac{25}{3}s. = 8s.\ ,,\ 4d.$$

$$\frac{5}{14} \text{ of a guinea} = \frac{5}{\cancel{14}_{2}} \times \cancel{21}^{3} = \frac{15}{2}s. = 7s.\ ,,\ 6d.$$

Therefore difference is $\quad 10d.$

Ex. 3. Find the amount of

$$\frac{1}{3} \text{ of } £1 + \frac{2}{7} \text{ of a guinea} - \frac{4}{9} \text{ of } 15s.$$

$$\frac{1}{3} \text{ of } 20 = 6s. \text{ „ } 8d.$$

$$\frac{2}{7} \text{ of } 21 = 6s.$$

Sum of these is 12*s.* „ 8*d.*

Subtract $\quad \frac{4}{\underset{3}{\cancel{9}}}$ of $\overset{5}{\cancel{15}}$, or $\frac{20}{3}s.$, or 6*s.* „ 8*d.*

The required amount is 6*s.*

Ex. 4. How many shillings should be given in exchange for $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}$ of a pound?

$$\frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}} = \frac{\frac{7}{12}}{\frac{5}{6}} = \frac{7}{\underset{2}{\cancel{12}}} \times \frac{\cancel{6}}{5} = £\frac{7}{10},$$

and $\qquad \frac{7}{10}$ of $20s. = 14s.$

Ex. 5. Out of £$4\frac{3}{8}$ one-third is paid to *A* and one-seventh to *B*; after this $\frac{4}{11}$ths of the remainder is paid to *A*, and the rest to *B*; find the sums respectively received by *A* and *B*.

When $\frac{1}{3}$ had been paid to *A*, and $\frac{1}{7}$ to *B*, there had been paid altogether $\frac{10}{21}$, and the fraction remaining was $\frac{11}{21}$.

$$A \text{ now has } \frac{4}{11} \text{ of } \frac{11}{21}, \text{ or } \frac{4}{21},$$

$$B \text{ has the remaining } \frac{7}{21}, \text{ or } \frac{1}{3}.$$

$$\text{In all } A \text{ has } \frac{1}{3} + \frac{4}{21}, \text{ or } \frac{11}{21}.$$

$$B \text{ has } \frac{1}{7} + \frac{1}{3}, \text{ or } \frac{10}{21}.$$

But $\quad \frac{11}{21}$ of $4\frac{3}{8} = \frac{11}{21} \times \frac{35}{8} = £2.$ „ $5s.$ „ $10d.$,

$$\text{and } \frac{10}{21} \text{ of } 4\frac{3}{8} = \frac{10}{21} \times \frac{35}{8} = £2. \text{ „ } 1s. \text{ „ } 8d.$$

Ex. 6. From $\frac{1}{2}$ of $\frac{1}{3}$ of a penny subtract $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of a shilling.

$$\left(\frac{1}{2} \times \frac{1}{3}d.\right) - \left(\frac{1}{4} \times \frac{1}{5} \times \frac{1}{\cancel{6}} \times \overset{2}{\cancel{12}}d.\right)$$

$$= \frac{1}{6} - \frac{1}{10} = \frac{5-3}{30} = \frac{2}{30} = \frac{1}{15}d.$$

Ex. 7. Reduce 3 qrs. 14 lbs. to the fraction of a ton.

$$3 \text{ qrs. } 14 \text{ lbs.} = 3\frac{1}{2} \text{ qrs.} = \frac{7}{2} \text{ qrs.},$$

and a ton contains 20×4 qrs.;

therefore $\quad \frac{7}{2} \times \frac{1}{20} \times \frac{1}{4} = \frac{7}{160}$ ton.

Ex. 8. Reduce $18s.$ „ $8d.$ to the fraction of half-a-crown.

$$18s. \text{ „ } 8d. = 18\frac{2}{3}s. = \frac{56}{3}s.,$$

$$\text{and half-a-crown} = \frac{5}{2}s.;$$

therefore $\quad \frac{56}{3} \div \frac{5}{2} = \frac{56}{3} \times \frac{2}{5} = \frac{112}{15}$, or $7\frac{7}{15}$ half-crowns.

Ex. 9. Reduce $\frac{3}{8}$ of $5s.$ to the fraction of $17s.$ „ $6d.$

$$\frac{3}{8} \text{ of } 5s. = \frac{15}{8}s.$$

$$\text{and } 17s. \text{ „ } 6d. = \frac{35}{2}s.;$$

therefore $\frac{15}{8} \times \frac{2}{35} = \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$, the required fraction of $17s.$ „ $6d.$

Ex. 10. Express 3 weeks, 4 days, 6 hours, as the fraction of a year of $365\frac{1}{4}$ days.

$$3 \text{ weeks, } 4 \text{ days, } 6 \text{ hours} = 25\frac{1}{4} \text{ days} = \frac{101}{4} \text{ days};$$

$$\text{and the year consists of } \frac{1461}{4} \text{ days;}$$

therefore $$\frac{101}{4} \times \frac{4}{1461} = \frac{101}{1461} \text{ of a year.}$$

Ex. 11. What fraction of £12 „ 7*s.* „ 6*d.* is $\frac{11}{17}$ of £3 „ 3*s.* „ 9*d.*?

In other words, reduce $\frac{11}{17}$ of £3 „ 3*s.* „ 9*d.* to the fraction of £12 „ 7*s.* „ 6*d.*

$$\frac{11}{17} \text{ of £3 „ 3s. „ 9d.} = \frac{11}{17} \times £3\frac{3}{16} = £\frac{11}{17} \times \frac{51}{16};$$

$$\text{and £12 „ 7s. „ 6d.} = £12\frac{3}{8} = £\frac{99}{8};$$

therefore $\frac{11}{17} \times \frac{51}{16} \times \frac{8}{99} = \frac{1 \times 3 \times 1}{1 \times 2 \times 9} = \frac{1}{6}$, the required fraction.

Ex. 12. Bring £3 „ 12*s.* „ 6*d.* to the fraction of £14 „ 3*s.* „ 4*d.*

$$\frac{3\frac{5}{8}}{14\frac{1}{6}} = \frac{\frac{29}{8}}{\frac{85}{6}} = \frac{29}{8} \times \frac{6}{85} = \frac{87}{340}.$$

Ex. 13. Compare the values of $\frac{1}{96}$ of a pound, $\frac{7}{36}$ of a shilling, and $\frac{1}{112}$ of a guinea.

$$\frac{1}{96} \text{ of a pound is } \frac{1}{96} \text{ of } 20s. = \frac{5}{24}s.$$

$$\frac{1}{112} \text{ of a guinea is } \frac{1}{112} \text{ of } 21s. = \frac{3}{16}s.$$

Therefore reducing to a common denominator the fractions $\frac{5}{24}s.$, $\frac{7}{36}s.$, $\frac{3}{16}s.$, they become respectively $\frac{30}{144}s.$, $\frac{28}{144}s.$, $\frac{27}{144}s.$, and the comparative values of the given fractions are as 30, 28, 27.

Exercise VII.

1. What fraction of a pound is 19*s.* „ 11¼*d.*? Give the reasons for the method employed.

2. If 26 francs are equivalent to a pound, what fraction of a shilling is a franc?

3. Find the value of the sum of the following fractions:

$$£\left(\frac{1}{2}+\frac{3}{4}\right),\ \left(\frac{1}{3}+\frac{3}{5}\right)s.,\ \left(\frac{1}{4}+\frac{4}{5}\right)d.$$

4. What fraction of 13*s*. „ $3\frac{3}{4}d.$ is 17*s*. „ 9*d.*?

5. From $\frac{5}{8}$ of a pound sterling take $\frac{1}{3}$ of $\frac{7}{8}$ of a shilling.

6. What fraction of 7 weeks is $\frac{2}{5}$ of a day?

7. Reduce $\frac{1}{8}$ of 7*s*. to the fraction of a crown.

8. Add together

$$\frac{3}{7} \text{ of } £15,\ \frac{1}{4} \text{ of } \frac{1}{2\frac{4}{5}} \text{ of } \frac{4}{5} \text{ of } £1.\ „\ 12s., \text{ and } \frac{4}{7} \text{ of } 3d.$$

9. What fractional part of £1. „ 6*s*. „ 3*d*. is 15*s*. „ 9*d.*?

10. Reduce 24 days, 2 hours, 8 minutes, to the fraction of a month of 30 days.

11. What fraction of 15*s*. „ $7\frac{1}{4}d.$ is 2*s*. „ $2\frac{3}{4}d.$?

12. What fractional part of three guineas is half-a-crown? and how much is $\frac{3}{20}$ of a day?

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13. Add together $\frac{1}{5}$ of a shilling, $\frac{2}{7}$ of a crown, $\frac{4}{9}$ of a guinea.

14. Reduce 20 feet, $7\frac{1}{2}$ inches to the fraction of a mile.

15. What part of £20 is half-a-guinea? and how much is $\frac{15}{16}$ of a cwt.?

16. Compare the values of $\frac{31}{40}$ of a pound, $\frac{61}{84}$ of a guinea, and $3\times4\frac{1}{2}$ shillings.

17. What fractions of a pound are $\frac{3}{14}$ of a penny, and $\frac{15}{56}$ of a guinea respectively?

18. Subtract $\frac{1}{7}$ of $3s.\ ,,\ 2\frac{1}{2}d.$ from $\frac{7}{5}$ of $\frac{3}{4}$ of a crown.

19. Find the value of the sum of $\frac{3}{5}$ of $\frac{7}{4}$ of £1 and $\frac{1}{3}$ of $\frac{5}{2}$ of $7s.\ ,,\ 6d.$

20. What is the value of $\dfrac{2}{3+\dfrac{3}{5+\dfrac{1}{4}}}$ of £2. „ 10s.?

21. What is the value of $\dfrac{2+\dfrac{1}{3}}{3+\dfrac{1}{2}}$ of £?

22. Compare the values of $\frac{3}{32}$ of £21, $\frac{23}{24}$ of a florin, and $\frac{47}{504}$ of a guinea.

23. What fractional part of a pound is

$$1\frac{1}{3}s. + \left\{\frac{3}{4} \text{ of } \frac{8}{5} \times \frac{2}{9} \div \frac{4}{15}\right\} \text{ of a shilling?}$$

24. What is the value of $\frac{4}{7}$ of $\frac{1}{3}$ of $\dfrac{13\frac{4}{5}}{3\frac{2}{7}}$ of a ton?

---

25. What fraction of a mile is 27 yards, 1 foot, 6 inches?

26. Bring $\frac{6}{7}$ of an ounce troy to the fraction of a lb. troy; also to the fraction of a lb. avoirdupois.

27. Bring $\frac{7}{21}$ of a day to the fraction of a week, and find the value of $\frac{11}{80}$ of an hour.

28. Find the value of $\dfrac{\frac{1}{3} \text{ of } \frac{2}{3} + \frac{1}{4} \text{ of } \frac{5}{6}}{\frac{1}{3}+\frac{3}{7}-\frac{8}{9}+\frac{71}{63}}$ of $\dfrac{6\frac{15}{16}}{4\frac{5}{8}}$ of $\frac{1}{3}$ of a square foot.

29. Reduce $\dfrac{\frac{1}{6} \text{ of } 1\frac{1}{5} - \frac{1}{5} \text{ of } \frac{2}{3}}{\frac{1}{2} - \frac{2}{3} + \frac{4}{9} - \frac{1}{18}} \div \frac{3}{4} \text{ of } \frac{4}{5} \text{ of } \frac{6}{8}$ of a rod to the fraction of a furlong.

30. What fraction of $\frac{1}{10}$ of a quarter is $\dfrac{3\frac{3}{4} - 2\frac{5}{9}}{\frac{1}{12} + \frac{7}{9} \text{ of } \frac{1}{4}} \div 3\frac{4}{13}$ of a peck?

31. Bring $\left(\dfrac{5\frac{1}{4} - \frac{1}{3} \text{ of } 2\frac{5}{6}}{\frac{3}{5} \times 4\frac{1}{6} + \frac{1}{12}} + \dfrac{2\frac{3}{7}}{4\frac{3}{7}}\right) \div 21\frac{28}{29} \text{ of } 3\frac{19}{206}$ cwt. to the fraction of $4\frac{1}{7}$ ton.

32. Bring $2\frac{39}{80}$ ton to the fraction of a quarter; and $\frac{5}{1496}$ of a mile to the fraction of a yard.

33. Find the value of $\frac{3}{29}$ of $\dfrac{\frac{1}{2}}{1 + \dfrac{\frac{1}{3}}{1 + \dfrac{\frac{1}{4}}{1 + \frac{1}{5}}}}$ of a square foot.

34. What fraction of a mile represents the same length as $\frac{3}{4}$ of an inch?

35. Bring 10 dwt. 23 grs. Troy to the fraction of an oz. Avoirdupois.

36. Express 7 oz. 3 drams Avoirdupois as a fraction of 1 lb. Troy.

CHAPTER VIII.

DECIMALS.

§ 64. From the law of notation, we see that in the ordinary decimal scale the value of any digit *decreases* in a tenfold degree for each place that it advances towards the right hand. Thus in the number 222 the 2 in the place of tens, which represents 20, is a tenth of the 2 in the place of hundreds, which represents 200; while the 2 in the place of units is a tenth of the 2 in the place of tens.

Now if we assume that this law shall hold good for positions to the *right* hand of the units place, we shall have the great advantage of being able to deal with fractions of a certain kind in precisely the same manner that we deal with whole numbers. These fractions will be tenths, hundredths, thousandths, &c.: that is, *they will be fractions which must always have ten, or some power of ten for their denominators.* Such fractions are called *Decimals.* We shall in this manner have a decimal scale of notation extended below unity, thus:

thousands	hundreds	tens	unit	tenths	hundredths	thousandths
4	3	2	1	2	3	4

It will only be necessary in writing whole numbers together with fractions of this peculiar kind, to mark clearly which figure is meant to stand in the place of *units*, and then the principle of *local value* will determine the relative magnitude of each of the figures standing to the right of the units place. It is customary to put a full stop, (a comma is sometimes used,) after the figure in the units place. This is called

the decimal point, and indicates that while all the figures to the left of it are ordinary whole numbers, all the figures to the right of it are decimal fractions. If there be no whole numbers, yet a decimal point may be written and figures may follow it, and thus decimal fractions may be expressed either with or without whole numbers. Also every figure standing on the right of a decimal point is called a decimal place.

Thus 32·3 means 3 *tens* 2 *units* and 3 *tenths*,
·45 means 4 *tenths* and 5 *hundredths*,
·006 means 6 *thousandths*.

[Observe that while in whole numbers 100 stands for one hundred, in decimals ·001 represents *one-thousandth*.]

The advantage derived from this simple developement of the law regulating the local value of digits is very great, for by it we are able to deal with the most minute fractions with as much ease as we can with whole numbers.

§ 65. Since we have explained that a decimal such as ·56 means 5 tenths and 6 hundredths, it will follow that ·560 means 5 tenths 6 hundredths and *no* thousandths; where the addition of the cipher to the right hand has made no alteration in the value of the decimal.

In fact $$\cdot 56 = \frac{56}{100}$$

and $$\cdot 560 = \frac{560}{1000} = \frac{56}{100};$$

from which we see that by adding a cipher to the right of a *decimal* fraction, we only multiply both numerator *and* denominator by 10, and consequently do not alter the value of the decimal at all. Whence we deduce that *the addition of any number of ciphers to the right hand of a decimal does not in any way alter its value.*

But if we place a cipher *before* the other figures of a decimal, and instead of ·56 write ·056, we see that by this we alter the *position* and therefore alter the *value* of every successive figure; that the *tenths* have become *hundredths*, and the *hundredths* have become *thousandths;* and that the value of the decimal has been decreased ten-fold.

So that, exactly contrary to what happens in whole numbers, the addition of ciphers to the right does *not* alter the

value of a decimal; the addition of ciphers to the left *does* alter the value by decreasing the value of the decimal ten-fold for every cipher added.

§ 66. We infer from this that as the value of a decimal is *decreased* ten-fold for every cipher added to the left hand, we do in fact *divide* a decimal by 10, by 100, by 1000, &c., as we shift the decimal point *one, two, three*, &c. places to the *left*; and that conversely by shifting the decimal point *one, two, three*, &c. places to the *right*, we *multiply* the decimal by 10, by 100, by 1000, &c. For instance, the expression 56·789 is divided by 10 if written 5·6789, is divided by 100 if written ·56789, and is divided by 1000 if written ·056789; whereas the expression ·007023 is multiplied by 10 if written ·07023, is multiplied by 100 if written ·7023, and is multiplied by 1000 if written 7·023.

§ 67. Although it is true that by means of decimals we can only express fractions which have ten or some power of ten for their denominator, yet we shall see, as we proceed, that we can in every case either reduce any given vulgar fraction to a decimal fraction exactly equivalent to it, or can at least find a decimal fraction which shall approximate to the given vulgar fraction so closely, as to differ from it by less than any given quantity.

This will be more fully explained when we come to the conversion of vulgar fractions into decimals, and to repeating or circulating decimals; we will now only explain that when any vulgar fraction can be *exactly* expressed by a decimal, that decimal is called *terminate* or *finite*; whereas, when it cannot be exactly so expressed, the decimal is called *interminate* or *infinite*.

To express any finite decimal as a vulgar fraction.

Since ·456 means 4 tenths, 5 hundredths, and 6 thousandths, we see that

$$\cdot 456 = \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$$

$$= \frac{400 + 50 + 6}{1000}$$

$$= \frac{456}{1000}.$$

Similarly ·007009 means 7 thousandths and 9 millionths;

hence,
$$\cdot007009 = \frac{7}{1000} + \frac{9}{1000000}$$
$$= \frac{7000 + 9}{1000000}$$
$$= \frac{7009}{1000000}.$$

Hence any finite decimal may be at once expressed as a vulgar fraction *by writing the given decimal as a whole number* (*i.e.* writing it without the decimal point.) *for the numerator of the vulgar fraction; and writing for the denominator* 1 *followed by as many ciphers as there are decimal places in the given decimal.*

Again, since 37·89 means 37 integers together with 8 tenths and 9 hundredths,

$$37\cdot89 = 37 + \frac{8}{10} + \frac{9}{100}$$
$$= 37 + \frac{89}{100}$$
$$= \frac{3789}{100}.$$

Whence we see that an expression consisting of whole numbers followed by decimals may be expressed, in a precisely similar manner, as a vulgar fraction.

§ 68. To read off, or express in words decimal fractions, read the decimal figures as if whole numbers, and to the last figure add the name of its order, determined by the place it occupies: thus

·734 is read *seven hundred and thirty-four* thousandths;

58·64327 is read *fifty-eight, together with sixty-four thousand three hundred and twenty-seven* hundred-thousandths;

·080905 is read *eighty thousand nine hundred and five* millionths.

Obs. Much confusion would be avoided by beginners if they would bear in mind that *decimals* are *fractions*, although fractions of a peculiar kind; and that whereas in vulgar fractions the denominator may be *any number what-*

ever, (because a vulgar fraction is explained to arise from the division of unity into *any* number of equal parts,) and consequently it is necessary in every case to write the denominator at full length, in finite decimal fractions on the other hand we can at once read off the denominator by inspection, and therefore we are not obliged to write it at length. Still, in all operations into which decimals enter, it must be remembered that we are only dealing with fractions with suppressed denominators.

Obs. The peculiar advantage of employing decimal fractions arises from this, that as such fractions are expressed by an extension of the ordinary denary scale of notation, the addition, subtraction, multiplication and division of such fractions will be performed by processes the same as in ordinary whole numbers, with only additional rules for placing the decimal points in the results. Again, we can at once *compare* such fractions, *i.e.* can tell which is the largest and which the least with the same ease as in whole numbers, since there is no difficulty in the reduction of decimals to a common denominator.

Exercise VIII.

1. Define a decimal fraction; and explain how the principle of *local value* may be extended to find the value of such fractions.

2. Explain the *advantages* of decimal fractions.

3. Express in words the following decimals and mixed numbers:

·283, ·5321, ’74895, ’821056, 27,8354, 34,0009, 43’101007.

4. In the following mixed numbers write the fractional part in decimals:

$$53\frac{9}{10};\quad 47\frac{73}{100};\quad 6\frac{69}{10000};\quad 1\frac{1}{1000000};\quad 3\frac{7000}{10000};$$

$$35\frac{721341}{1000000};\quad 9\frac{400537}{1000000000}.$$

5. Express as vulgar fractions ·7; ·07; ·007; ·000007; ’327; 3’27; 32’7; ·45697; 456·97; ·893; ·0000893.

6. Express as decimal fractions the following: seventy-three *thousandths;* one hundred and ninety-seven *ten thou-*

sandths ; one *millionth* ; two hundred and sixty-one *hundred thousandths* ; one thousand and one *ten millionths*.

7. Express as vulgar fractions in their lowest terms: ·5; ·25; ·75; ·125; ·05; ·025; ·2; ·002; ·375; ·0635; ·005005; 47·256.

8. Multiply ·379 successively by 10, by 100, by 1000.

9. Divide ·0703 successively by 10, by 1000, by 10000.

10. Show that the value of a decimal fraction is not altered by the addition of ciphers to the right hand.

11. Express as vulgar fractions in their lowest terms, ·365; ·125; ·0035; ·012; 1·75; 8·3625.

12. Multiply each of the quantities ·0007453, 48·95621, and 8·76430071 successively by one thousand, by ten thousand, and by one hundred thousand; and divide each of 531·674, ·000317 and 9020·30401 successively by one million, by ten million, and by one hundred million.

CHAPTER IX.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF DECIMALS.

§ 69. Decimals, or integers and decimals mixed, may be added together precisely as in whole numbers, care being taken so to arrange the figures that all the decimal points fall exactly under one another. This will ensure that *tenths* fall under *tenths*, *hundredths* under *hundredths*, &c. The reason of this arrangement will appear from the following consideration: if this rule were *not* observed, tenths would fall under hundredths, or hundredths under thousandths, as the case might be; and we should be attempting to add together fractions which had not common denominators. But if we arrange the decimal points all exactly beneath one another, tenths fall under tenths, hundredths under hundredths, &c.; in other words, by so arranging them we at once bring the several fractions to a common denominator, and can proceed to add them together. The decimal point, in the answer, will fall exactly beneath the decimal points in the quantities to be

added. When the sum of any figures exceeds 10, 20, &c., *carrying* to the next denomination will be performed exactly as in whole numbers, whether the given quantities are all decimals or are mixed integers and decimals. For as the value of each figure decreases ten-fold as we proceed from left to right, the rules of ordinary addition are immediately applicable.

For instance, let it be required to add together the following quantities ·5, ·06, ·007; also ·8, ·78, ·678; also 3·007, 42·6, 5·3975: arranging these severally with the decimal points beneath one another, we have

$$\begin{array}{r} \cdot 5 \\ \cdot 06 \\ \cdot 007 \\ \hline \cdot 567 \end{array}$$

where it is obvious that the sum of 5 *tenths*, 6 *hundredths* and 7 *thousandths*, must be expressed as ·567;

in the next instance,

$$\begin{array}{r} \cdot 8 \\ \cdot 78 \\ \cdot 678 \\ \hline 2\cdot 258 \end{array}$$

we see, after writing in the answer 8 in the place of *thousandths*, that 7 *hundredths* and 8 *hundredths* added together make 15 *hundredths;* but 15 *hundredths* are 1 *tenth* and 5 *hundredths;* writing 5 in the place of *hundredths*, and *carrying* one to the place of *tenths*, we obtain 22 *tenths;* but 22 tenths are properly written as 2 *integers* and 2 *tenths.*

Again, where integers and decimals are mixed,

$$\begin{array}{r} 3\cdot 007 \\ 42\cdot 6 \\ \cdot 3975 \\ \hline 46\cdot 0045 \end{array}$$

writing 5 in the place of *ten thousandths*, the sum of 7 *thousandths* and 7 *thousandths* is 14 *thousandths;* writing 4 in the place of *thousandths*, and carrying 1 to the place of *hundredths*, we obtain 10 as the sum in the *hundredths'* place; but 10 *hundredths* are 1 *tenth;* carrying 1 to the place of *tenths*, we have 10 *tenths;* but as 10 *tenths* are one *unit*, we carry 1 to the place of integers, and write 6 in the place of *units*, and 4 in the place of tens.

We might show the correctness of these results by writing the given decimals as vulgar fractions, and finding their sum in each instance by the rules of addition in vulgar fractions; *e.g.*

$$
\begin{aligned}
\cdot 5+\cdot 06+\cdot 007 &= \frac{5}{10}+\frac{6}{100}+\frac{7}{1000}\\
&= \frac{500}{1000}+\frac{60}{1000}+\frac{7}{1000}\\
&= \frac{567}{1000}\\
&= \cdot 567
\end{aligned}
$$

$$
\begin{aligned}
\cdot 8+\cdot 78+\cdot 678 &= \frac{8}{10}+\frac{78}{100}+\frac{678}{1000}\\
&= \frac{800}{1000}+\frac{780}{1000}+\frac{678}{1000}\\
&= \frac{2258}{1000}\\
&= 2\cdot 258
\end{aligned}
$$

$$
\begin{aligned}
3\cdot 007+42\cdot 6+\cdot 3975 &= \frac{3007}{1000}+\frac{426}{10}+\frac{3975}{10000}\\
&= \frac{30070+426000+3975}{10000}\\
&= \frac{460045}{10000}\\
&= 46\cdot 0045
\end{aligned}
$$

§ 70. In subtraction of decimals, or of integers and decimals mixed, for reasons precisely similar the decimal points must be arranged to fall exactly beneath one another; and then the smaller quantity can be subtracted from the larger in the same manner as in whole numbers, *thousandths* being taken from *thousandths*, *hundreths* from *hundredths*, *tenths* from *tenths*. The decimal point in the answer will fall exactly beneath the decimal points in tho subtrahend and minuend. If the number of figures in the subtrahend should exceed the number in the minuend, ciphers may be added (or supposed to be added) to the *right* of the decimal figures in the minuend, as this will not alter the value (§ 66), and the subtraction may proceed as in whole numbers.

For example, let it be required to subtract ·756 from ·897; and ·8765 from ·93; and ·907 from 37·6; arranging these with the decimal points beneath one another, and subtracting as in whole numbers, we have

$$\begin{array}{r} \cdot 897 \\ \underline{\cdot 756} \\ \cdot 141 \end{array}$$

where the difference between 7 *thousandths* and 6 *thousandths* is 1 *thousandth*, between 9 *hundredths* and 5 *hundredths* is 4 *hundredths*, between 8 *tenths* and 7 *tenths* is 1 *tenth*.

Again, writing ·93 as ·9300, and subtracting as in whole numbers, we have

$$\begin{array}{r} \cdot 9300 \\ \underline{\cdot 8765} \\ \cdot 0535 \end{array}$$

Also, in the third instance, 37·6 may be written 37·600, and we have

$$\begin{array}{r} 37\cdot 600 \\ \underline{\cdot 907} \\ 36\cdot 693 \end{array}$$

These results may also be proved by vulgar fractions as follows:

$$\begin{aligned} \cdot 897 - \cdot 756 &= \frac{897}{1000} - \frac{756}{1000} \\ &= \frac{141}{1000} \\ &= \cdot 141 \end{aligned}$$

$$\begin{aligned} \cdot 93 - \cdot 8765 &= \frac{93}{100} - \frac{8765}{10000} \\ &= \frac{9300 - 8765}{10000} \\ &= \frac{535}{10000} \\ &= \cdot 0535 \end{aligned}$$

$$\begin{aligned} 37\cdot 6 - \cdot 907 &= \frac{376}{10} - \frac{907}{1000} \\ &= \frac{37600 - 907}{1000} \\ &= \frac{36693}{1000} \\ &= 36\cdot 693 \end{aligned}$$

§ 71. *Multiplication of Decimals.*

We have stated that for every place we shift the decimal point to the *right*, we increase the value of the decimal *ten-fold;* for every place we shift it to the *left*, we decrease the value *ten-fold*. Now in multiplying two decimals together, since the law of *local value* holds with regard to the digits composing the decimals, the process of multiplication will be performed exactly as in ordinary whole numbers; the only matter requiring consideration will be the proper position of the decimal point in the product.

Let it be required to multiply 18·56 by 1·932.

If we shift the decimal point to the *right* in the multiplicand *two* places, and in the multiplier *three* places, so that both become whole numbers, we shall thereby increase the multiplicand 100-fold, and the multiplier 1000-fold.

Hence the product we shall obtain will be 100000-fold too great.

Therefore in this product we must mark off *five* decimal places, or shift the decimal point *five* places back to the *left;* this will divide the product by 100000, and give the correct result.

But if it be required to multiply ·1856 by ·01932, by shifting the decimal point to the *right* in the multiplicand *four* places, and in the multiplier *five* places, we shall increase the multiplicand *ten thousand-fold,* and the multiplier an *hundred thousand-fold*, and shall obtain a product a *thousand million* times too great. We must therefore divide that product by 1000000000, or must shift the decimal point *nine* places to the *left*, in order to obtain the correct result.

Hence we deduce the following practical rule for the multiplication of decimals: *Multiply the decimals together as in whole numbers; and point off in the product as many decimal places as there are in the multiplier and multiplicand together; prefixing ciphers, if necessary, to the left of the product.*

The process will stand as follows:

```
   18·56
   1·932
   -----
    3712
   5568
 16704
 1856
--------
35·85792
```

and in the second instance

$$\begin{array}{r} \cdot 1856 \\ \cdot 01932 \\ \hline 3712 \\ 5568 \\ 16704 \\ 1856 \\ \hline \cdot 003585792. \end{array}$$

The correctness of these results may be proved by vulgar fractions; for writing 18·56 as $\frac{1856}{100}$ and 1·932 as $\frac{1932}{1000}$, and multiplying these vulgar fractions together, we have

$$18{\cdot}56 \times 1{\cdot}932 = \frac{1856}{100} \times \frac{1932}{1000}$$
$$= \frac{3585792}{100000}$$
$$= 35{\cdot}85792.$$

Again $$\cdot 1856 \times \cdot 01932 = \frac{1856}{10000} \times \frac{1932}{100000}$$
$$= \frac{3585792}{1000000000}$$
$$= \cdot 003585792.$$

§ 72. *Division of Decimals.*

Since by multiplying or dividing both the numerator and denominator of a fraction by the same number, we do not alter its value (§ 48), it follows that by shifting the decimal point the same number of places to the right or the left in both Dividend and Divisor (i.e. by multiplying or dividing both by 10, by 100, &c. as may be required), we shall not alter the value of the Quotient.

Now in the Division of Decimals, an operation in which mistakes are more frequently made than in any other part of arithmetic, the essential requisite is to determine correctly the local value of the first figure in the quotient. We shall most easily do this if we shift the decimal point till it stands on the right of the extreme left-hand significant figure of the Divisor, shifting likewise the decimal point in the Dividend the same number of places. We thus shall have the divisor always in the form of a whole number of one figure, followed

usually by some decimal places: and the local value of the first figure in the quotient will be the same as that of the first, or of the second significant digit (whether integral or decimal) on the left hand of the altered dividend: it will be of the value of the *first* significant digit when all the figures of the divisor can be contained in an equal number of the figures of the dividend; it will be of the value of the *second* significant digit, when the figures of the divisor cannot be contained in an equal number of figures in the dividend, but require that an additional figure should be used.

To illustrate this rule, take the following examples:

1. ·736644 ÷ 234·6.

Shifting the decimal point two places to the left in both dividend and divisor, we have

·00736644 ÷ 2·346.

And as the first four significant figures in the dividend are divisible by the four figures of the divisor, the first figure of the quotient will be of the same local value as the 7, *i.e.* will be in the place of *thousandths*. The operation will stand as follows:

2·346) ·00736644 (·00314
3284
9384
0000

2. 302·85 ÷ ·000673.

Shifting the decimal point *four* places to the right in both cases, we have

3028500 ÷ 6·73;

and as the first three figures of the dividend are *not* divisible by the three figures of the divisor, the first figure of the quotient will be of the same local value as the *second* figure of the dividend, i. e. will be in the place of *hundreds of thousands;* the operation will be

6·73) 3028500 (450000
3365
0000

3. 34809·6 ÷ 940·8.

Shifting the decimal place by the rule, we have

348·096 ÷ 9·408;

whence the first figure in the quotient will be in the place of *tens:* the operation will be

$$9{\cdot}408)\ 348{\cdot}096\ (37$$
$$65\,856$$
$$00\,000^*$$

In each case the accuracy of the result may be tested by Vulgar Fractions as follows:

$$(1)\quad \cdot736644 \div 234{\cdot}6 = \frac{736644}{1000000} \div \frac{2346}{10}$$
$$= \frac{736644}{1000000} \times \frac{10}{2346}$$
$$= \frac{314}{100000}$$
$$= \cdot00314.$$

$$(2)\quad 302{\cdot}85 \div \cdot000673 = \frac{30285}{100} \div \frac{673}{1000000}$$
$$= \frac{30285}{100} \times \frac{1000000}{673}$$
$$= 45 \times 10000$$
$$= 450000.$$

* The advantage of the Rule given above is that by it the computer determines the value of each figure in the quotient from the very commencement of his work, and therefore knows how far to carry it to obtain sufficient accuracy. In *contracted division* it will be found most important to determine in this manner the value of the first figure of the quotient *at once;* and in ordinary division the rule is neater than those commonly given. A form now often taught is as follows: "*Remove the decimal point to the end of the divisor: "remove the decimal point in the dividend as many places to the right "as it has been moved in the divisor: in the quotient, insert the deci- "mal point when the decimal point in the dividend is reached in the "course of the division.*" With a little thought the principle upon which the process depends may be reasoned out from this; but it is open to the objection of not determining the position of the decimal point in the quotient *at once*, and consequently of not being so useful in contracted division. Sometimes this rule is modified a little, and stated in a slightly different form as follows: "*Count as many places "after the decimal point in the dividend as there are decimal places "in the divisor, adding ciphers if necessary; put a mark after them; "and as soon as all those figures are brought down, put in your deci- "mal point in the answer.*" Thus stated it becomes a perfect "cram" rule, and as such will perhaps commend itself to a certain class of teachers; nevertheless by it the reason of the process is quite obscured, while the operation itself is decidedly clumsy.

(3) $34809{\cdot}6 \div 940{\cdot}8 = \dfrac{348096}{10} \div \dfrac{9408}{10}$

$$= \frac{348096}{9408}$$

$$= 37.$$

§ 73. It sometimes happens that the division does not terminate, and we have to add ciphers to the right of the dividend. This addition of ciphers to the *right* does not alter the value of a decimal (§ 66); we can therefore proceed with the division as if the number of ciphers were without end; and can carry on the operation either till there is no remainder (when the division terminates), or till the remainder is a fraction so small as to be inconsiderable. In practice, unless greater accuracy should be specially required, it is seldom necessary to obtain more than 4, or at most 5, decimal places in the quotient. The following examples will illustrate the method of adding ciphers:

(4) $\cdot0020925 \div \cdot000864$.

8·64) 20·92500000 (2·421875
3645
1890
1620
7560
6480
4320
0000

(5) $1 \div \cdot0197$.

1·97) 100· (50·7614 &c.
1500
1210
280
830
42

To find the real value of the remainder at any step, we notice at once that in this example it is impossible that the remainder should be 42 in whole numbers; and that the only reason for which it might perhaps *appear* to be so is, because,

after establishing the decimal point in the quotient, we have been working as if with ordinary whole numbers, without further reference to the position of the decimal point in the various remainders. Moreover it is to be observed that by shifting the decimal place in divisor and dividend we have *not* indeed altered the value of the quotient, but we *have* altered the position of the decimal point in the line from which the remainder at first arises, and have therefore increased or diminished the remainder by as many places as we have shifted the decimal point to the right or to the left. The position of the decimal point in the first remainder will be established by considering what is the real value of the subtrahend; in the above example 1·97 × 50 gave the product 98·50, which was the subtrahend to be taken from 100·0; and the value of the remainder was 1·50; this being determined, we can retain the decimal point in its proper position in each of the subsequent remainders: but in the last remainder we must shift the decimal point back again as many places right or left, as it was originally shifted left or right in the dividend. To illustrate this, example (5) and an additional example shall be exhibited *with the decimal point retained throughout*, in order to determine the value of the remainder:

```
(5)        1·97) 100·0  (50·7614 &c.
                 1·500
                  ·1210
                  ·00280
                  ·000830
                  ·000042
```

And as the decimal place in the dividend was shifted two places to the *right*, it must be shifted back two places to the *left* in the remainder; which thus becomes ·00000042.

(6) 8·5 ÷ 2354·7.

```
2·3547)  ·0085      (·003609
         ·00143590
         ·0000230800
         ·0000018877
```

And as the decimal point in the dividend was shifted three places to the *left*, it must be shifted back three places to the *right* in the remainder, which thus becomes ·0018877.

The value of the remainder, expressed, as in an ordinary example in long division, by a vulgar fraction, would be ·000042 ÷ 1·97 (which is the same thing as ·00000042 ÷ ·0197); whence the vulgar fraction would be $\frac{42}{1000000} \times \frac{100}{197}$, or $\frac{42}{1970000}$, or $\frac{21}{985000}$. Similarly in Ex. 6 the vulgar fraction representing the value of the remainder would be $\frac{18877}{10000000} \times \frac{10}{23547}$, or $\frac{18877}{23547000000}$.

§ 74. In the multiplication and division of decimals, when the number of decimal places given is large, and yet accuracy not required beyond 4 or 5 decimal places in the answer, the labour of extended multiplication and division may be avoided by a *contraction* of the ordinary process. Accordingly we shall here set the contracted form side by side with the ordinary process, and then explain the method of performing the operation.

Let it be required to multiply ·456798 by ·23456 correctly to *five* decimal places in the answer.

Contracted Form.	*Ordinary Process.*
·456798	·456798
·23456	·23456
·09136	2740788
1370	2283990
183	1827192
23	1370394
2	913596
·10714	·10714653888

To explain the contracted method, observe that it makes no difference whether we multiply by the extreme *left* hand or the extreme *right* hand figure of the multiplier first, provided we establish the decimal point in its right place, and keep the other rows of multiplication in their proper order. We will therefore commence multiplying by the 2, the extreme *left* hand figure of the multiplier. Next, since multiplying *four* decimal figures by *one* decimal figure will give us *five* decimal places in the result, and it is only required to obtain five places in the answer, it will suffice in this case to begin multiplying the 2 into the 7, which is the *fourth* figure of the multiplicand. We must however *carry* from the product of the rejected figures; (always carrying to the first figure a-

down in each row of multiplication as many *units* as are equal to the *nearest number of tens* derived from the multiplication of the last two rejected figures of the multiplicand, (*i.e.* carrying 1 for any number from 5 to 14; carrying 2 for any number from 15 to 24, and so on: where we reckon 5 as nearer to 10 than to 0; 15 as nearer to 20 than to 10; 25 as nearer to 30 than to 20, &c.). Hence the process will be,—twice 8 is 16; twice 9 is 18, and 1 is 19; but 19 is nearer to 20 than to 10, therefore carry 2; then say twice 7 is 14, and two, 16; set down 6, and finish the line of multiplication in the ordinary way. To establish the decimal point, observe that as four decimal figures have been multiplied by one decimal figure, there must be 5 decimal places in the result; therefore add a cipher and prefix the decimal point. For the next row multiply by 3, rejecting this time 7 also from the multiplicand, but carrying 2, as the number of units equal to the nearest number of tens derived from the multiplication of the rejected 9 and 7; 3 times 6, eighteen, and two, 20; place the 0 under the last figure in the upper row of multiplication, and finish the line in the ordinary way. Rejecting every time one figure from the multiplicand, in the next row multiply the 4 into the 5, carrying 3; (for 4 times 7 is 28; 4 times 6, twenty-four, and 2, twenty-six; and 26 is nearer to 30 than to 20; so carry 3). In the next row for a similar reason 3 has likewise to be carried; and multiplying 5 into 4 and carrying 3, we obtain 23. In the last row 6 times 4 is 24, of which the 2 only is set down. These rows added together as they stand will give the required product correct to 5 decimal places.

As however it is not always easy to fix the decimal point correctly in the first line when multiplying thus by the figures in inverted order, the process may be effected and exhibited in a somewhat different form as follows. Shift the decimal point in the *multiplier* so many places to the right hand that it becomes a whole number; and shift the decimal point in the *multiplicand* the same number of places to the left hand: the product will not be altered by this, because while the multiplier has been multiplied, the multiplicand has been divided by the same number. In the example given the multiplier will by this process become 23456, and the multiplicand ·00000456798. Now when the multiplicand is multiplied by the figure standing in the place of units in the multiplier, the product will always contain the same number of decimal places that there are in the multiplicand. Hence if the first

figure required in the product be the 5th place of decimals, the figure 6 standing in the units' place in the multiplier need only be multiplied into the 5th place of decimals in the multiplicand; the figure 5 in the tens' place need only be multiplied into the 6th place of decimals in the multiplicand; the figure 4 in the hundreds' place, into the 7th place of decimals; and so on. Therefore place the units figure of the multiplier under that place of decimals in the multiplicand which is the last figure to be retained in the product, (in this case place the 6 under the 5th place of decimals,) and write the other figures of the multiplicand in an *inverted order* to the right hand of it: by this means each figure in the multiplier will stand under that figure in the multiplicand into which it must be multiplied in order to produce the first figure that is to be retained in the product. Now multiply the several figures of the multiplier, beginning with the right hand figure, each into the figure of the multiplicand immediately above it, but *carrying* from the preceding figures as above directed; set down the several results so that the first figures to be retained in them are in the same vertical column, and add in the ordinary way. The operation would stand as follows:

```
·00000456798
      65432
-----------
     ·09136
       1370
        183
         23
          2
     ------
     ·10714
```

Ex. 2. Multiply ·007853 by ·00476 correctly to *seven* places of decimals.

Shifting the decimal points, and proceeding as above directed, the process will stand thus;

```
·00000007853
        674
-----------
   ·0000314
         55
          4
   --------
   ·0000373
```

Ex. 3. Multiply ·0057913 by 3796·8 correctly to *four* places of decimals.

·00057913
86973
173739
40539
5212
347
·0046
21·9883

The method of performing contracted division may be best seen from the following examples:

Ex. 1. Let it be required to divide 2·569141797 by 7·5284 correctly to *five* places of decimals:

Common Method.	*Contracted Method.*
7·5284) 2·569141797 (·34126	7·5284) 2·569141797 (·34126
310621	31062
94857	948
195739	196
451717	46
13	1

In the contracted form, after the first figure in the quotient has been found in the usual manner, and the first reminder obtained, instead of bringing down the next figure, cut off from the divisor the extreme right hand figure, and divide by the remaining figures. At each successive step in the division cut off another figure from the right hand of the divisor, and continue the division with the remaining figures. It is necessary however to *carry* from the rejected figure, in the same manner as in contracted multiplication.

It will be observed in this process that by thus cutting off a figure at each step from the decimal divisor, we do not alter the relative value of the figures which are left. Hence it is allowable to reject these superfluous figures, and only employ just so many as will produce in the quotient the required number of decimal places.

Ex. 2. Let it be required to divide 3 by ·643528 correctly to *five* places of decimals.

Shifting the decimal point in divisor and dividend, according to the rule given in ordinary division, whereby the position of the decimal point in the quotient is at once correctly determined, we observe that in this case, as there are *more* figures in the divisor than are required in the quotient, we may at once cut off the 8, the extreme right hand figure of the divisor, *carrying* however from the multiplication of 4 into 8, and saying 4 times 8 is 32, carry 3; 4 times 2 is 8, and 3 is 11, &c.

Common Method.	*Contracted Method.*
6·43528) 30·00000 (4·6618	6·4352,8) 30·0000 (4·6618
4 258880	42589
3977120	3978
1169520	117
5259920	53
111696	2

Ex. 3. Let it be required to divide ·197241937 by ·254 correctly to *five* places of decimals.

In this case we shall not be able to use the contracted form at first, as there are *fewer* figures in the divisor than are required in the quotient. We must therefore perform the first three rows of division in the ordinary way, or we should not obtain 5 figures in the quotient: after that, instead of bringing down more figures, we can proceed in the last two rows by the contracted method:

```
2·54) 1·97241937 (·77654
        1944
         1661
          137
           10
            0
```

Exercise IX.

1. Add together the following decimals:

1. ·0103, ·205, ·36997, ·008.
2. 2·63, ·263, ·0263, ·000263.
3. 516·3, 36·51, 1·563, ·03561.
4. ·01, 3·001, 0·1, ·30103.

5. 594·7261, ·03, 8316·02, 193, ·0003795, 7·2493167.
6. ·005, 8·7, 597·8576, ·6, 1327, ·00001.
7. 27·00382, ·002, ·01041, 53·201, 37·0028, 2913·083.
8. ·0031, 59·4, ·36, 68, 1·007, 729·3917, 950·938.
9. 56952·135, 551·1345, 22·01, ·00023, 99, 921·75, 78·247.
10. 7·2493167, 193·000816, 59·270031, 1·7253, 217·580273, 5378·2176, 67231·45.

2. Subtract

1. 3·07 from 6·501
2. 2·9989 from 3.
3. ·0090806 from 39·857.
4. ·876534 from 1·21314.
5. 56·5376 from 65·49.
6. ·000387 from 7.
7. 31·49723 from 31·4975
8. 1869·5713954 from 1869·5714.
9. 566·54322 from 597.
10. 81·99 from 173·47873.

3. Multiply together the following, proving the results by vulgar fractions:

1. ·0027 by ·014.
2. 32·56 by ·00457.
3. ·764 by 3·56.
4. ·0089 by ·652.
5. ·305687 by ·03024.
6. ·007853 by ·00476.
7. 35·0645 by 281·315.
8. 5·76305 by 101·746.
9. 317·243 by ·00295.
10. 286 by ·0000831.
11. ·3854172 by 571000.
12. 2579·357 by 725·3864.

4. Divide, proving in each case the accuracy of the result by vulgar fractions:

1. 17·084592 by ·024.
2. 1237·0519 by ·5425.
3. 762·151 by ·00325.
4. 56·25 by ·0045.
5. ·019 by 190.
6. 1·95 by ·00013.
7. ·03679 by 2·83.
8. 165·434 by 36·2.
9. ·027472 by 3·434.
10. 17·171717 by 343·4.
11. 57·856077 by ·823.
12. 6·1848924 by ·09316.
13. 5·83122295 by 9·53375.
14. 76·391955 by ·0000920385.
15. 46·91999995675 by 5353·10895.
16. ·8728 by 87·28.
17. 486·1349989925 by 165916·38225.
18. 31494·0704 by 1321·06.
19. 124·59993 by 3194·87.
20. 37·492127554 by ·04051451.
21. 3 by ·876 to 3 places of decimals.
22. 7 by 796·3 to 5 places of decimals.

5. Find by contracted multiplication the product of:

(1) ·01245 by ·825 correct to *six* places of decimals.
(2) 37·06205 by ·34005 correct to *five* places of decimals.
(3) 33·166248 by 1·4142136 correct to *five* places of decimals.
(4) ·27056 by ·37025 correct to *six* places of decimals.
(5) 3·1729432 by 8·316259 correct to *four* places of decimals.

6. Find by contracted division the quotient, correct to *five* places of decimals of:

(1) 1·6866591 by ·4471618.
(2) 85·643825 by 6·321.
(3) 6001·58373 by 1732·508.
(4) 7·2117562 by 2·257432.
(5) 573429·13 by 813·764.
(6) 31·47 by 839·27656.

CHAPTER X.

REDUCTION OF DECIMALS.

§ 75. *To explain how to reduce a vulgar fraction to a decimal, without altering its value.*

Since a decimal fraction must have 10 or some power of 10 for a denominator, if we take some fraction not a decimal, *e.g.* $\frac{9}{32}$, and endeavour to convert it into a decimal, we shall have to find means of altering its denominator into 10, 100, 1000, or some other power of 10.

Now if we multiply the numerator and denominator of the given fraction by 10, by 100, by 1000, &c., we shall obtain a series of fractions, viz. $\frac{90}{320}$, $\frac{900}{3200}$, $\frac{9000}{32000}$, &c.; each of which will be equal to $\frac{9}{32}$; but each of whose denominators is exactly divisible by 32, with quotient 10, 100, 1000, &c. If therefore any one of the *numerators* 90, 900, 9000, &c., be exactly divisible by 32, we can convert that fraction whose numerator and denominator are both exactly divisible by 32, into a fraction having some power of ten for its denominator, *i.e.* into a *decimal fraction*. We must now try by actual division which is the first of the numerators 90, 900, 9000, &c., which can be divided by 32 without remainder

```
32) 90 (2        32) 900 (28        32) 9000 (281
    26                260                 260
                        4                  40
                                            8

32) 90000 (2812           32) 900000 (28125
    260                        260
     40                         40
      80                         80
      16                         160
                                 000
```

We find upon the fifth trial that 900000 is divisible by 32 without remainder. Whence we have

$$\frac{9}{32}=\frac{900000}{3200000}=\frac{28125}{100000}=\cdot 28125.$$

Now in practice we need not write down all these trial divisions separately; for the last case contains all that went before. It will therefore be sufficient to divide the numerator by the denominator, fixing the decimal point as in ordinary division, and proceeding as far as we please in the operation as if the number of ciphers were unlimited: for instance, if it be required to reduce $\frac{7}{16}$ to a decimal fraction we might proceed thus;

```
1·6) ·70000 (·4375
       60
       120
        80
        00
```

Obs. The above simple examples will suffice to illustrate the principle on which the process of converting a vulgar into a decimal fraction depends. And when the numerator and denominator of the given vulgar fraction contain a good many figures, it will generally be found best to perform the operation as above; *e.g.* it will be seen, by dividing the numerator by the denominator according to the ordinary rule of division, that $\frac{5016}{78375}=\cdot 064$, and that $\frac{735}{89612}=\cdot 008202$ approximately. We need not however always employ long division, if in practice we can discover any easier method of finding an equivalent fraction whose denominator is some power of ten: for instance, when the denominator is some power of 5, we may use multiplication instead of division, and may say

$$\frac{3}{25}=\frac{12}{100}=\cdot 12;\quad \frac{11}{125}=\frac{88}{1000}=\cdot 088;$$

$$\frac{376}{15625}=\frac{3008}{125000}=\frac{24064}{1000000}=\cdot 024064.$$

Or we may perform the division by successive steps, dividing the denominator by some convenient factor, and then dividing

the numerator decimally by the same: *e.g.* in the examples we first took, we might have said

$$\frac{9}{32}=\frac{2{\cdot}25}{8}={\cdot}28125,$$

$$\frac{7}{16}=\frac{1{\cdot}75}{4}={\cdot}4375.$$

§ 76. When a vulgar fraction can be exactly expressed as a decimal, the result is said to be a *terminating* decimal; but it is not every vulgar fraction that can be reduced to a terminating decimal. For if the denominator of the given vulgar fraction when reduced to its lowest terms should have other prime factors than 2 or 5, then that vulgar fraction cannot be *exactly* expressed as a decimal.

The reason for this is as follows: 2 and 5 are the prime factors of 10; *i.e.* are the only numbers that divide 10 without remainder; and by annexing ciphers to the numerator, we multiply it each time successively by 10. Now any number that measures another, must also measure its product into any whole number (§ 37). Hence if the prime factors of the denominator be 2 or 5, they will measure 10, and therefore measure 10 multiplied into the whole number which is the numerator; but if the prime factors be not 2 or 5, they will not measure the numerator multiplied by 10, and the division will not terminate.

Hence we can ascertain whether a vulgar fraction can be expressed exactly as a decimal by the following rule: *reduce the given vulgar fraction to its lowest terms, and resolve its denominator into its prime factors: if those prime factors be only* 2 *and* 5, *it can be expressed by a terminating decimal; otherwise, it cannot.*

Hence after any vulgar fraction has been reduced to its lowest terms, if it be expressed as a decimal which is *terminate*, the *number* of figures which that decimal contains must be equal to the *greatest number* of times that either of the prime factors 2 or 5 is repeated in the denominator: for 10 must be repeated as many times as a factor in the numerator as 2 or 5 occurs as a factor in the denominator, in order to reduce the vulgar fraction to a decimal: *e.g.*

$$\frac{1}{4}=\frac{1}{2\times 2}=\frac{10\times 10}{2\times 2\times 100}=\frac{25}{100}=\cdot 25.$$

$$\frac{1}{125}=\frac{1}{5\times 5\times 5}=\frac{10\times 10\times 10}{5\times 5\times 5\times 1000}=\frac{8}{1000}=\cdot 008.$$

RECURRING DECIMALS.

§ 77. It is one of the *disadvantages* of decimal fractions that when attempting to reduce a vulgar fraction to an equivalent decimal fraction, we may sometimes obtain an *interminate* result.

The difference however between the given vulgar fraction and the resulting interminate decimal may be rendered *less* than any number we please to name; and thus by continuing the process far enough, any required degree of accuracy may be obtained.

We may explain this more at length as follows:

We have seen that if the numerator, when multiplied by a sufficiently high power of ten, be exactly divisible by the denominator, the given vulgar fraction can be transformed into an exactly equivalent decimal fraction. But if the numerator cannot be multiplied by any power of ten so as to be exactly divisible by the denominator, a somewhat different process will show that we can nevertheless obtain a decimal fraction as *nearly as possible* equivalent to the given vulgar fraction: and this process will serve to illustrate the first case also.

Taking first the same example as before, viz. $\frac{7}{16}$ we have

$$\frac{7}{16}=\frac{1}{10}\text{ of }\frac{70}{16}=\frac{1}{10}\left(4+\frac{6}{16}\right)$$

$$=\frac{4}{10}+\frac{1}{10}\text{ of }\frac{6}{16}=\frac{4}{10}+\frac{1}{10}\text{ of }\frac{1}{10}\text{ of }\frac{60}{16}$$

$$=\frac{4}{10}+\frac{1}{100}\left(3+\frac{12}{16}\right)=\frac{4}{10}+\frac{3}{100}+\frac{1}{100}\text{ of }\frac{12}{16}$$

$$= \frac{4}{10} + \frac{3}{100} + \frac{1}{100} \text{ of } \frac{1}{10} \text{ of } \frac{120}{16}$$

$$= \frac{4}{10} + \frac{3}{100} + \frac{1}{1000}\left(7 + \frac{8}{16}\right)$$

$$= \frac{4}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{1000} \text{ of } \frac{8}{16}$$

$$= \frac{4}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{1000} \text{ of } \frac{1}{10} \text{ of } \frac{80}{16}$$

$$= \frac{4}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{10000} \text{ of } 5$$

$$= \frac{4}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{5}{10000}$$

$$= \cdot 4375.$$

Next $$\frac{1}{7} = \frac{1}{10} \text{ of } \frac{10}{7} = \frac{1}{10}\left(1 + \frac{3}{7}\right)$$

$$= \frac{1}{10} + \frac{1}{10} \text{ of } \frac{3}{7} = \frac{1}{10} + \frac{1}{100} \text{ of } \frac{30}{7}$$

$$= \frac{1}{10} + \frac{1}{100}\left(4 + \frac{2}{7}\right)$$

$$= \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} \text{ of } \frac{20}{7}$$

$$= \frac{1}{10} + \frac{4}{100} + \frac{2}{1000} + \frac{1}{10000} \text{ of } \frac{60}{7}$$

$$= \frac{1}{10} + \frac{4}{100} + \frac{2}{1000} + \frac{8}{10000} + \frac{1}{10000} \text{ of } \frac{4}{7}$$

$$= \cdot 1428 + \frac{1}{10000} \text{ of } \frac{4}{7}$$

and since $\frac{1}{10000}$ of $\frac{1}{7}$ is less than $\frac{1}{10000}$, it appears that the decimal fraction already found differs from the given vulgar fraction by a quantity which is less than *one ten-thousandth part of unity:* and by continuing the operation, we can find a decimal fraction which will differ less and less from the original vulgar fraction.

§ 78. *To explain the reason of the recurrence of the figures of the quotient in the same order, when reducing a vulgar to an interminate decimal fraction.*

In attempting to reduce a vulgar fraction to a decimal, the *remainder* at each step of the division must always be *less* than the divisor, *i.e.* than the denominator of the vulgar fraction; and the number of remainders different from each other which can arise, can only be a number less than the units in the divisor. If therefore the remainder never become 0, by carrying on the division far enough, one remainder must occur again, and as a cipher is added to every remainder, when the dividend becomes the same as has occurred before, the quotient will necessarily be again the same, and the process from that point will be repeated.

For example in reducing $\frac{1}{7}$ to a decimal, proceeding as in ordinary division, the number of times that 7 will be contained

in 10 is 1 with a remainder 3,
in 30 is 4 2,
in 20 is 2 6,
in 60 is 8 4,
in 40 is 5 5,
in 50 is 7 .... 1.

Now we observe that we have obtained every possible remainder except 0; consequently the remainder after the next step must be either 0, or one of the remainders that have occurred already. The next remainder being 3, the whole process will recur again from the beginning, and we shall have $\frac{1}{7} = \cdot 142857142857$, &c.

Such a decimal is called a *circulating* or *repeating* decimal: and when the same series of figures occurs from the beginning, it is called a *pure* circulating decimal; but if some of the figures do not repeat, and these are followed by some which do repeat, such a case is called a *mixed* circulating decimal.

It is usual to denote a circulating decimal by placing a dot over the first and last of the recurring figures; and the recurring period is called a *simple* or a *compound repetend* according as it consists of one or more figures: *e.g.* the pure circulating decimal ·3333, &c., which consists of the simple repetend 3, is written $\cdot\dot{3}$; the mixed circulating decimal ·3574597459, &c., which has the compound repetend 7459, is written $\cdot 35\dot{7}45\dot{9}$.

We see from the above considerations that the *greatest* number of figures which it is possible for the period of a circulating decimal to contain is *one less* than the number of units in the denominator of the vulgar fraction from which it springs.

§ 79. *To reduce a circulating decimal to an equivalent vulgar fraction.*

Let it be required to convert $\cdot\dot{5}\dot{7}$ into a vulgar fraction.

Remembering that we effect the multiplication of a decimal by 10, by 100, &c., by shifting the decimal place one, two, &c. places towards the *right* hand, we may say—

Let x be the vulgar fraction equivalent to $\cdot\dot{5}\dot{7}$.

$$\text{And if } x = \cdot 5757, \text{ \&c.}$$

$$\text{then } 100x = 57\cdot 5757, \text{ \&c.}$$

$$\left.\begin{array}{l}\text{by subtracting the}\\ \text{upper from the}\\ \text{lower line}\end{array}\right\}\quad 99x = 57$$

$$\left.\begin{array}{l}\therefore \text{ dividing each of}\\ \text{these equals by } 99\end{array}\right\}\quad x = \frac{57}{99} = \frac{19}{33}.$$

Again, to find the vulgar fraction equivalent to $\cdot 765\dot{3}41\dot{9}$.

$$\text{Let } x = \cdot 765\dot{3}41\dot{9}$$

$$1000x = 765\cdot 34193419, \text{ \&c.}$$

$$10000000x = 7653419\cdot 3419, \text{ \&c.}$$

$$\left.\begin{array}{l}\text{by subtracting}\\ \text{the 2nd from}\\ \text{the 3rd line}\end{array}\right\}\quad 9999000x = 7652654$$

$$\therefore\ x = \frac{7652654}{9999000} = \frac{3876327}{4999500}.$$

Again, let it be required to convert $\cdot 007\dot{2}\dot{8}$ into an equivalent vulgar fraction.

$$\text{Let } x = \cdot 007\dot{2}\dot{8}$$

$$1000x = 7\cdot\dot{2}\dot{8}$$

$$100000x = 728\cdot\dot{2}\dot{8}$$

$$\left.\begin{array}{l}\text{Subtracting the}\\ \text{2nd from the}\\ \text{3rd line}\end{array}\right\}\quad 99000x = 721$$

$$\therefore\ x = \frac{721}{99000}.$$

Now, by observing that $\cdot\dot{5}\dot{7} = \frac{57}{99}$,

$$\text{that } \cdot 765\dot{3}41\dot{9} = \frac{7653419 - 765}{9999000}$$

$$\text{that } \cdot 007\dot{2}\dot{8} = \frac{728 - 7}{99000}.$$

We may obtain the following practical rule for converting a circulating decimal into its equivalent vulgar fraction:

Place for the numerator of the vulgar fraction the circulating decimal written as a whole number, minus *the figures which do not recur; and for the denominator as many nines as there are* recurring *figures, followed by as many ciphers as there are* non-recurring *figures.*

§ 80. *To reduce any quantity or fraction of one denomination to the decimal of another denomination.*

Let it be required to express 17*s.* „ $5\frac{1}{4}$*d.* as the decimal of £1.

The process will be first to express the *fractional* part of a penny as a *decimal* of a penny: next placing the 5 as a whole number before this decimal, to divide that result by 12, in order to reduce it to the decimal of a shilling: then placing the 17 as a whole number before this decimal, to divide that result by 20 in order to reduce it to the decimal of a pound: This will be written as follows:

$$\begin{array}{r|l}
4 & 1\cdot \\ \cline{2-2}
12 & 5{\cdot}25 \text{ pence.} \\ \cline{2-2}
20 & 17{\cdot}4375 \text{ shillings.} \\ \cline{2-2}
 & \cdot 871875 \text{ of a pound.}
\end{array}$$

It will be seen from this that whatever we should divide by in whole numbers in order to bring pence into shillings, or shillings into pounds, that we must likewise divide by in this case, only marking off correctly the decimal results.

Let it be required to express 2 cwt. „ 3 qrs. „ 13·832 lbs. as the decimal of a ton.

Dividing severally by 28, 4, and 20, in order to bring lbs. into cwt., cwt. into qrs., and qrs. into tons, the process will be

$$\begin{array}{r|l} 28\begin{cases}4\\7\end{cases} & \begin{array}{l}13{\cdot}832\\\hline 3{\cdot}458\\\hline\end{array}\\ 4 & 3{\cdot}494\\\hline 20 & 2{\cdot}8735\\\hline & \;\;{\cdot}143675\end{array}$$

Conversely, *to find the value in a lower denomination of any decimal of a higher denomination*, we must *multiply* successively by the same factors that we should employ in whole numbers: *e.g.* find the value of £·81875: the process is

$$\begin{array}{r} {\cdot}81875\\ 20\\\hline 16{\cdot}37500\\ 12\\\hline 4{\cdot}500\\ 4\\\hline 2{\cdot}0 \end{array}$$

Here multiplying the denomination pounds by 20, to bring it into shillings, we obtain 16 shillings, and three hundred and seventy-five *thousandths* of a shilling. Multiplying this ·375 of a shilling by 12, to bring it into pence, we obtain 4 pence and five *tenths* of a penny. Multiplying ·5 of a penny by 4 to bring it into farthings, we obtain 2 farthings. The answer therefore is 16*s*. „ 4½*d.*

Similarly, if it be required to find the value of ·3945 of a day, we should multiply by 24 to find the number of hours; and then multiply the resulting decimal part of an hour by 60, to find the number of minutes; and again by 60 to find the number of seconds: thus, (multiplying by 24 in one line by the "back figure" system, § 31, *5*)

$$\begin{array}{r} {\cdot}3945\\ 24\\\hline 9{\cdot}4680\\ 60\\\hline 28{\cdot}080\\ 60\\\hline 4{\cdot}80 \end{array}$$

Hence the result is 9 hours „ 28 min. „ 4·8 seconds.

§ 81. Some examples worked at length, in order to exhibit the processes employed, are now subjoined: in which it will be observed that the fractions $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, being of frequent occurrence, are not worked out each time at length; it may be once for all remarked that

$$\frac{1}{8}=\cdot125,\ \frac{1}{4}=\cdot25,\ \frac{1}{2}=\cdot5,\ \frac{3}{4}=\cdot75;$$

and therefore these vulgar fractions may be at once expressed by their equivalent decimals.

Ex. 1. Express £3 „ 13*s.* „ 6½*d.* as the decimal of £5.

$$\begin{array}{r|l}
12 & 6\cdot5 \\ \hline
20 & 13\cdot541\dot{6} \\ \hline
5 & 3\cdot67708\dot{3} \\ \hline
 & \cdot73541\dot{6}
\end{array}$$

therefore the required decimal of £5 is $\cdot73541\dot{6}$.

Ex. 2. What decimal of a mile is 3 fur., 100 yds., 2 feet, 3 inches?

$$\begin{array}{r|l}
12 & 3 \\ \hline
3 & 2\cdot25 \\ \hline
\left\{\begin{array}{l}20 \\ 11\end{array}\right. & \begin{array}{l}100\cdot75 \\ \hline 5\cdot0375\end{array} \\ \hline
8 & 3\cdot4579\dot{5}\dot{4} \\ \hline
 & \cdot4322443\dot{1}\dot{8}
\end{array}$$

Ex. 3. Express the difference between £3 „ 3*s.* „ 3¾*d.* and £2 „ 5·3125*s.* as the decimal of 15*s.*

$$\begin{array}{r|l}
12 & 3\cdot75 \\ \hline
 & \cdot3125
\end{array}$$

$$\begin{array}{rl}
 & \text{£}\quad s. \\
\therefore \text{ from} & 3 \text{ „ } 3\cdot3125 \\
\text{subtract} & \underline{2 \text{ „ } 5\cdot3125} \\
 & 18s.
\end{array}$$

Now to reduce the difference, which is 18*s.*, to the decimal of 15*s.*

$$\begin{array}{l}
15)\ 18\cdot0\ (1\cdot2 \\
\quad\ \ 30 \\
\quad\ \ 00
\end{array}$$

∴ 1·2 is the required decimal.

Ex. 4. Compare the values of £·775 and 7·75 shillings.

£	*s.*
·775	7·75
20	12
15·500	9·00
12	
6·0	

Hence the values are 15*s.* „ 6*d.* and 7*s.* „ 9*d.*; and the value of the first is *twice* as great as that of the second.

Ex. 5. Find the difference between ·31595 of a guinea, and 5·12295 of a shilling; and reduce the difference to the decimal of a dollar whose value is 4*s.* 3*d.*

Multiplying by 21 by the "back figure" system (§ 31, 5), we have

Guinea.
·31595
21
6·63495

Now from 6·63495 of a shilling
Subtract 5·12295 of a shilling.
Difference 1·512

Next divide the difference, viz. 1·512 of a shilling by 4·25.

4·25) 1·512 (·356
2370
2550
0000

Ex. 6. Multiply 17 acres, 3 roods, 19 perches by ·325, and by ·0325.

17 „ 3 „ 19
4
71
40
2859 perches
·325
14 295
57 18
857 7
40) 929·175 perches
4) 23 roods „ 9·175 perches
5 acres „ 3 roods „ 9·175 perches.

Next to multiply 2859 perches by ·0325 is to divide the above result by 10, hence

$$\begin{array}{l} 40\underline{)\ 92\cdot9175} \text{ perches} \\ \quad 2 \text{ roods „ } 12\cdot9175 \text{ perches.} \end{array}$$

Ex. 7. Find the value of $3\cdot\dot{3}$ of $\dfrac{4\cdot\dot{4}}{\cdot\dot{7}3\dot{5}}$ of 1 sq. foot „ 3 sq. inches.

$3\cdot\dot{3} \times 4\cdot\dot{4} \div \cdot\dot{7}3\dot{5}$ of 1 sq. foot „ 3 sq. inches

$= 3\frac{3}{9} \times 4\frac{4}{9} \div \dfrac{735}{999} \times 147$ sq. inches

$= \dfrac{10}{3} \times \dfrac{\overset{8}{\cancel{40}}}{\cancel{9}} \times \dfrac{\overset{111}{\cancel{999}}}{\underset{147}{\cancel{735}}} \times 147$ sq. inches

$= \dfrac{10}{3} \times 8 \times 111$ sq. inches

$= 10 \times 8 \times 37$ sq. inches

$= 2960$ sq. inches

$= 2$ sq. feet „ 80 sq. inches.

Ex. 8. Find the value of $2\cdot8680\dot{5}$ of $3s. + \cdot8\dot{3}$ of $4s. - 1\cdot8$ of $5s.$

$2\cdot8680\dot{5}$ of $3s. = 2\dfrac{86805-8680}{90000}$ of 36 pence

$= 2\dfrac{\overset{3125}{\cancel{78125}}}{\underset{3600}{\cancel{90000}}} \times 36$

$= \dfrac{10325}{3600} \times 36 = \dfrac{10325}{100}$

$= 103\cdot25 = 103\frac{1}{4}$ pence.

$\cdot8\dot{3}$ of $4s. = \dfrac{83-8}{90} \times 48$ pence

$= \dfrac{75}{90} \times 48 = \dfrac{5}{6} \times 48$

$= 40$ pence.

$1\cdot8$ of $5 = 1\frac{4}{5} \times 60$ pence

$= \dfrac{9}{5} \times 60$

$= 108$ pence.

Hence $103\frac{1}{4} + 40 - 108$ pence

$= 35\frac{1}{4}$ pence

$= 2s.$ „ $11\frac{1}{4}d.$

Ex. 9. Find the difference between ·70323 of a pound, and 3·5646 of a shilling; and reduce 7*s.* „ $8\frac{1942}{10000}$*d.* to the decimal of half a guinea.

$$\begin{array}{lr} & \pounds \\ & \cdot70323 \\ & 20 \\ \hline & 14\cdot06460 \\ \text{Subtract} & 3\cdot5646 \\ \hline & 10\cdot5 \end{array}$$

∴ the difference is 10*s.* 6*d.*

Next, writing the fraction $\frac{1942}{10000}$ as ·1942, we have to reduce 7*s.* „ 8·1942*d.* to the decimal of half a guinea; i.e. to bring 92·1942 pence to the decimal of 126 pence.

$$\begin{array}{l} 126)\ 92\cdot1942\ (\cdot7317 \\ \quad\quad\ 3\ 99 \\ \quad\quad\ \ 214 \\ \quad\quad\ \ \ 882 \\ \quad\quad\ \ \ 000 \end{array}$$

Ex. 10. Reduce $\pounds\frac{\cdot036}{1875}$ to the fraction of a farthing; and divide £·36 by ·001875.

$$\pounds\frac{\cdot036}{1875} = \frac{\overset{12}{\cancel{36}}}{\underset{\underset{25}{\cancel{50}}}{\cancel{1000}}} \times \frac{1}{\underset{625}{\cancel{\cdot1875}}} \times \cancel{20} \times 12 \times \overset{2}{\cancel{4}} = \frac{144\times2}{5\times3125} = \frac{288}{15625}.$$

$$\begin{array}{ll} \text{Also} & 1\cdot875)\ 360\cdot000\ (\pounds192 \\ & \quad\quad\quad 17250 \\ & \quad\quad\quad\ \ 3750 \\ & \quad\quad\quad\ \ 0000 \end{array}$$

Ex. 11. Find the sum of ·65 of £4 „ 10*s.*, and ·0125 of £5 „ 13*s.* „ 4*d.*; and reduce the whole to the decimal of £3.

$$\begin{array}{l} \cdot65 \\ \ \ 90 \\ \hline 58\cdot50 \text{ shillings.} \end{array} \qquad\qquad \begin{array}{r} s.\quad d. \\ 113 \text{ „ } 4 \\ 12 \\ \hline 1360 \\ \cdot0125 \\ \hline 6800 \\ 16320 \\ \hline 17\cdot0000 \text{ pence.} \end{array}$$

Hence the sum is 58*s.* „ 6*d.* + 1*s.* „ 5*d.* = 59*s.* „ 11*d.*

$$\begin{array}{r|l} 12 & 11\cdot000 \\ \hline 60 & 59\cdot91\dot{6} \\ \hline & \cdot9986\dot{1} \text{ decimal of £3.} \end{array}$$

Obs. In reducing any given quantities to the decimal of a higher denomination, a method similar to that used in the rule of "Practice" may sometimes be conveniently adopted: the process will be more clearly understood after "Practice" has been studied: meanwhile in order to explain the method, one or two examples are subjoined.

Ex. 12. Reduce 19 cwt. „ 3 qrs. „ 10 lbs. to the decimal of a ton.

1 cwt.	$\frac{1}{20}$	= ·05	of a ton
		19 (multiply by)	
19 cwt.		= ·95	of a ton
2 qrs.	$\frac{1}{2}$ of 1 cwt.	= ·025	....
1 qr.	$\frac{1}{2}$ of 2 qrs.	= ·0125	....
4 lbs.	$\frac{1}{7}$ of 1 qr.	= ·0017857 &c.	....
4 lbs.	$\frac{1}{7}$ of 1 qr.	= ·0017857 &c.	....
2 lbs.	$\frac{1}{2}$ of 4 lbs.	= ·0008928 &c.	....
		·9919642 &c.	of a ton.

Ex. 13. Reduce 3 weeks „ 3 days „ 13 hours „ 36 minutes to the decimal of a lunar month.

3 weeks	$\frac{3}{4}$	= ·75	of a month.
3 days	$\frac{1}{7}$ of 3 weeks	= ·10714285 &c.	....
12 hours	$\frac{1}{6}$ of 3 days	= ·01785714 &c.	....
1 hour	$\frac{1}{12}$ of 12 hours	= ·00148809 &c.	....
30 min.	$\frac{1}{2}$ of 1 hour	= ·00074404 &c.	....
6 min.	$\frac{1}{5}$ of 30 min.	= ·00014880 &c.	....
		·87738092 &c.	of a month.

Obs. In converting shillings, pence, and farthings into florins, cents, and mils; and conversely, in finding the value of florins, cents, and mils in ordinary money, it is sufficient to remember that we are dealing with a "decimal coinage," i.e. that if we express shillings, pence, and farthings as the decimal of a pound, thereby expressing them as *tenths*, *hundredths*, and *thousandths* of a pound, we do in reality convert them into florins, cents, and mils; and conversely, that if we express any given number of florins, cents, and mils as the decimal of a pound, and find the value of that decimal, we do in fact

express the value of the florins, cents, and mils in shillings, pence, and farthings.

Thus, let it be required to convert 16*s.* „ $7\frac{1}{2}$*d.* into florins, &c.

$$\begin{array}{r|l} 12 & 7{\cdot}5 \\ \hline 20 & 16{\cdot}625 \\ \hline & £{\cdot}83125 \end{array}$$

and since a florin is *one-tenth*, a cent *one-hundredth*, and a mil *one-thousandth* of a pound, £·83125 = 8 florins „ 3 cents „ $1\frac{1}{4}$ mils.

Conversely, if it be required to express 9 florins „ 9 cents„ 8 mils as shillings, &c., we write 9 florins „ 9 cents „ 8 mils = £·998; and the value of this decimal is found in the ordinary manner, thus:

$$\begin{array}{r} {\cdot}998 \\ 20 \\ \hline 19{\cdot}96 \\ 12 \\ \hline 11{\cdot}52 \\ 4 \\ \hline 2{\cdot}08 \end{array}$$

Whence 19*s.* „ 11*d.* „ $2\frac{2}{25}$*far.* is the value required.

We may however convert florins, cents, and mils into shillings, pence, and farthings *by inspection*, if we attend to the following principles:

The florins must be *doubled* for shillings; then, since 5 cents are $\frac{5}{100}$ or $\frac{1}{20}$ of £1, or 1*s.*, we observe that whenever the number of cents is 5 or upwards, we must subtract 5 from that number and add 1 to the number of shillings already obtained. Any remainder that there may now be from the cents, i.e. the *hundredths*, may be expressed with the mils as *thousandths;* (e.g. 4 cents, 7 mils = £·047 = £$\frac{47}{1000}$;) and as a farthing is £ $\frac{1}{960}$, these thousandths, being in *value* somewhat *less* than farthings, will be in *number* somewhat *more* than the farthings equivalent to the remaining cents and mils.

Now it is found in practice, that we may reckon these thousandths *as* farthings if, when the number of thousandths

be 12 and upwards, we subtract 1 from them; if, when the number be 36 and upwards, we subtract 2.

Thus, to bring 7 florins „ 8 cents „ 9 mils to shillings, &c.

7 florins = 14*s*.; 8 cents = 5 cents + 3 cents = 1*s*. + 3 cents;

Add 1*s*. to the 14*s*. already obtained; and we have

15*s*. + 3 cents + 9 mils;

but 3 cents + 9 mils = 39 thousandths;

from this number, since it is larger than 36, subtract 2; i.e. say

39 thousandths = 37 farthings = $9\frac{1}{4}d$.

Hence the result is 15*s*. „ $9\frac{1}{4}d$.

This method may be extended to ordinary decimals of £1, by noticing that when the *fourth* place of decimals is 5 or upwards, it must be reckoned as an additional figure in the *third* place; and any places of decimals beyond the fourth may be neglected, as only influencing the fractional part of a farthing.

Thus, to find the value in shillings, &c. of £·7169.

Here, since the fourth figure is more than 5, increase the third figure by 1, and read ·717: double the first 7, and say 14 shillings; from the 17 thousandths subtract 1, and say 16 farthings, or 4*d*.;

Therefore the result is 14*s*. „ 4*d*., which is correct within a fraction of a farthing.

Again, to find the value of £·358994.

Here, after adding 1 to the figure in the *third* place, because of the 9 in the *fourth* place, and neglecting the figures in the *fifth* and *sixth* places, we double the 3, and to the 6*s*. thus obtained we add 1*s*. on account of the 5 in the place of hundredths; then, remembering that the 8 in the place of thousandths is to be taken as 9, we say 9 farthings are $2\frac{1}{4}d$.;

Hence 7*s*. „ $2\frac{1}{4}d$. is the result, correct within a fraction of a farthing.

The method of converting *by inspection* pounds, shillings, pence and farthings, into florins, cents and mils, i.e. into the decimal of a pound correct to the third figure, would be as follows:

Halve the given number of shillings for the first place of decimals, and reckon the odd shilling (if there be one) as 5 in the second place; bring the pence and farthings into farthings, and if the number of farthings be between 12 and 35 (inclu-

sive) increase it by *one*, if the number be 36 and upwards increase it by *two*, and add this number to the second and third places of decimals, or, if it be a number less than ten, to the third place.

For example, let it be required to turn 9*s.* 11$\frac{3}{4}$*d.* into florins, cents and mils.

Halve 9, and the result is 4, with 1 over; write 4 in the first place of decimals, followed by 5 in the second place for the odd shilling: then 11$\frac{3}{4}$*d.* = 47 farthings; and this, increased by 2 since it is more than 36, must be added as 49 to the second and third places of decimals: thus

$$\begin{array}{r} \cdot45 \\ \underline{49} \\ \cdot499 \end{array}$$

i. e. 4 florins, 9 cents, 9 mils.

The decimal of £1, worked out at length, would be ·499458$\dot{3}$.

Similarly, taking 16*s.* 1$\frac{3}{4}$*d.* we should halve 16, and write the 7 farthings as 7 in the *third* place; thus, ·807: the decimal, if worked at length, would be ·807291$\dot{6}$.

Similarly, 7*s.* 3$\frac{1}{2}$*d.* would be thus expressed as a decimal of £1,

$$\begin{array}{r} \cdot35 \\ \underline{15} \\ \cdot365 \end{array}$$ (worked at length, ·36458$\dot{3}$).

Again, 4*s.* 9$\frac{1}{2}$*d.* = ·240 (worked at length, ·23958$\dot{3}$).

It is manifest that decimals thus obtained to three places are sufficiently accurate for addition, subtraction, or division: and for multiplication by any number not exceeding 10: but for multiplication by higher numbers, every additional figure in the multiplier would require an additional place of decimals in the multiplicand.

Exercise X.

REDUCTION OF DECIMALS.

1. Reduce to the decimal of £1 the following sums: viz.

(1) 3*s.* „ 6$\frac{1}{2}$*d.* (2) 4*s.* „ 9*d.* (3) 17*s.* „ 7$\frac{1}{2}$*d.* (4) 19*s.* „ 4$\frac{1}{2}$*d.* (5) 11*s.* „ 11$\frac{3}{4}$*d.*

2. Find the value in shillings, pence, &c., of the following: viz.

(1) £·375. (2) £·1$\dot{6}$. (3) £·9291$\dot{6}$. (4) £·68125. (5) £·78975.

3. Reduce $\frac{4}{7}$ of a guinea to the decimal of £1; also to the decimal of £5.

4. Reduce 8·775 shillings to the decimal of a moidore (27*s.*).

5. Express 3 qrs. „ 3 lbs. „ 1 oz. „ $12\frac{4}{5}$ drs. as the decimal of 1 cwt.

6. Find the value of ·7385 of a mark (13*s.* „ 4*d.*).

7. Bring 4*s.* „ $11\frac{3}{4}$*d.* to the decimal of £1; and find the value of £·009765; and express 5*s.* „ $8\frac{3}{4}$*d.* as the decimal of £7.

8. Find the value of $\cdot 089\dot{2}8571\dot{4}$ of 7*s.*

9. Express 3·74976 minutes as the decimal of a week.

10. Reduce 12 hrs. „ 55′ „ $23\frac{1}{13}$″ to the decimal of a day.

11. Express 1 florin „ 6 cents „ $8\frac{3}{4}$ mils as shillings, pence, &c.

12. Reduce 6*s.* „ 6*d.* to florins, cents and mils.

13. Find by *inspection*, neglecting the fractions of a farthing, the value of each of the following decimals of £1, viz.: (1) ·7416. (2) ·8793. (3) ·3142. (4) ·5839. (5) ·0608.

14. Write down *by inspection* the value in shillings, &c. of (1) 7 florins „ 6 cents „ 5 mils. (2) 9 florins „ 9 cents „ 9 mils. (3) 3 florins „ 6 cents „ 8 mils. (4) 14 florins „ 7 cents „ 1 mil. (5) 6 florins „ 2 cents „ 3 mils. (6) 5 florins „ 8 cents „ 7 mils.

15. By *inspection* obtain the florins, cents and mils equivalent to (1) 8*s.* „ $2\frac{1}{4}$*d.*; (2) 17*s.* „ $2\frac{3}{4}$*d.*; (3) 3*s.* „ $11\frac{1}{2}$*d.*; (4) 19*s.* „ $10\frac{3}{4}$*d.*; (5) 12*s.* „ $8\frac{1}{4}$*d.*; (6) 15*s.* „ $9\frac{3}{4}$*d.*

EXERCISE XI.

MISCELLANEOUS QUESTIONS IN DECIMAL FRACTIONS.

1. Reduce the following vulgar fractions to decimals:

$$\frac{3}{16},\ \frac{9}{40},\ \frac{130}{625},\ \frac{17}{125},\ \frac{106}{125},\ \frac{1\frac{1}{10}}{62\frac{1}{2}},\ \text{and } 6\tfrac{3}{2} \text{ of } \frac{1}{5}+\frac{17}{5}.$$

2. Reduce the following expression to a decimal:

$$\frac{63}{125}+\frac{510}{625}+\frac{45}{640}+\frac{39}{800}.$$

3. Reduce the following fractions to circulating decimals:

$$\frac{13}{990},\ \frac{17}{275},\ \frac{11}{1665},\ \frac{129}{550},\ \frac{6401}{4950},\ 46\tfrac{104}{333},\ \text{and}\ \frac{20555}{33300}.$$

4. Reduce the following expression to a decimal:

$$\frac{5}{12}+\frac{102}{220}+\frac{4}{42}+\frac{5}{22}+\frac{22}{45}.$$

5. Reduce the following decimals to vulgar fractions:

·5, ·005, ·025, ·0025, 7·5, ·075, ·004, ·4, ·01015625, ·71575, ·0071575.

6. Multiply ·013 by ·00016, 32·56 by ·0457, ·764 by ·356, ·07 by ·0762, 3·05 by ·203, ·07853 by ·0476.

7. Divide 1·25 by ·0025, 14·4 by ·012, 19·5 by ·00013, 76·2151 by ·325, 12370·519 by 5·425, 1708·4592 by ·24.

8. Divide $\cdot\dot{6}$ by $\cdot\dot{0}\dot{9}$, $\cdot\dot{0}\dot{4}$ by $\cdot\dot{3}8461\dot{5}$, ·7 by $\cdot\dot{1}4285\dot{7}$, $234\cdot\dot{6}$ by $7\cdot\dot{7}$.

9. Reduce 4*s*. „ 9*d*. to the decimal of £1.
2·1*s*. to the decimal of a guinea.
4·2*s*. to the decimal of three guineas.
2*s*. „ 6*d*. to the decimal of 13*s*. „ 4*d*.

10. Multiply ·03574 by 7·46, ·1787 by 3·73, ·014296 by ·01492.

11. Reduce 3 oz. „ 12 dwts. to the decimal of a pound troy.

12. Find the value of $\frac{2}{3}$ of a guinea + $\frac{9}{16}$ of a crown + $\frac{9}{10}$ of 7*s*. „ 6*d*. − $\frac{2}{3}$ of 2*d*.; and express it as the decimal of 16*s*.

13. Reduce the following expression to a decimal:

$$4\tfrac{1}{6}+4\tfrac{7}{11}+\frac{20}{21}+2\tfrac{3}{11}+4\tfrac{8}{45}.$$

14. Reduce 3 oz. „ $0\frac{16}{21}$ dr. to the decimal of a lb. avoirdupois.

15. Find the value of ·548671875 of one day.

16. What is the difference between $\frac{2}{3}$ of $5\frac{1}{2}$ metres and $3\frac{1}{2}$ of $9\frac{1}{6}$ yards, 12 yards being equal to 11 metres?

17. Add together $\frac{1}{3}$ of $\frac{5}{7}$, $4\frac{1}{2}$, $\dfrac{1\frac{2}{3}}{7}$, and $\dfrac{\frac{5}{6}}{2\frac{1}{2}}$ and reduce the result to a circulating decimal.

18. Reduce 7 wks. „ 1 d. „ 10 h. „ 12′ „ 14″ to the decimal of $3\frac{1}{2}$ months.

19. Reduce 1 m. „ 550 yds. to the decimal of a league (3 miles). Also reduce 2 m. „ 3 p. „ 0 yds. „ 2 ft. „ 6 in. to the decimal of 2 miles.

20. Find the value of 6·1188 of 1 m. „ 530 yds.

21. Find the value of $\frac{7}{8}$ of $1\frac{1}{14}$ of 3 acres − 10·04375 sq. yds. + $\cdot 11\dot{3}\dot{6}$ of $3\frac{1}{2}$ sq. ft.

22. Reduce 3 bush. „ $7\frac{1}{2}$ gallons to the decimal of a quarter.

23. Find the value of ·625 of 5*s.* + ·75 of $1\frac{1}{2}$*d.* − ·65625 of 1*s.* + ·175 of a pound − ·375 of 10*s.* „ 6*d.*

24. Find the greatest common measure of 1353·6 and 231·48.

25. Reduce 17*s.* „ $6\frac{1}{4}$*d.* to florins, cents, and mils; and find the value in shillings, pence, &c., of 8 florins „ 3 cents „ 7 mils.

26. Multiply the *sum* of 5 florins „ 3 cents „ 5 mils, 8 florins „ 9 cents „ 6 mils, and 4 florins „ 3 cents „ 4 mils, by 30; and express the value of the result in pounds, shillings, &c.

27. Divide the difference between £317 „ 7 florins „ 7 cents, and £212 „ 6 florins „ 0 cents „ 2 mils, between 14 persons; stating the value of each person's share in pounds, shillings, &c.

28. Shew that 3 times the difference between 7 florins „ 5 cents „ 7 mils, and 4 florins „ 3 cents „ 2 mils, is the same as twice the sum of 3 florins „ 8 cents „ $6\frac{1}{2}$ mils, and 1 florin „ 0 cent „ 1 mil.

THE METRIC SYSTEM.

Examples must be worked by decimals from the equivalents given in the Tables: for instance, if we assume that

$$£1 = 25{\cdot}2215 \text{ francs},$$

$$\text{then } 10s. = 12{\cdot}61075 \text{ francs},$$

$$1s. = 1{\cdot}261075 \text{ francs},$$

$$1d. = {\cdot}105089 \text{ francs}.$$

Conversely,

1 franc $= 9\cdot51384$ pence $= 9d.$ „ $2\cdot055$ *far.*

5 francs $= 47\cdot5692$ pence $= 3s.$ „ $11d.$ „ $2\cdot276$ *far.*

10 francs $= 95\cdot1384$ pence $= 7s.$ „ $11d.$ „ $0\frac{1}{2}$ *far.*

20 francs $= 190\cdot2768$ pence $= 15s.$ „ $10d.$ „ $1\frac{1}{10}$ *far.*

Hence for small sums we may with tolerably close approximation reckon that

1 franc $= 9\frac{1}{2}d.$

5 francs $= 4s.$

10 francs $= 8s.$

20 francs $= 16s.$

If we might assume that £1 = 25 francs, we should bring pounds into francs by multiplying by 25, *i.e.* by adding 2 ciphers and dividing the result by 4, (cf. § 31: 2,) and we should bring shillings into francs by multiplying by $\frac{5}{4}$. Conversely, francs into pounds by dividing by 25, *i.e.* multiplying by 4 and dividing the result by 100, (cf. § 31: 7,) or, which is the same thing, by multiplying by $\cdot04$; and francs into shillings by multiplying by $\frac{4}{5}$ or $\frac{8}{10}$.

For length, we find by the tables that a metre = 1 yd. „ 0 ft. „ $3\cdot3708$ inches.

Whence, by expressing the inches as a decimal of a yard, we obtain 1 metre $= 1\cdot0936\dot{3}$ yards.

Whence we may say approximately that a metre $= 1\frac{1}{12}$ yds.; while the kilometre, the measure generally used for distance, is about 6 yards short of 5 furlongs, or about $\frac{5}{8}$ of a mile.

Conversely, by dividing 1 by $1\cdot09363$, it will be found that 1 yard $= \cdot91438$ metre; whence the other denominations of feet, inches, &c. can be determined.

For weight, we are told that the kilogram is equal to 2 lbs. „ 3 oz. „ $4\cdot383$ drams; or, converting the ounces and drams into the decimal of a lb., to $2\cdot204621$ lbs. avoir.

Now let it be required to express a lb. in grammes; we should say, using the contracted form of division,

```
2·204621) 1000·0000 (453·5927
           1181516
             79205
             13066
              2043
                59
                15
                 0
```

Hence the lb. avoir. = 453·5927 grammes.

Also, in order to obtain the ton, we multiply this number by 2240, the number of lbs. in a ton;

```
  453·5927
      2240
 ---------
 1814 37080
 9071 854
90718 54
-----------
1016047·6480
```

Hence the ton = 1016047·648 grammes,
= 1016·047648 kilog.

Whence in practice it will be sufficient to take the ton as equivalent to 1016 kilog.

Had it been required to express directly a ton in the denomination of kilogs, we should have obtained the same result by dividing 2240, the number of lbs. in a ton, by 2·204621.

```
2·204621) 2240·000 (1016·047
           35 379
           13 333
              105
               17
                2
```

[Besides the French, several of the German coins and measures ought perhaps to be noticed. A "Thaler" being very nearly equal to 3*s.*, it is sufficient to multiply any number of thalers by ·15 to find the equivalent pounds sterling. In lineal measure the German "Ruthe" is about $12\frac{1}{3}$ feet; and the "meile" is nearly $4\frac{3}{4}$ miles. While in cubic measure the "Schacht-ruthe" is nearly equivalent to 6 cubic yards.—There are however no authorised tables of comparison: and these somewhat rough approximations are not intended to serve for scientific, but only for ordinary calculations.]

EXERCISE XII.

1. Express a yard as a decimal of a metre; and a mile in the denomination of kilometres.

2. Express a dekametre in the denomination of yards; and a myriametre in the denomination of miles.

3. In 50 miles, how many kilometres? in 8 myriametres how many miles?

4. Express a furlong in metres. What is the least number of metres that contains an exact number of inches?

5. A traveller on the Continent paid the following Hotel bills in English money, reckoning 25 francs to the sovereign: (1) Hotel de France 22 frs., (2) Hotel de la Couronne 81·50, (3) Hotel de la Ville 91·50, (4) Hotel Belle Vue 252·25, (5) Hotel du Lac 69·50, (6) Schweizerhof 217·20. Find in each case the exact sum he paid.

6. Pay in francs the following English bills, reckoning 25 francs to the pound: (1) 17*s*. 6*d*., (2) £1. 8*s*. 3*d*., (3) £1. 13*s*. 9*d*., (4) £4. 16*s*., (5) £6. 16*s*., (6) £5., (7) £43. 8*s*.

7. Reckoning the pound sterling as 25·2215 francs, what is the value of 25000 francs? and what sum in francs is £50?

8. In accepting in English money the payment of a debt of 10000 francs, how many pounds, &c. did the creditor gain or lose by reckoning 25 instead of 25·2215 francs to the pound?

9. Express an acre in the denomination of hectares.

10. The docks at Hâvre covering 30 hectares, find their area in acres.

11. In a field of 13 acres, 3 roods, 17 poles, how many hectares?

12. If the area of a lake be 7·0457 acres, what is the equivalent in hectares?

13. How many gallons in a cask containing 100 litres?

14. In 57 gallons, $3\frac{1}{2}$ pints, how many hectolitres?

15. How many pounds, shillings, &c. would 4·732 hectolitres cost at 12 florins, 2 cents 5 mils per gallon?

16. If we take the litre as $1\frac{3}{4}$ pints, and allow that the bottles of the Litre Wine Company hold $\frac{1}{8}$th more than the English reputed quart bottles, find how much short of a quart the attenuated English bottle now holds?

17. In 100 kilolitres how many bushels?

18. Express the lb. Troy in hectograms; and the lb. avoirdupois in kilograms.

19. What weight in avoirdupois is equivalent to 42·75 kilograms?

20. Find in myriagrams the weight of a gun of 4 tons, 19 cwt. 2 qrs. 19 lbs.

21. A gallon of water weighs 10 lbs.: express the weight of a litre of water in kilograms.

22. Express in kilograms the excess of 263 myriagrams over 2 tons, 9 cwt. 3 qrs. 24 lbs.

23. A French gun projected a shot of 31·6874 kilograms a distance of 5000 metres: express the weight and the distance in English measure.

24. An English gun threw a shot of 134 lbs. $5\frac{1}{2}$ oz. a distance of $3\frac{3}{8}$ miles: express the weight and the range in kilograms and metres.

25. The pendulum vibrating seconds in the latitude of London is 39·1393 inches: express that length referred to the metre.

26. The metre being the ten-millionth part of a line drawn along the meridian from the pole to the equator, what is the earth's circumference in miles?

27. If the earth's polar diameter be 1271·057 metres, and the equatorial diameter 7924 miles, find the polar diameter in miles, and the equatorial diameter in metres.

28. Express a cubic yard as a stere.

In 5028·45915 cubic feet, how many steres?

29. An embankment contained 245·95994 steres: how many cubic yards and cubic feet were there in it?

30. In 3651 thalers, how many pounds, shillings, &c.?

31. Find the equivalents in English measures of 273 German "meiles," 87 "ruthe," and 139 "Schacht-ruthe."

CHAPTER XI.

PRACTICE.

§ 82. The rule of Practice does not involve any principles beyond the rules of compound multiplication and division, fractions and decimals. It depends for its use on the readiness with which the above rules can be applied, and on the dexterity and quickness which the calculator acquires by practice. It is the rule by which, when the price of an unit of any denomination is given, we can find the price of any quantity of the same kind of goods.

The method of its application is as follows: when the cost is required say of 90 yards at the rate of £2 „ 16*s.* „ 6*d.* per yard, instead of reducing £2 „ 16*s.* „ 6*d.* to pence, multiplying the pence so obtained by 90, and then dividing that result again by 12 and 20 to bring the result back into pounds, shillings and pence, we should *by practice* first multiply 90 by 2, to find what the 90 yards cost at £2 per yard; and we should then take the remainder of the money given as the cost price, and subdividing it so that each part may be a simple fraction of that which preceded it, we should find the cost separately of the 90 yards at each of these rates, and then add together the several results.

The form of the process is as follows:

10*s.*	$\frac{1}{2}$	£90	= cost of 90 yards at £1 per yard
		2 (multiply by)	
		180	= cost of 90 yards at £2 per yard
5*s.*	$\frac{1}{2}$	45	= .... 10*s.* ....
1*s.*	$\frac{1}{5}$	22 „ 10	= .... 5*s.* ....
6*d.*	$\frac{1}{2}$	4 „ 10	= .... 1*s.* ....
		2 „ 5	= .... 6*d.* ....
		£254 „ 5	= cost of 90 yards at £2 „ 16*s.* „ 6*d.* per yard.

§ 83. The common difficulty in working examples in practice

consists in finding out what are the proper fractional parts into which to subdivide either the money which is the price, or the quantities of the goods purchased; the tables given below will supply this information.

An Aliquot part, (aliquoties, *a certain number of times*), sometimes called a submultiple, may be thus defined:

One number or fraction is said to be an aliquot part of another number or fraction when the first is contained an exact number of times in the second.

The first table contains the aliquot parts of £1, so arranged that the simple aliquot parts, or the aliquot part of an aliquot part, may be found at once by inspection. The figures written by themselves or before a semicolon are *shillings;* those after a semicolon are *pence* or fractions of a penny.

		Half.	Third.	Fourth.	Fifth.	Sixth.	Eighth.	Tenth.	Twelfth.	Twentieth.	Fortieth.
Of £1.		10	6; 8	5	4	3; 4	2; 6	2	1; 8	1	; 6
Half	10	5	3; 4	2; 6	2	1; 8	1; 3	1	;10	;6	; 3
Third	6;8	3; 4		1; 8	1;4		;10	;8		;4	; 2
Fourth...	5	2; 6	1: 8	1; 3	1	;10	; 7½	;6	; 5	;3	; 1½
Fifth	4	2	1; 4	1		; 8	; 6		; 4		
Sixth	3;4	1; 8		;10	;8		; 5	;4		;2	; 1
Eighth ..	2;6	1; 3	;10	;7½	;6	; 5	; 3¾	;3	; 2½	;1½	; ¾
Tenth ...	2	1	; 8	; 6		; 4	; 3		; 2		
Twelfth..	1;8	;10		; 5	;4		; 2½	;2		;1	; ½
Twentieth	1	; 6	; 4	; 3		; 2	; 1½		; 1		
Fortieth..	;6	; 3	; 2	;1½		; 1	; ¾		; ½		

Here the simple aliquot parts of a pound, the half, third, &c., are found in the *left* hand column, and in the *upper* column; the aliquot part of an aliquot part is found in the square *opposite to* the one and *under* the other part; thus opposite to *eighth* and under *fourth* is found ; 7½; which shows that 7½*d.* is the *eighth* of a *fourth* of a pound.

If now the price of 3107 yards at 7½*d.* per yard be required, by dividing 3107 by 4 and the result by 8, we obtain the *eighth* of the *fourth* of £3107, and so find the cost required; thus

$$\begin{array}{r} 4)\ 3107 \\ \hline 8)\ 776\ „\ 15 \\ \hline £97\ „\ 1s.\ „\ 10\tfrac{1}{2}d. \end{array}$$

The following tables of aliquot parts will be found useful for reference:

Of a Shilling.

1*d.*	...	is	...	$\frac{1}{12}$
$1\frac{1}{2}$	...	are	...	$\frac{1}{8}$
2	...	„	...	$\frac{1}{6}$
3	...	„	...	$\frac{1}{4}$
4	...	„	...	$\frac{1}{3}$
6	...	„	...	$\frac{1}{2}$

Of a Pound Troy.

6 oz.	...	are	...	$\frac{1}{2}$
4	...	„	...	$\frac{1}{3}$
3	...	„	...	$\frac{1}{4}$
2	...	„	...	$\frac{1}{6}$
1 oz. 10 dwts.		„	...	$\frac{1}{8}$
1	...	is	...	$\frac{1}{12}$

Of an Ounce Troy.

10 dwts.		are	...	$\frac{1}{2}$
6 dwts. 16 gr.		„	...	$\frac{1}{3}$
5	...	„	...	$\frac{1}{4}$
4	...	„	...	$\frac{1}{5}$
3 dwts. 8 gr.		„	...	$\frac{1}{6}$
2 dwts. 12 gr.		„	...	$\frac{1}{8}$
2	...	„	...	$\frac{1}{10}$
1 dwt. 16 gr.		„	...	$\frac{1}{12}$

Of an Acre.

2 roods	...	are	...	$\frac{1}{2}$
1 rood	...	is	...	$\frac{1}{4}$
20 poles	...	are	...	$\frac{1}{8}$
16	...	„	...	$\frac{1}{10}$

AVOIRDUPOIS.

Of a Ton.

10 cwt.	...	are	...	$\frac{1}{2}$
5	...	„	...	$\frac{1}{4}$
4	...	„	...	$\frac{1}{5}$
2	...	„	...	$\frac{1}{10}$
1	...	is	...	$\frac{1}{20}$

Of a Cwt.

2 qrs.	...	are	...	$\frac{1}{2}$
1	...	is	...	$\frac{1}{4}$
16 lbs.	...	are	...	$\frac{1}{7}$
14	...	„	...	$\frac{1}{8}$
8	...	„	...	$\frac{1}{2}$ of $\frac{1}{7}$
7	...	„	...	$\frac{1}{2}$ of $\frac{1}{8}$
$3\frac{1}{2}$	...	„	...	$\frac{1}{4}$ of $\frac{1}{8}$
2	...	„	...	$\frac{1}{7}$ of $\frac{1}{8}$
$1\frac{3}{4}$	...	„	...	$\frac{1}{8}$ of $\frac{1}{8}$

Of a Quarter.

14 lbs.	...	are	...	$\frac{1}{2}$
7	...	„	...	$\frac{1}{4}$
4	...	„	...	$\frac{1}{7}$
$3\frac{1}{2}$	...	„	...	$\frac{1}{8}$

Of a Pound.

8 oz.	...	are	...	$\frac{1}{2}$
4	...	„	...	$\frac{1}{4}$
2	...	„	...	$\frac{1}{8}$
1	...	is	...	$\frac{1}{16}$

§ 84. The following examples worked at length will exhibit the usual form of questions in practice:

Ex. 1. Find the cost of 6 cwt. „ 3 qrs. „ $7\frac{1}{2}$ lbs. at £7 „ 13*s.* „ 6*d.* per cwt.

		£ s. d.	
2 qrs.	$\frac{1}{2}$	7 „ 13 „ 6	= cost of 1 cwt.
		6	
		46 „ 1 „ 0	= ...... 6 cwt.
1 qr.	$\frac{1}{2}$	3 „ 16 „ 9	= ...... 2 qrs.
{4 lbs.	$\frac{1}{7}$	1 „ 18 „ $4\frac{1}{2}$	= ...... 1 qr.
{$3\frac{1}{2}$ lbs.	$\frac{1}{8}$	5 „ $5\frac{11}{14}$	= ...... 4 lbs.
		4 „ $9\frac{9}{16}$	= ...... $3\frac{1}{2}$ lbs.
		£52 „ 6 „ $4\frac{95}{112}$	= ...... 6 cwt. 3 qrs. „ $7\frac{1}{2}$ lbs.

Here as £7 „ 13*s.* „ 6*d.* is the cost of 1 cwt. we multiply that sum by 6 to obtain the cost of 6 cwt.; then, as 2 qrs. are $\frac{1}{2}$ of 1 cwt., $\frac{1}{2}$ of the top line will be the cost of 2 qrs.; and as 1 qr. would cost $\frac{1}{2}$ of the cost of 2 qrs., we divide the cost of 2 qrs. by 2. Next 4 lbs. being $\frac{1}{7}$ and $3\frac{1}{2}$ lbs. being $\frac{1}{8}$ of 1 qr., we divide the cost of 1 qr. by 7 and 8 successively. These results being added together, give the cost of 6 cwt. „ 3 qrs. „ $7\frac{1}{2}$ lbs.

In cases where there are fractional parts of a penny remaining after division, it is easier *not* to reduce them to farthings, but to leave them as fractions of a penny. If greater accuracy should be thought necessary, the fraction in the answer may be reduced to farthings. Thus the answer of this example may be written £52 „ 6*s.* „ 4 *d.* „ $3\frac{11}{28}$ *far.*

[N.B. If any fractional parts of a farthing remain, never write the farthings as a fraction of a penny, to be followed again by another fraction, which is *meant* to represent (but does not represent) the fraction of a farthing. But write the farthings as whole numbers of a separate denomination, followed by their own fraction.]

To avoid these fractions, we may use, as in the next example, decimals of a penny.

Ex. 2. Required the price of 9 yds. „ 1 ft. „ $5\frac{1}{2}$ in. at £2 „ 5*s.* „ $7\frac{1}{2}$*d.* per yard.

		£ s. d.	
1 foot	$\frac{1}{3}$	2 „ 5 „ $7\frac{1}{2}$	= cost of 1 yard.
		9	
		20 „ 10 „ 7·5	= .... 9 yards.
{4 inches	$\frac{1}{3}$	15 „ 2·5	= .... 1 foot.
{$1\frac{1}{2}$ inches	$\frac{1}{8}$	5 „ $0·8\dot{3}$	= .... 4 inches.
		1 „ 10·812	= .... $1\frac{1}{2}$ inches.
		£21 „ 12 „ 9·645	= ...... 9 yds. „ 1 ft. „ $5\frac{1}{2}$ in.

Here the cost of 1 yard was multiplied by 9 to obtain the cost of 9 yards; and $\frac{1}{3}$ the cost of 1 yard was taken for the cost of 1 foot; then $\frac{1}{3}$ of the cost of 1 foot, and $\frac{1}{8}$ of the cost of 1 foot were taken for the cost of 4 inches and of $1\frac{1}{2}$ inches respectively; and these results were added together to obtain the entire cost required.

§ 85. The solution of questions in practice may often be simplified by various artifices, of which the following are given as the most commonly useful:

1. Taking aliquot parts of aliquot parts whose value is already found: thus,

Find the price of 3729 articles at £7 „ 2*s.* „ $9\frac{3}{4}$*d.*

s. *d.*		£	
2 „ 6	$\frac{1}{8}$	3729	would be the cost of the 3729 articles at £1 each.
		7	
		26103	= cost of the articles at £7 each.
$3\frac{3}{4}$	$\frac{1}{8}$	466 „ 2 „ 6	= .... 2*s.* „ 6*d.* each.
		58 „ 5 „ $3\frac{3}{4}$	= .... $3\frac{3}{4}$*d.* each.
		£26627 „ 7 „ $9\frac{3}{4}$	= .... £7 „ 2*s.* „ $9\frac{3}{4}$*d.* each.

Since the 3729 articles at £1 each would obviously cost £3729 if we multiply that sum by 7, we obtain the cost at £7. As 2*s.* „ 6*d.* is $\frac{1}{8}$ of £1, by dividing 3729 by 8 we find the cost at 2*s.* 6*d.* Next we see by the table that $3\frac{3}{4}$*d.* is the *eighth* of an *eighth* of £1; hence by dividing the cost at 2*s.* „ 6*d.* by 8, we obtain the cost at $3\frac{3}{4}$*d.*

2. Taking aliquot parts of the *multiplied* quantity instead of the original quantity: thus,

Required cost of 108 articles at £7 „ 17*s.* „ 6*d.*

		108
		7
17*s.* „ 6*d.*	$\frac{1}{8}$	756
		94 „ 10
		£850 „ 10*s.*

Here 17*s.* „ 6*d.*, which is $\frac{7}{8}$ of £1, is $\frac{1}{8}$ of £7.

3. Taking the price as if somewhat *higher* than that which is given, and subtracting a convenient aliquot part: thus,

Find the price of 218 cwt. at £5 „ 18*s*. „ 4*d*.

Here £5 „ 18*s*. „ 4*d*. = £6 *less* 1*s*. „ 8*d*.

Therefore find the cost at £6, and *subtract* the cost at 1*s*. „ 8*d*.

1*s*. „ 8*d*.	$\frac{1}{12}$	218	= price of 218 cwt. at £1 each.
		6	
		1308	= .... £6 each.
Subtract		18 „ 3*s*. „ 4*d*.	= .... 1*s*. „ 8*d*. each.
		£1289 „ 16*s*. „ 8*d*.	= .... £5 „ 18*s*. „ 4*d*. each.

4. Introducing a subsidiary aliquot part to facilitate calculation, and erasing it from the result when the other parts have been found from it: *e. g.*

Find the cost of a silver-gilt goblet weighing 3lbs. „ 4 oz. „ 15 grs. at £2 „ 7*s*. „ 8*d*. per ounce.

3 lbs. „ 4 oz. = 40 oz.

		£ *s*. *d*.	
1 dwt.	$\frac{1}{20}$	2 „ 7 „ 8	
		40	
		95 „ 6 „ 8	= cost of 3 lbs. „ 4 oz.
		2 „ $4\frac{3}{5}$	= cost (to be erased) of 1 dwt.
12 grs.	$\frac{1}{2}$	1 „ $2\frac{3}{10}$	= cost of 12 grs.
3 grs.	$\frac{1}{4}$	$3\frac{23}{40}$	= cost of 3 grs.
		£95 „ 8 „ $1\frac{9}{16}$	= cost of 3 lbs. „ 4 oz. „ 15 grs.

In this example, as the denomination of *dwts.* does not occur in the question, and as a *grain* is $\frac{1}{480}$ part of an *ounce*, it is convenient to introduce the price of 1 *dwt.*; not, however, to affect the answer, but only to derive from it the price of 15 *grs.* When this has been done, the cost of the 1 *dwt.* must be erased, before the products forming the answer are added together.

5. When the price is an *even number of shillings*, multiply the number of articles by *half* the number of shillings, cut off the *units* figure of the result, and *double* it: reckon this doubled figure as the *shillings*, and the rest of the result as the *pounds* of the answer.

The reason of this will be easily understood from the following example:

The cost of 313 articles at 18*s*. each, is

$$\frac{£313\times\overset{9}{\cancel{18}}}{\underset{10}{\cancel{20}}}=£\frac{2817}{10}$$

$$=£281\tfrac{7}{10}$$
$$=£281\tfrac{14}{20}$$
$$=£281 \text{ „ } 14s.$$

Hence if the cost of 648 articles at 38 shillings each be required, the result may be obtained by multiplying 648 by 19, cutting off and doubling the units figure for the shillings, and taking the rest of the product for pounds; thus

$$\begin{array}{r} 648 \\ 19 \\ \hline 1231,2 \end{array}$$

therefore £1231 „ 4*s*. answer.

6. When both the factors in the question contain fractional parts which are complicated, it is a good plan to turn the money factor into pounds and the *decimal* of a pound, and then to take the aliquot parts of the other factor: *e.g.* let it be required to find the price of 11 tons „ 17 cwt. „ 1 qr. „ 19 lbs. at £7 „ 7*s*. „ 4½*d*. per ton.

Here, reducing 7*s*. „ 4½*d*. to the decimal of a pound, we find it is £·36875.

Hence the cost of a ton is £7·36875.

10 cwt.	½	7·36875
		11
		81·05625
{5 cwt.	½	3·68437, &c.
{2 cwt.	⅕	1·84218, &c.
1 qr.	⅛	·73687, &c.
{14 lbs.	½	·09210, &c.
{ 4 lbs.	⅐	·04605, &c.
1 lb.	¼	·01315, &c.
		·00328, &c.
		87·47425, &c.

therefore, determining the shillings, pence and farthings by *inspection*, £87 „ 9*s*. „ 5¾*d*. is the cost required.

7. It is often convenient to obtain the required cost by expressing each of the quantities as a decimal of the highest

denomination given, and then multiplying one by the other even when the decimals are not finite. But it must be borne in mind that if in such cases the operation be performed in the ordinary way, the decimal result will only be approximately correct; and that at least as many figures as there are in the longest of the two factors, whether multiplier or multiplicand, must be rejected as incorrect from the right hand of the product. Here however the value of the "contracted form" of multiplication becomes apparent, as giving far greater accuracy with no greater labour. This the following example will illustrate:

Required the cost of 19 cwt. „ 3 qrs. „ 17 lb. at £5 „ 11*s.* 2*d.* per cwt.

Reducing these quantities to the decimal of a cwt. and of £1 respectively, we find that it is required to calculate the cost of 19·90178571 &c. cwt. at £5·558$\dot{3}$ each.

It might at first sight appear that if we multiplied 19·902 by 5·558, we should obtain a product correct to 6 places of decimals: whereas upon trial it will turn out that only the first figure is absolutely, and the second nearly correct; the others so incorrect as to be valueless. But by using "contracted multiplication" we should certainly obtain, (without using more figures in the actual multiplication,) 3 places of decimals absolutely correct, while the 4th place would never be too much or too little by more than one. In the two processes as set side by side below, the contracted method gives 4 places of decimals absolutely correct, the ordinary method only the first figure correct.

19·902	·00019901785
5·558	338555
159216	995089
99510	99509
99510	9951
99510	1592
110·615316	60
	6
	110·6207

Hence the required cost, (determining the shillings and pence by *inspection*,) is £110 12*s.* 4$\frac{3}{4}$*d.*

Exercise XIII.

EXAMPLES IN PRACTICE.

Find the cost of

1. 204 at 1*s.* „ 8*d.*
2. 324 at £1 „ 3*s.* „ 4*d.*
3. 800 at £2 „ 0*s.* „ 1½*d.*
4. 640 at £7 „ 4*s.* „ 7½*d.*
5. 320 at £3 „ 0*s.* „ 3¾*d.*
6. 582 at £12 „ 10*s.* „ 2½*d.*
7. Find the cost of 150 oranges at 9½*d.* per dozen.
8. The price of 265 sheep at £63 „ 3*s.* „ 1½*d.* per score.
9. The value of 85 articles at £8 „ 17*s.* „ 6*d.* for every 40.
10. The cost of 111 things at £11 „ 11*s.* „ 11*d.* for every 11.
11. The value of 425⅝ ounces of gold at £3 „ 17*s.* „ 10½*d.* per ounce.
12. The cost of 1243⅞ cwt. at £3 „ 14*s.* „ 2*d.* per cwt.

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13. Required the cost of 17 cwt. „ 1 qr. „ 19 lbs. at £1 „ 5*s.* „ 2*d.* per cwt.
14. Cost of 19 cwt. „ 3 qrs. „ 11 lbs. at £2 „ 9*s.* „ 8*d.* per cwt.
15. Cost of 37 cwt. „ 3 qrs. „ 2 lbs. at £3 „ 14*s.* „ 7½*d.* per cwt.
16. Cost of 72 cwt. „ 3 qrs. „ 17 lbs. at 6*s.* „ 1½*d.* per quarter.
17. Cost of 4 cwt. „ 2 qrs. „ 12 lbs. at £4 „ 13*s.* „ 4*d.* per quarter.
18. Cost of 5 cwt. „ 1 qr. „ 23 lbs. at 7½*d.* per lb.
19. Required the rent of 250 acres „ 3 roods „ 28 poles at £2 „ 15*s.* „ 6*d.* per acre.
20. Required the cost of 30 yards „ 1 foot „ 1½ inches at £6 „ 3*s.* „ 9*d.* per yard.

21. Find the price of 7 lbs. „ 5 oz. „ 12 dwts. „ 12 grs. at £4 „ 2*s.* „ 4*d.* per lb.

22. Find the cost of 2 tons „ 4 cwt. „ 1 qr. „ 3½ lbs. at £7 „ 9*s.* „ 4½*d.* per cwt.

23. Required the cost of a solid block containing 3 cub. yards „ 3 cub. feet „ 192 cub. inches at £13 „ 7*s.* „ 4½*d.* per cub. foot.

24. What is the price of 4 quarters „ 3 bushels „ 1 peck „ 1½ gallons at 13*s.* „ 6*d.* per bushel?

25. Find, (taking the aliquot part of an aliquot part,) the price of 45 acres „ 3 roods „ 24 poles at £96 „ 2*s.* „ 8½*d.* per acre.

26. Find, (using only one aliquot part,) the cost of 18 lbs. „ 7 oz. „ 4 dwts. at £5 „ 5*s.* „ 8*d.* per lb.

27. Find the cost of 3 quarters „ 3 bushels „ 3 pecks at £6 „ 16*s.* „ 8*d.* per quarter (artifice 3).

28. Find, (introducing a subsidiary aliquot part,) the cost of making a road whose length is 3 miles „ 30 poles „ 5 yards at £72 „ 17*s.* „ 6*d.* per mile.

29. Find, (using decimals,) the price of 10 lbs. „ 11 oz. „ 16 dwts. „ 16 grs. of gold, at £3 „ 17*s.* „ 10½*d.* per oz.

30. Find the price of the following goods, each at an *even* number of shillings; viz.:

(1) 3019 at 18*s.* each.

(2) 517 at £1 „ 18*s.* each.

(3) 2466 at 16*s.* per dozen.

(4) 620 dozen at £2 „ 4*s.* per score.

31. Find, (by decimal multiplication,) the value of 256 lbs. „ 2 oz. „ 15½ dwts. of plate at 4*s.* „ 9⅜*d.* per oz.

32. Find, in the same manner, the price of 518 acres „ 3 roods „ 7⅘ poles at £118 „ 7*s.* „ 6*d.* per acre.

33. Cost of 8 cwt. „ 1 qr. „ 7 lbs. „ 6½ oz. at £3 „ 3*s.* „ 5½*d.* per cwt.

34. Cost of 17 dwts. „ 15⅝ grs., at £5 „ 7 florins „ 2 cents „ 5 mils per lb. Troy.

35. The value of a crop of grass growing on 3 roods „ 31 poles „ 15½ sq. yds., at £5 „ 13*s.* „ 9¾*d.* per acre.

36. The value of 7 sq. chains „ 13 poles „ 12 sq. yds., at £117 „ 1*s.* „ 2¾*d.* per acre.

CHAPTER XII.

PROPORTION, COMMONLY CALLED THE RULE OF THREE.

§ 86. It is necessary to have a clear idea of the sense in which the words *Ratio* and *Proportion* are used, though confused notions concerning them are very common; and this is perhaps in some degree to be attributed to the fact that the Definition of Ratio, as generally given, is expressed in terms that are not sufficiently explicit[1].

Let us first endeavour to grasp the idea which our words are intended to convey. We are about to institute a comparison between two numbers, in order to determine *how many times* one of them is greater or is less than the other.

Yet with the common confusion that arises from loosely-expressed ideas, beginners often take this to be the same thing as enquiring *how much* one number is greater or less than another. This is the error against which we specially raise a warning. It is true that in establishing the relative

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[1] Euclid's definition of Ratio (Bk. v. Def. 3) is thus translated: "Ratio is a mutual relation of two *magnitudes* of the same kind to "one another in respect of *quantity*." While in Algebra Ratio is commonly defined as "the relation which one *quantity* bears to "another with respect to *magnitude*." By thus using the two terms magnitude and quantity, which in common language are taken as synonymous, as if they could mutually explain each other, a defective notion of the thing meant is conveyed. This indistinctness in the Algebraical definition is generally removed by adding the words "the comparison being made by considering what multi-"ple, part, or parts the first is of the second:" and it would be well if it could be removed from the Geometrical Definition, by amending the translation. In the Greek, Euclid's definition is as follows: *Λόγος ἐστὶ δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικότητα πρὸς ἄλληλα ποιὰ σχέσις.* In this the word *πηλικότης* has been rendered by *quantity*, whereas it would be better translated by some word expressing *how-many-times* one magnitude contains another. Euclid's own meaning is made evident by the fourth definition, "Magnitudes "are said to have a ratio to one another, which when *multiplied* "(*πολλαπλασιαζόμενα*) can exceed one the other."

magnitude of two numbers, there are two ways in which it is *possible* to make the comparison: either by *subtracting* one from the other, and seeing *how much* greater the one is than the other; or else by *dividing* one by the other, and seeing *how many times* it is greater than the other; but it must be remembered that it is the second method *only* which is contemplated when the word "Ratio" is used.

Take the numbers 6 and 2; 6 is greater than 2 by 4; but this is a comparison which has nothing at all to do with *ratio;* the *ratio* of 6 to 2 is found by *dividing* 6 by 2, and saying that 6 is 3 *times* greater than 2.

We therefore require in our definition a word which shall express *the number of times* one number is greater or less than another. Accordingly if, following the analogy of the word multiplicity from *multiplex*, manifold, we adopt the word *quantuplicity*, explaining it to mean *what number of times* one magnitude contains or is contained by another, we may give our definition thus: "Ratio is the mutual relation of two "magnitudes of the same kind with reference to quantu-"plicity[1];" that is to say, the ratio between two quantities is determined by considering *how many times* the first is greater or less than the second.

From this it is clear that quantuplicity, and therefore ratio, can only subsist between either abstract numbers, or else concrete quantities which are *of the same kind:* we may divide one abstract number by another abstract number; or we may enquire how many times one quantity is greater or less than another quantity of the same kind, comparing one length with another length, or one weight with another weight; but it would be absurd to compare a pound weight with a yard, or to enquire how many times a quart were contained in a mile; adopting Euclid's test, "such quantities cannot be multiplied "so as to exceed one the other."

[*Obs.* It is very important to bear in mind, that in determining the ratio which one quantity bears to another, we are seeking to establish *how many times* the first contains or is contained by the second; and that this comparison can only be made between quantities which are *of the same kind.*]

[1] In arithmetic, ratio has been sometimes defined to be "the relation which one number bears to another with respect to *quotity;*" the same attempt being made to evade the ambiguous use of the word quantity.

§ 87. **A ratio is usually written with two dots, one above the other, placed between its two terms; thus the ratio of 6 to 2 is written 6 : 2, where the first term is called the *antecedent*, the second the *consequent*.**

But to determine the value of a ratio, the antecedent is written as the numerator, and the consequent as the denominator of a fraction; for this will determine how many times the first contains or is contained in the second, since a fraction (see Def. 2, § 44, page 72) is a simple manner of expressing the division of the numerator by the denominator; and the magnitude of the fraction thus determines the value of the ratio. The ratio of 6 to 2 is expressed by the fraction $\frac{6}{2}$, or $\frac{3}{1}$, which denotes that 6 is *three times greater than* 2; whereas the ratio of 2 to 6 would be expressed by the fraction $\frac{2}{6}$, or $\frac{1}{3}$; which denotes that 2 is *one-third of* 6, *i.e.* is *three times less than* 6. By this method ratios can be treated *arithmetically*, their values determined, and so compared with one another.

§ 88. *Proportion is the relation of equality subsisting between ratios.*

If we have two numbers as 6 and 2, of which we know the first is 3 times greater than the second, and two other numbers 15 and 5, of which the first is again 3 times greater than the second, the *ratio* of 6 to 2 is *equal* to the ratio of 15 to 5, and these four terms are said to constitute a proportion. Such a proportion is usually written $6 : 2 :: 15 : 5$, or $6 : 2 = 15 : 5$; and since each ratio may be written as a fraction, the proportion may be written $\frac{6}{2} = \frac{15}{5}$.

The first and last terms of a proportion are called the *extremes*, the two middle terms the *means*.

Although the two terms of a ratio, if they be not abstract numbers, must be quantities of *the same kind*, it is not necessary that all the four terms of a proportion should be of the same kind: it will be sufficient that the quantities in the first ratio be of one kind, and the quantities in the second ratio of one kind; thus 3 cwt. : 12 cwt. = 7 inches : 28 inches; and generally we may say, that any four quantities are proportionals *when the first is the same number of times greater or less than the second that the third is greater or less than the*

fourth; in other words, *when the first is the same multiple part or parts of the second that the third is of the fourth.*

§ 89. Since a proportion expresses the equality of ratios, and the value of these ratios may be denoted by fractions, the properties of ratios are made to depend directly upon the properties of fractions.

Hence, if we take any two ratios which are equal to one another, for instance 4 : 12 and 7 : 21, where it is true that 4 : 12 :: 7 : 21, we may say

$$\frac{4}{12} = \frac{7}{21};$$

next, reducing these fractions to equivalent fractions having a common denominator, we have

$$\frac{4 \times 21}{12 \times 21} = \frac{7 \times 12}{21 \times 12},$$

or

$$4 \times 21 = 7 \times 12;$$

and as the numbers taken were not particular but general, we deduce the following general rule, viz. that in every proportion, *the product of the extremes is equal to the product of the means.*

From this it follows as a necessary consequence that if the product of the *means* be divided by *one* extreme, the quotient will be the *other* extreme; or if the product of the *extremes* be divided by *one* mean, the quotient will be the *other* mean.

It is important to notice this, because upon this property of proportion depends the solution of questions in the Rule of Three; where three known terms of a proportion are given, to find a fourth unknown term.

Now, without going deeply into Algebra, we may say *generally* that if a, b, c, d are four quantities such that $a : b :: c : d$, we are at liberty to assume that the product of the extremes is in every case equal to the product of the means, or that $ad = bc$; whence by dividing each of these equal quantities successively by d, by a, by c, and by b, we have

$$a = \frac{bc}{d}, \quad d = \frac{bc}{a},$$

$$b = \frac{ad}{c}, \quad c = \frac{ad}{b};$$

that is to say, if a, d, b, c be in turn the unknown term, they could each be found, provided the other three terms were known.

In the method of arranging the terms of a proportion, or of "stating the sum," as we now proceed to explain it, it will be observed that we never allow only *three* terms to be written in any proportion, but always *four*, three of which are known, and one unknown. Also, that the *place* occupied by the unknown term is perfectly immaterial; it may be last, or first, or third, or second; for so long as the terms are correctly arranged, and the product of the extremes taken as equal to the product of means, the right result is sure to be obtained.

It will be best now to illustrate the foregoing explanations by examples.

The simplest form under which an example in the "Rule of Three" can stand is as follows:

Ex. 1. *If* 17 *barrels of beer cost* £51, *what number of barrels can be bought for* £93?

Of two different quantities of the same article, bought at the same rate, the one will be *as many times* greater or less than the other as the sum of money expended in the first case is greater or less than the sum expended in the latter case; hence we can form a proportion, and say

$$17 \text{ barrels} : \textit{Ans.} \text{ barrels} :: £51 : £93;$$

we next multiply the *means* together and the *extremes* together, remembering that these two products are *equal;* whence

$$\textit{Ans.} \times 51 = 17 \times 93;$$

we then divide *equals* by the *same* number; *i.e.* divide both sides of this equality by 51, and *cancelling*, or dividing both numerator and denominator of the fraction by 17, we get

$$\textit{Ans.} = \frac{\cancel{17} \times 93}{\underset{3}{\cancel{51}}}$$

$$= 31 \text{ barrels.}$$

Observe that as every proportion consists of *four* terms, so in every statement it will be found that there are *three* known and *one* unknown term; over this unknown term in the statement, which is usually represented by "*Ans.*" it is well to write the denomination which it is meant to represent.

It is immaterial whether the unknown term be placed first, second, third, or fourth in the statement, so long as the other terms be arranged correctly: thus, in the example given, we

should have been at liberty to make either of the four following statements:

Ans. barrels[1] : 17 barrels :: £93 : £51,

17 barrels : *Ans.* barrels :: £51 : £93,

£93 : £51 :: *Ans.* barrels : 17 barrels,

£51 : £93 :: 17 barrels : *Ans.* barrels,

but in each case by multiplying together the *extremes* and *means* we should obtain

$$Ans. \times 51 = 17 \times 93.$$

Be careful not to set down in the statement

17 barrels : £51 :: *Ans.* barrels : £93.

For although by multiplying together extremes and means we should here also obtain $Ans. \times 51 = 17 \times 93$; nevertheless we should be involved in two false ratios: as it is absurd to enquire *how many times* some *barrels* are greater or less than some *pounds*.

Not only must the quantities in the respective ratios be of the same *kind*, but they must be brought to the same *denomination*, if they be not so in the question. Suppose it had been asked "If 17 barrels cost 51 *guineas*, what number of barrels could be bought for 93 *pounds?*" It is clear that the statement already given would not in this case produce a correct result; and that it is not sufficient that the terms 51 and 93 should be of the same *kind*, namely money; but either the guineas must be expressed in the denomination of pounds, or the pounds in the denomination of guineas, or both in the denomination of shillings, before we could form a proper *ratio*.

When after having stated the proportion, we begin to multiply together extremes and means, we are involved in the seeming absurdity of being required to multiply together two such concrete quantities as *barrels* and *pounds:* an unknown number of barrels multiplied by 51 pounds being said to be equal to 17 barrels multiplied by 93 pounds. But it is to be remembered, that the ratio of 51 *pounds* to 93 *pounds* is the same as the ratio of the abstract number 51 to the abstract

[1] What is meant by such expressions as "*Ans.* barrels," "*Ans.* shillings," "*Ans.* stone," &c. is that the unknown quantity which is the *Ans.*, is in the denomination of barrels, shillings, stone, &c.

number 93. We may therefore be supposed to consider only the abstract numerical value of the terms; and by the principles of proportion we shall find the numerical value of the unknown term.

Ex. 2. *A bankrupt's debts are £5620, and his effects are only £1405; what dividend will he pay?*

In this case it is clear, that as many times as his debts are greater than his effects, so many times will the pound be greater than the dividend or shillings he can pay in the pound.

Hence, expressing the pound in the denomination of shillings, we shall have

$$\overset{\text{debts}}{5620} : \overset{\text{effects}}{1405} :: 20s. : Ans. \text{ shillings},$$

$$Ans. \times 5620 = 1405 \times 20,$$

$$Ans. = \frac{1405 \times \cancel{20}}{\cancel{5620}_{281}}$$

$$= 5 \text{ shillings}.$$

Ex. 3. *If* $12\frac{1}{2}$ *tons cost £3 „ 15s., what will 5 cwt. cost?*

Instead of reducing each of the given quantities to their lowest terms, fractions may be commonly used with advantage; care being taken to express the quantities in each ratio in the *same* denomination. Thus, in the example given, expressing 5 cwt. as a fraction of a ton, and 15s. as the fraction of £1, we have

$$12\tfrac{1}{2} \text{ tons} : \tfrac{1}{4} \text{ ton} :: £3\tfrac{3}{4} : £Ans.,$$

$$\frac{25}{2} \times Ans. = \frac{1}{4} \times \frac{15}{4},$$

$$Ans. = \frac{1}{4} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{\underset{5}{\cancel{25}}}$$

$$= £\frac{3}{40}$$

$$= 1s. „ 6d.$$

Ex. 4. *If* $\frac{7}{8}$ *of a share in a speculation be worth £21 „ 17s. „ 6d., what would* $\frac{3}{8}$ *of a share be worth?*

$$\frac{7}{8} : \frac{2}{3} :: 21\tfrac{7}{8} : Ans.,$$

$$\frac{7}{8} \times Ans. = \frac{2}{3} \times \frac{175}{8},$$

$$Ans. = \frac{2}{3} \times \frac{\overset{25}{\cancel{175}}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{7}}$$

$$= \frac{50}{3} = 16\tfrac{2}{3}$$

$$= £16 \text{ ,, } 13s. \text{ ,, } 4d.$$

Ex. 5. *Find a fourth proportional to the numbers* 2·7, 2·425, 10·8.

This example will serve to shew how decimals may be used in the solution of any questions in proportion.

$$2·7 : 2·425 :: 10·8 : Ans.,$$

$$Ans. \times 2·7 = 2·425 \times 10·8,$$

$$Ans. = \frac{2·425 \times \overset{4}{\cancel{10·8}}}{\cancel{2·7}}$$

$$= 9·700,$$

whence 9·7 is the fourth term of the proportion.

Ex. 6. *If* 123·5 *lbs. cost* 7·4 *shillings, what number of lbs. can be bought for* £3·33?

$$123·5 \text{ lbs.} : Ans. \text{ lbs.} :: £\frac{7·4}{20} : £3·33,$$

$$Ans. \times \frac{3·7}{10} = 123·5 \times 3·33,$$

$$Ans. \times ·37 = 123·5 \times 3·33,$$

$$Ans. = \frac{123·5 \times 3·33}{·37}$$

$$= \frac{123·5 \times \overset{9}{\cancel{333}}}{\cancel{37}}$$

$$= 1111.5 \text{ lbs.}$$

Ex. 7. *If, after paying an income-tax of 7d. in the pound, a person has £776 ,, 13s. ,, 4d. remaining, what is his actual gross income?*

Every pound of 240 pence is reduced, by paying a tax of 7*d*., to 233 pence, and the whole income is reduced in the same ratio; hence

$$240d. : 233d. :: \text{original income} : \text{reduced income},$$
$$:: Ans. : 776\tfrac{2}{3},$$
$$233 \times Ans. = 240 \times \frac{2330}{3},$$
$$Ans. = \overset{80}{\cancel{240}} \times \frac{\overset{10}{\cancel{2330}}}{\cancel{3}} \times \frac{1}{\cancel{233}}$$
$$= £800.$$

Ex. 8. *The net rental of an Estate, after deducting 6d. in the pound for Income-Tax, and then 5 per cent. more from the portion which remained for the expense of collecting, is £694 ,, 13s. ,, 9d.; what is the gross rental?*

By paying 6*d*. in the pound, £100 would pay £2 ,, 10*s*., and be reduced to £97 ,, 10*s*. This £97 ,, 10*s*. would now pay a tax of 5 per cent., *i.e.* would pay £4 ,, 17*s*. ,, 6*d*., and be thereby further reduced to £92 ,, 12*s*. ,, 6*d*. And the original income would be reduced in the same ratio: hence

$$£100 : £92\tfrac{5}{8} :: \text{original income} : \text{reduced income},$$
$$:: Ans. : 694\tfrac{11}{16},$$
$$\frac{741}{8} \times Ans. = 100 \times \frac{11115}{16},$$
$$Ans. = \overset{50}{\cancel{100}} \times \frac{\overset{15}{\cancel{11115}}}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{\cancel{741}}$$
$$= £750.$$

Ex. 9. *Given that the diameter of a circle is to the circumference as* $1 : 3\frac{1}{7}$; *find the number of revolutions made by a wheel whose radius is 1 foot 9 inches in a journey of 8 miles.*

When the radius is 1 foot 9 inches, the diameter is 3 feet 6 inches, and we are told that

$$\text{diameter} : \text{circumference} :: 1 : 3\tfrac{1}{7};$$

therefore $$3\tfrac{1}{2} : \text{circumference} :: 1 : 3\tfrac{1}{7},$$
$$\text{circumference} = \frac{7}{2} \times \frac{22}{7}$$
$$= 11.$$

The question is now only how many times a length of 11 feet is contained in 8 miles; hence, bringing miles to feet, we have

$$Ans. = \frac{8 \times \overset{160}{\cancel{1760}} \times 3}{\cancel{11}}$$
$$= 3840 \text{ times.}$$

Ex. 10. *Given that the areas of circles are as the squares of their diameters; there are two circular ponds, whose diameters are as* 11 : 12, *and the area of the first is* $3\frac{1}{2}$ *acres. Find the area of the second pond.*

$$\text{area of first pond : area of second pond} :: 11^2 : 12^2,$$
$$3\tfrac{1}{2} \text{ acres} : Ans. \text{ acres} :: 121 : 144,$$
$$121 \times Ans. = 144 \times \frac{7}{2},$$
$$Ans. = \overset{72}{\cancel{144}} \times \frac{7}{\cancel{2}} \times \frac{1}{121}$$
$$= \frac{504}{121}$$
$$= 4 \text{ acres} \,,,\, 0 \text{ roods} \,,,\, 26\tfrac{54}{121} \text{ perches.}$$

Ex. 11. *A clock gains* $3\frac{1}{4}$ *minutes in* 15 *seconds under the* 24 *hours; at noon it is* 2 *minutes too slow: when will it indicate true time?*

Take 15 seconds from 24 hours, and there remain 23 hrs. „ 59 min. „ 45 sec. The question therefore is, "*if a clock gain* $3\frac{1}{4}$ *minutes in* 23 *hours* „ 59 *min.* „ 45 *sec., in what time will it gain* 2 *minutes?*"

$$3\tfrac{1}{4} \text{ min.} : 2 \text{ min.} :: 23\tfrac{239}{240} \text{ hours} : Ans. \text{ hours},$$
$$\frac{13}{4} \times Ans. = 2 \times \frac{5759}{240},$$
$$Ans. = \cancel{2} \times \frac{5759}{\underset{\underset{30}{\cancel{60}}}{\cancel{240}}} \times \frac{\cancel{4}}{13}$$
$$= \frac{443}{30}$$
$$= 14 \text{ hours} \,,,\, 46 \text{ minutes};$$

therefore the clock will indicate true time at 46 minutes past 2 A.M. the following day.

Ex. 12. *Find the value of* 3 *cwt.* „ 2 *qrs.* „ 11 *lbs.*, *if* 27 *cwt.* „ 19 *lbs. cost* £37 „ 16*s.* „ $4\frac{1}{2}$*d.*

When the quantities given in the question are *compound*, we may first reduce the compound quantities in each ratio to the decimal of the highest denomination in either, and may then (in a manner similar to that suggested in "Practice") use contracted multiplication and division of decimals, retaining four places of decimals in the quotient, whereby a sufficient degree of accuracy will be attained.

Thus, first we must express the given weights as decimals of 1 cwt., and the price as a decimal of £1;

$\begin{cases}4\\7\end{cases}$	11· 2·75	$\begin{cases}4\\7\end{cases}$	19· 4·75
4	2·39285	4	0·6785
	3·59821, &c., cwt.		27·16963, &c., cwt.,

and by *inspection* £37 „ 16*s.* „ $4\frac{1}{2}$*d.* = £37·818.

Hence 27·16963 : 3·59821 :: 37·818 : *Ans.*,

$$27·16963 \times Ans. = 3·59821 \times 37·818,$$

$$Ans. = \frac{3·59821 \times 37·818}{27·16963}.$$

```
·00359821
    81873
---------
  1079463
   251875        2·7169,6) 13·6077 (5·0086
    28786                    229
      360                     13
      288                      1
---------
 136·0772
```

Hence £5·0086, which by *inspection* = £5 „ 0*s.* „ $2\frac{1}{4}$*d.*, is the cost required.

Ex. 13. *Required cost of* 55 *cwt.* „ 7 *lbs.*, *if* 6 *cwt.* „ 2 *qrs.* „ $17\frac{1}{2}$ *lbs. cost* £41 „ 7*s.* „ 3*d.*

Reducing these compound quantities to the decimals of the highest denominations, we have

$$55·0625 : 6·65625 :: Ans. : 41·3625,$$

$$6·65625 \times Ans. = 55·0625 \times 41·3625,$$

$$Ans. = \frac{55·0625 \times 41·3625}{6·65625}.$$

$$
\begin{array}{r}
\cdot 00413625 \\
52\cdot 6055 \\
\hline
20681250 \\
2068125 \\
24818 \\
827 \\
207 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
6\cdot 65625)\,2277\cdot 5227\,(342\cdot 1630 \\
2806477 \\
143977 \\
10852 \\
4196 \\
202 \\
2
\end{array}
$$

Hence £342 „ 3*s*. „ 3*d*. is the required cost.

§ 90. It often happens that there are more than *three* known terms given in the questions proposed; and it is common to class such questions under a distinct rule called "double rule of three," or "compound proportion":—they may, however, be all reduced to simple proportion; and examples will be now subjoined exhibiting the process, of every step in which the reason will be explained.

Ex. 14. *If* 9 *men in* 8 *days consume* 12 *stone of flour, how many stone will serve* 24 *men for* 30 *days?*

The process will be as follows:

9 men × 8 days : 24 men × 30 days :: 12 stone : *Ans.* stone.

$$Ans. \times 9 \times 8 = 24 \times 30 \times 12.$$

$$Ans. = \frac{\overset{3}{\cancel{24}} \times \overset{10}{\cancel{30}} \times 12}{\cancel{9} \times \underset{3}{\cancel{8}}}$$

$$= 10 \times 12$$

$$= 120 \text{ stone.}$$

In making this *statement*, it will at first appear as if we began by actually multiplying together such concrete quantities as men and days: we point out, however, that the 9 does not really represent 9 actual men, but the *daily consumption* of 9 men; and the reason why we may multiply this by 8 days, or repeat the *daily* consumption of the 9 men 8 *times* over, will be easily understood, if we say

Let 1 represent the consumption of 1 man in 1 day,
then 9 will represent .... 9 men in 1 day,
and 9×8 .... 9 .... 8 days.

Similarly, 24×30 will represent the consumption of 24 men in 30 days; but we are told that the quantity consumed by the first set of 9 men in 8 days was 12 stone; and this must bear the same ratio to the quantity consumed by the second set of 24 men in 30 days, as 9×8 does to 24×30: hence writing the terms as a proportion, we have

$$\overset{\text{first cause}}{9 \times 8} : \overset{\text{second cause}}{24 \times 30} :: \overset{\text{first effect}}{12} : \overset{\text{second effect}}{\textit{Ans.}}$$

Here likewise it will be observed, that it is immaterial *where* in the statement the unknown term is placed: it is only necessary to arrange the terms so that the first *cause* bears to the second *cause* the same ratio that the first *effect* bears to the second *effect;* the *causes* will be the agencies of the living agents, so many times repeated, the *effects* will be the food consumed, the work performed, the money spent, &c. Hence the Rule will be "Multiply the number of *living* "*agents*[1] in each case by the *time* (the months, days, hours, &c.) "they respectively work for: place these as the *first* and *second* "terms of the proportion: place in the *third* term the *work* "done by the agents in the *first* term, and in the *fourth* term "the *work* done by the agents in the *second* term." Having thus made the statement, multiply together *extremes* and *means;* only for the convenience of cancelling do not actually multiply together the various factors, but set them down with the sign of multiplication between them, placing the *Ans.* with its factors, according to custom, on the *left*-hand side of the equality, and the other factors on the *right.* Then divide the factors on the *right* by the factors standing with the *Ans.* on the *left;* which in effect divides each side of the equation by the same factors.

Ex. 15. *If* 8 *men in* 5 *days of* 6 *hours each can mow* 20 *acres, how many acres can be mowed by* 12 *men in* 4 *days of* 11 *hours each?*

Here the 8 men worked for 5 *times* 6 hours, or 30 hours in all: and the 12 men worked for 4 *times* 11 hours, or 44 hours:

[1] The few cases in which there are no *living* agents, will be noticed below, where, as in Examples 16, &c., it will be shewn how to distinguish the agencies from the effects produced.

the *hourly* work therefore of the 8 men will be repeated 30 times, and that of the 12 men, 44 times: not multiplying up these factors however, for the convenience of cancelling, we write

8 men × 5 dys. × 6 hrs. : 12 men × 4 dys. × 11 hrs. :: 20 acres : *Ans.* acres,

$$8 \times 5 \times 6 \times Ans. = 12 \times 4 \times 11 \times 20,$$

$$Ans. = \frac{\overset{2}{\cancel{12}} \times \cancel{4} \times 11 \times \overset{4}{\cancel{20}}}{\underset{2}{\cancel{8}} \times \cancel{5} \times \cancel{6}}$$

$$= 44 \text{ acres}.$$

Ex. 16. *If goods weighing* 17 *tons be conveyed* 60 *miles on a railway for* £12 „ 15*s*., *how far on a canal may goods weighing* 11 *tons be carried for* £8 „ 5*s*., *it being supposed that the rate of carriage is half as much again by rail as by water?*

In this case there is no "*living* agent": but the sum of money paid for carriage depends on (1st) the weight carried, (2nd) the distance. The weight and the distance are therefore the *causes* of the payment of a certain sum of money the *effect*.

Also, as the rate by rail is $1\frac{1}{2}$ times the rate by water, the goods which cost £8 „ 5*s*. by *canal*, would have cost £8 „ 5*s*. + £4 „ 2*s*. „ 6*d*., or £12 „ 7*s*. „ 6*d*., by *railway*: hence

17 tons × 60 miles : 11 tons × *Ans.* miles :: £$12\frac{3}{4}$: £$12\frac{3}{8}$.

$$\frac{51}{4} \times 11 \times Ans. = 17 \times 60 \times \frac{99}{8},$$

$$Ans. = \cancel{17} \times 60 \times \frac{\overset{9}{\cancel{99}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\underset{3}{\cancel{51}}} \times \frac{1}{\cancel{11}}$$

$$= \frac{60 \times 9}{2 \times 3}$$

$$= 90 \text{ miles}.$$

Obs. The use of fractions will usually be found to render the working of examples in "Rule of Three" much shorter than the process of bringing the various terms to their lowest denomination.

Ex. 17. *If* 18 *men working* 9 *hours a day take* 6 *days to build a stone wall* 90 *yards long,* 10 *feet* 6 *inches high, and*

1 foot 4 inches thick, how many weeks will it take 17 men working 11 hours daily to build such a wall 2720 feet long, 8 feet 3 inches high, and 1 foot 9 inches thick?

Here observe that the first set of men work 6 *days*, the other set an unknown number of *weeks*; therefore if we find the answer in *days*, we must afterwards bring that result into *weeks*. Also, that the first wall was 90 *yards* long, the second 2720 *feet*; therefore we must reduce these to the same denomination, and multiply the 90 yards by 3, to bring them into feet. Further, that instead of reducing all the dimensions to *inches*, we write the inches as fractions of a foot.

$$18 \times 6 \times 9 : 17 \times \textit{Ans.} \times 11 :: (90 \times 3) \times 10\tfrac{1}{2} \times 1\tfrac{1}{3} : 2720 \times 8\tfrac{1}{4} \times 1\tfrac{3}{4},$$

$$17 \times \textit{Ans.} \times 11 \times 90 \times 3 \times \frac{21}{2} \times \frac{4}{3} = 18 \times 6 \times 9 \times 2720 \times \frac{33}{4} \times \frac{7}{4},$$

$$\textit{Ans.} = 18 \times 6 \times \cancel{9} \times \overset{\overset{10}{\cancel{160}}}{\cancel{2720}} \times \frac{\overset{11}{\cancel{33}}}{\cancel{4}} \times \frac{\cancel{7}}{\cancel{4}} \times \frac{1}{\cancel{17}} \times \frac{1}{11} \times \frac{1}{\underset{10}{\cancel{90}}} \times \frac{1}{\cancel{3}} \times \frac{\cancel{2}}{\underset{3}{\cancel{21}}} \times \frac{3}{\underset{2}{\cancel{4}}}$$

$$= 18 \times 6 \times \tfrac{1}{2}$$

$$= 54 \text{ days},$$

and as they must be supposed to work only 6 days in the week, the answer will be 9 weeks.

§ 91. The questions given in proportion are not always set in these perfectly simple terms; sometimes both care and ingenuity are required to prepare the question, before it is *stated* as a proportion. We shall accordingly now proceed to give examples where the questions are not put exactly in the straightforward manner in which the previous examples have been given; yet we shall show that a little previous consideration will enable us to reduce each question to a plain proportion.

Ex. 18. *If 12 oxen and 35 sheep eat 6 tons 7 cwt. of hay in 4 days, how much will it cost per week to feed 4 oxen and 6 sheep, the price of hay being £3 „ 15s. per on, and 2 oxen being supposed to eat as much as 5 sheep?* (S.-H., 1 Nov., 1851).

If	the consumption of	2 oxen	=	that of	5 sheep,
then	....	12	=	....	30 ....,
and	....	4	=	....	10 ....

Also the cost of $6\frac{7}{20}$ tons of hay at £$3\frac{3}{4}$ per ton will be

$$\frac{127}{\overset{}{\underset{4}{\cancel{20}}}} \times \frac{\overset{3}{\cancel{15}}}{4} = £\frac{381}{16}.$$

Hence the question becomes "*If it cost* £$\frac{381}{16}$ *to feed* 30 *sheep* + 35 *sheep for* 4 *days, how much will it cost to feed* 10 *sheep* + 6 *sheep for* 7 *days?*"

$$65 \times 4 : 16 \times 7 :: \frac{381}{16} : Ans.,$$

$$65 \times 4 \times Ans. = \cancel{16} \times 7 \times \frac{381}{\cancel{16}},$$

$$Ans. = 7 \times 381 \times \frac{1}{65} \times \frac{1}{4}$$

$$= \frac{2667}{260}$$

$$= £10 \text{ ,, } 5s. \text{ ,, } 1\tfrac{11}{13}d.$$

Ex. 19. *If a piece of work can be done in* 50 *days by* 35 *men working at it together, and if, after working together for* 12 *days,* 16 *of the men were to leave the work; find the number of days in which the remaining men could finish the work.* (S.-H., Jan., 1851.)

If the 35 men could do the work in 50 days, they do $\frac{1}{50}$ daily; and in 12 days they do $\frac{12}{50}$, and leave $\frac{38}{50}$ undone; and this is finished by $35-16$ men, *i.e.* by 19 men, in an unknown time.

Hence the question becomes "*If* 35 *men can do* $\frac{12}{50}$ *of a piece of work in* 12 *days, how many days will* 19 *men take to do* $\frac{38}{50}$ *of the work?*"

$$35 \times 12 : 19 \times Ans. :: \frac{12}{50} : \frac{38}{50}$$

$$:: 12 : 38,$$

$$19 \times 12 \times Ans. = 35 \times 12 \times 38,$$

$$Ans. = \frac{35 \times 12 \times 38}{19 \times 12}$$

$$= 35 \times 2$$

$$= 70 \text{ days.}$$

Ex. 20. *If 15 men, 12 women, and 9 boys can complete a piece of work in 50 days, what time would 9 men, 15 women, and 18 boys take to do four times as much, the parts done by each in the same time being as the numbers 3, 2, 1?* (S.-H., March, 1851).

Here we may convert the agency of the men and the women into that of boys, and say

if	the work of 1 man	= that of 3 boys,	
	.... 15 men	= .... 45 .....,	
also	.... 12 women	= 24	

Hence the first set of workers were equivalent to $45+24+9$ boys, or to 78 boys; similarly the second set were equivalent to $27+30+18$ boys, or to 75 boys. Hence the question is reduced to this: "*If 78 boys complete a piece of work in 50 days, how many days will 75 boys take to do four such pieces of work?*"

$$78 \times 50 : 75 \times Ans. :: 1 : 4,$$

$$Ans. \times 75 = 78 \times 50 \times 4,$$

$$Ans. = \frac{78 \times \overset{2}{\cancel{50}} \times 4}{\underset{3}{\cancel{75}}}$$

$$= 26 \times 2 \times 4$$

$$= 208 \text{ days.}$$

Ex. 21. *If the rate of wages depend upon the price of wheat, and 18 men working for 4 weeks receive £43 „ 4s. when wheat is 64 shillings a quarter; find the price of wheat when 16 men working for 5 weeks obtain £67 „ 10s.*

Since 18 men in 4 weeks receive £43 „ 4*s*. or 864 shillings, it follows that the weekly wage of each man was

$$\frac{864}{18 \times 4}, \text{ or } 12s.$$

Also in the second case the weekly wage was

$$\frac{1350}{16 \times 5}, \text{ or } \frac{135}{8} s., \text{ or } 16\tfrac{7}{8} s.;$$

and the wage was higher or lower as the price of wheat was greater or less; hence

$$12 : 16\tfrac{7}{8} :: 64 : Ans.,$$

$$12 \times Ans. = \frac{135}{\cancel{8}} \times \overset{8}{\cancel{64}},$$

$$Ans. = \frac{135 \times 8}{12}$$
$$= 90s.$$

Ex. 22. *Four men working 8 hours a day take 22 days to pave a road 440 yards long and 35 feet broad; how many days will four men, two of whom work 8 hours and two 10 hours a day, take to pave a road 1575 yards long and 36 feet 6 inches broad?* (S.-H., Jan., 1855).

Here the latter set of four men worked two for 8 and two for 10 hours a day; therefore *on the average* they worked for 9 hours a day; and we have

$$4 \times 22 \times 8 : 4 \times Ans. \times 9 :: 440 \times 3 \times 35 : 1575 \times 3 \times 36\tfrac{1}{2},$$

$$4 \times Ans. \times 9 \times 440 \times 3 \times 35 = 4 \times 22 \times 8 \times 1575 \times 3 \times \frac{73}{2},$$

$$Ans. = \frac{\cancel{22} \times 8 \times \overset{45}{\cancel{1575}} \times \cancel{3}}{9 \times \underset{20}{\cancel{440}} \times \cancel{3} \times \cancel{35}} \times \frac{73}{2}$$

$$= \frac{8 \times 45 \times 73}{9 \times 20 \times 2}$$

$$= 73 \text{ days.}$$

Ex. 23. *One horse is a power which can raise 33000 lbs. through 1 foot in 1 minute; what must be the horse power of an engine in order to raise 4125 tons through 3 yards in 7 hours?*

Since the given *unit of power* is the effort requisite to raise 33000 lbs. through the space of a *foot* in a minute of time, to raise 33000 lbs. through 3 *yards* would be to raise 33000 lbs. through 9 *feet*, or would be to raise a weight equivalent to 9 *times* 33000 lbs. through a foot in a minute.

Hence expressing 4125 *tons* as *lbs.*, and 7 *hours* as *minutes*, we have

$$\overset{\text{horse}}{1} \times \overset{\text{min.}}{1} : \overset{\text{horses}}{Ans.} \times \overset{\text{minutes}}{(7 \times 60)} :: \overset{\text{lbs.}}{33000} \times \overset{\text{foot}}{1} : \overset{\text{lbs.}}{(4125 \times 20 \times 112)} \times \overset{\text{feet}}{3 \times 3},$$

$$Ans. \times 7 \times 60 \times 33000 = 4125 \times 20 \times 112 \times 3 \times 3,$$

$$Ans. = \frac{4125 \times 20 \times 112 \times 3 \times 3}{7 \times 60 \times 33000}$$

$$= 6 \text{ horses.}$$

Ex. 24. *If* 60 *cannon which fire* 5 *rounds in* 8 *minutes kill on an average* 350 *men every* 75 *minutes, how many cannon firing* 7 *rounds in* 9 *minutes will kill* 980 *men in* 25 *minutes?*

Each of the first set of cannon which fired 5 rounds in 8 minutes, fired one round in $\frac{5}{8}$ of a minute; similarly each of the second set fired 1 round in $\frac{7}{9}$ of a minute. Hence

$$60 \times \frac{5}{8} \times 75 : Ans. \times \frac{7}{9} \times 25 :: 350 : 980,$$

$$Ans. \times \frac{7}{9} \times 25 \times 350 = 60 \times \frac{5}{8} \times 75 \times 980,$$

$$Ans. = 60 \times \frac{5}{8} \times 75 \times 980 \times \frac{9}{7} \times \frac{1}{25} \times \frac{1}{350}$$
$$= 405 \text{ cannon.}$$

Ex. 25. *If the sixpenny loaf weigh* 4·35 *lbs. when wheat is at* 5·75*s. per bushel, what ought to be paid for* 49·3 *lbs. of bread, when wheat is* 18·4*s. per bushel?*

Wheat at 5·75 gives 4·35 lbs. for 6*d.*,

.... 1 lb. ... $\frac{6}{4 \cdot 35}d.$

Wheat at 18·4 gives 49·3 lbs. for *Ans. d.*,

.... 1 lb. ... $\frac{Ans.}{49 \cdot 3}d.$,

and the price per lb. is greater or less according as the price per bushel is greater or less; hence

$$5 \cdot 75 : 18 \cdot 4 :: \frac{6}{4 \cdot 35} : \frac{Ans.}{49 \cdot 3},$$

$$5 \cdot 75 \times \frac{Ans.}{49 \cdot 3} = 18 \cdot 4 \times \frac{6}{4 \cdot 35},$$

$$Ans. = 18 \cdot 4 \times \frac{6}{4 \cdot 35} \times 49 \cdot 3 \times \frac{1}{5 \cdot 75}$$
$$= \frac{184 \times 6 \times 493}{435 \times 5 \cdot 75}$$
$$= \frac{544272}{2501 \cdot 25}$$
$$= 217 \cdot 6 \text{ pence}$$
$$= 18s. \text{ ,, } 1d. \text{ ,, } 2\tfrac{2}{5} far.$$

Ex. 26. "*If eight best variegated silk scarfs, measuring each three cubits in breadth and eight in length cost a hundred nishcas; say quickly, merchant, if thou understand trade, what a like scarf, three and a half cubits long and half a cubit wide, will cost, in terms of drammas, pannas, cacinis, and cowry shells?*"

(One nishca = 16 drammas, one dramma = 16 pannas, one panna = 4 cacinis, one cacini = 20 cowry-shells).
(S.-H., Feb., 1857.)

$$\overset{\text{scarfs}}{8} \times \overset{\text{cubits}}{3} \times \overset{\text{cubits}}{8} : \overset{\text{scarf}}{1} \times \overset{\text{cubits}}{3\tfrac{1}{2}} \times \overset{\text{cubit}}{\frac{1}{2}} :: \overset{\text{nishcas}}{100} : \overset{\text{nishcas}}{Ans.},$$

$$Ans. \times 8 \times 3 \times 8 = \frac{7}{2} \times \frac{1}{2} \times 100,$$

$$Ans. = \frac{7}{2} \times \frac{1}{2} \times \overset{25}{\cancel{100}} \times \underset{2}{\frac{1}{\cancel{8}}} \times \frac{1}{3} \times \frac{1}{8}$$

$$= \frac{175}{192} \text{ nishca,}$$

and the value of $\frac{175}{192}$ of a nishca is 14 drammas, 9 pannas, 1 cacini, $6\frac{2}{3}$ cowry-shells.

The example just given will be found in Colebrooke's *Translation from the Sanskrit of Bháscara's Lilavati*, and from the same source the following is taken:

Ex. 27. "*If the hire of carts to convey thirty benches twelve fingers thick, the square of four wide, and fourteen cubits long, a distance of one league be eight drammas, tell me, my friend, what should be the cart-hire for bringing fourteen benches, which are four less in every dimension, a distance of six leagues?*"

(Four times six fingers are a cubit.)

$$\overset{\text{ben.}}{30} \times \overset{\text{thick.}}{12} \times \overset{\text{width}}{16} \times \overset{\text{length}}{(14 \times 24)} \times \overset{\text{leag.}}{1} : \overset{\text{ben.}}{14} \times \overset{\text{thick.}}{8} \times \overset{\text{width}}{12} \times \overset{\text{length}}{(10 \times 24)} \times \overset{\text{league}}{6} :: \overset{\text{drammas}}{8} : Ans.,$$

$$Ans. \times 30 \times 12 \times 16 \times 14 \times 24 \times 1 = 14 \times 8 \times 12 \times 10 \times 24 \times 6 \times 8,$$

$$Ans. = \frac{14 \times 8 \times 12 \times 10 \times 24 \times 6 \times 8}{30 \times 12 \times 16 \times 14 \times 24}$$

$$= 8 \text{ drammas.}$$

§ 92. There are a certain class of questions in which the terms so depend upon one another, that any *increase* in the

one produces a proportional *decrease* in the other. These questions are commonly classed under a separate rule, called "The Rule of Three Inverse." A large number however of the questions ordinarily placed under this rule, may be solved with greater ease by the method indicated above; all those at any rate where the work done, *i.e. the effect* produced, is the same in both cases. When such instances occur, we shall represent the ratio of the first effect to the second effect by 1 : 1. We shall illustrate this by the following examples:

Ex. 28. *If* 18 *men perform a piece of work in* 7 *days, how many men will perform it in* 21 *days?*

Here the work done is *the same* in each case; without therefore enquiring whether the number of men must be greater or less who would take 21 days to do it, we state

$$\overset{\text{men}}{18} \times \overset{\text{days}}{7} : \overset{\text{men}}{Ans.} \times \overset{\text{days}}{21} :: 1 : 1,$$

$$Ans. \times 21 = 18 \times 7,$$

$$Ans. = \frac{18 \times 7}{21}$$

$$= 6 \text{ men.}$$

Ex. 29. *If a person can travel a certain distance in* 14 *days when the days are* 9 *hours long, how many days will he take to travel the same distance when the days are* 12 *hours long?*

$$\overset{\text{days}}{14} \times \overset{\text{hours}}{9} :: \overset{\text{days}}{Ans.} \times \overset{\text{hours}}{12} :: 1 : 1,$$

$$Ans. \times 12 = 14 \times 9,$$

$$Ans. = \frac{14 \times 9}{12}$$

$$= 10\tfrac{1}{2} \text{ days.}$$

Ex. 30. *If I lend a friend* £200 *for* 15 *months, for how long ought he upon another occasion to lend me* £300 *to requite the obligation?*

Since the obligation is the same in both cases, we have to enquire in how many months the interest on £300 is equivalent to the interest on £200 for 15 months. Hence

$$200 \times 15 : 300 \times Ans. :: 1 : 1,$$

$$Ans. \times 300 = 200 \times 15,$$

$$Ans. = \frac{200 \times 15}{300}$$

$$= 10 \text{ months.}$$

Sometimes however there are cases which require further explanation. These are cases in which *more* of one kind requires *less* of another; where if one quantity be *doubled* the other must be *halved*. For instance, when the size of the loaf depends upon the price of wheat, if the price of wheat be *doubled*, the size of the loaf is *halved*: if the price of wheat be *trebled*, the size of the loaf would be *one-third* of what it was originally. In stating such a sum we must be very careful not to write *the greater* : *the less* :: *the less* : *the greater*; for "four quantities are proportional when the first is the same multiple, part, or parts of the second that the third is of the fourth;" therefore according as the greater or less quantity is placed as the antecedent in the first ratio, so in the second ratio we must be careful to arrange the greater or less quantity likewise for the antecedent. Thus when we have ascertained that *more* of one quantity requires *less* of another, and have arranged as the first and second terms of the proportion those two which are quantities of the same kind, it will then only be necessary to remember that of the next two terms we must put in the third term of the proportion *the greater*, if the first term be *greater* than the second, or the *less*, if the first term be *less* than the second. We now add examples to illustrate this; although, as the *proportion* is always necessarily *direct*, we demur to the name of "The Rule of Three Inverse."

Ex. 31. *When wheat was at 16s. the bushel, the two penny loaf weighed $7\frac{1}{2}$ ounces: what should be the weight of the loaf when wheat is at 9s.?*

Here the *higher* the price of wheat, the *smaller* the loaf; so if we take the two *prices* for the first ratio, and arrange them 16 : 9, placing the greater for the antecedent, we must then arrange the two *weights* for the next ratio, likewise placing the greater for the antecedent; but the greater weight is that when wheat is cheaper, *i. e.* is the unknown number of ounces which is the answer; hence

$$16s. : 9s. :: Ans. : 7\tfrac{1}{2},$$

$$9 \times Ans. = 16 \times \frac{15}{2},$$

$$Ans. = 16 \times \frac{15}{2} \times \frac{1}{9}$$

$$= 13\tfrac{1}{3} \text{ ounces.}$$

Had we arranged the first ratio 9 : 16, placing the *smaller* price as the antecedent, we then should have said that the smaller weight was that when wheat was dearer, and placing the *smaller* weight as the antecedent of the second ratio, should have stated

$$9 : 16 :: 7\tfrac{1}{2} : \textit{Ans.}$$

from which we should have obtained the same result.

Ex. 32. *Supposing the length of the English mile to be to that of the Roman mile as* 11 : 10, *and that a Roman army could march for* 8 *hours daily at the average rate of* $2\frac{3}{4}$ *miles per hour; how many English miles would a Roman army march in* 17 *days?*

This is in other words to enquire how many English miles are contained in $8 \times \frac{11}{4} \times 17$ Roman miles. Now the English mile is the *greater in length;* therefore in the same distance there will be a *smaller number* of English than of Roman miles. Therefore

the number of Eng. miles : the number of Rom. miles :: 10 : 11,

$$\textit{Ans.} : 8 \times \frac{11}{4} \times 17 :: 10 : 11,$$

$$\textit{Ans.} \times 11 = 8 \times \frac{11}{4} \times 17 \times 10,$$

$$\textit{Ans.} = 8 \times \frac{11}{4} \times 17 \times 10 \times \frac{1}{11}$$
$$= 340 \text{ English miles.}$$

Ex. 33. *A heap of corn when measured with the imperial bushel was found to contain* 462 *bushels; what would be the result if it were measured with the old Winchester bushel, supposing that the Winchester bushel contains* 7·7 *gallons, and the imperial bushel* 8 *such gallons?*

The *smaller* the measure which is used, the *greater* the number of bushels which will result; that is to say there will be a greater number of Winchester bushels and a smaller number of imperial bushels in the same heap; hence

$$\overset{\text{the smaller}}{7{\cdot}7} : \overset{\text{the larger}}{8} :: \overset{\text{the smaller}}{462} : \overset{\text{the larger}}{\textit{Ans.}},$$

$$7{\cdot}7 \times \textit{Ans.} = 8 \times 462,$$

$$
\begin{aligned}
\textit{Ans.} &= \frac{8 \times 4620}{77} \\
&= \frac{8 \times 420}{7} \\
&= 8 \times 60 \\
&= 480 \text{ Winchester bushels.}
\end{aligned}
$$

Ex. 34. *If in a piece of gold* 22 *carats fine there be* 27 *ounces of alloy, what weight of alloy would there be in the same sized piece of gold that was only* 18 *carats fine?*

The unit of gold is divided into 24 equal parts, called carats; then pure gold being called 24 carats fine, gold 22 carats fine is 22 parts pure gold and 2 parts alloy; gold 18 carats fine is 18 parts pure gold and 6 parts alloy. In England *standard* gold is 22 carats fine, and *jeweller's* gold is 18 carats fine. Hence

$$
\begin{aligned}
&\overset{\text{larger}}{22} : \overset{\text{smaller}}{18} :: \overset{\text{larger}}{\textit{Ans.}} : \overset{\text{smaller}}{27}, \\
&18 \times \textit{Ans.} = 22 \times 27, \\
&\textit{Ans.} = \frac{22 \times 27}{18} \\
&\quad = 33 \text{ ounces alloy.}
\end{aligned}
$$

Ex. 35. "*If a female slave* 16 *years of age bring thirty-two nishcas, what will one aged* 20 *cost? If an ox which has been worked a second year sell for four nishcas, what will one which has been worked six years cost?*"

The price of slaves and cattle being regulated by their age, and the maximum value of female slaves being fixed at 16 years, and of oxen after 2 years work, the price of the older will be *less*, and of the younger *greater*. Hence

$$
\begin{aligned}
&\overset{\text{less}}{16} : \overset{\text{greater}}{20} :: \overset{\text{less}}{\textit{Ans.}} : \overset{\text{greater}}{32}, \\
&20 \times \textit{Ans.} = 32 \times 16, \\
&\textit{Ans.} = 25\tfrac{3}{5} \text{ nishcas.}
\end{aligned}
$$

$$
\begin{aligned}
&\overset{\text{less}}{2} : \overset{\text{greater}}{6} :: \overset{\text{less}}{\textit{Ans.}} : \overset{\text{greater}}{4}, \\
&6 \times \textit{Ans.} = 8, \\
&\textit{Ans.} = 1\tfrac{1}{3} \text{ nishca.}
\end{aligned}
$$

Exercise XIV.

1. If a score of sheep cost £43, what would be the price of a flock of 3580 sheep?

2. What would be the price of 7632 articles at $3\frac{3}{4}d.$ for every 144 of them?

3. If 1 cwt. „ 1 qr. cost £3 „ 3*s.* „ 4*d.*, what is the cost 15 cwt. „ 2 qrs. „ $3\frac{1}{2}$ lbs.?

4. If $1\frac{1}{2}$ lb. cost 4*d.*, find the price of 2 tons „ 8 cwt.

5. What will be the cost of $2\frac{1}{4}$ tons of merchandise, if 3 cwt. „ 27 lbs. cost £35 „ 18*s.* „ 10*d.*?

6. If $\frac{2}{7}$ of an Estate be worth 1000 guineas, what will be the worth of ·75 of it?

7. A bankrupt's debts amount to £739 „ 10*s.*, and his assets to £640 „ 18*s.*; how much in the pound can he pay?

8. When a bankrupt's debts are £204 „ 16*s.*, and he pays 17*s.* „ 6*d.* in the pound; what are his assets?

9. If with assets amounting to £603 „ 15*s.* a bankrupt pay his creditors 14*s.* „ $4\frac{1}{2}d.$ in the pound; what were his debts?

10. A bankrupt's assets are £225, out of which he pays 5*s.* in the pound on half his debts, and 4*s.* on the other half; find the amount of his debts.

11. A creditor receives upon a debt of £272 a dividend of 11*s.* „ 6*d.* in the pound; afterwards he receives a further dividend upon the deficiency of 3*s.* „ 9*d.* in the pound; what does the creditor receive on the whole?

12. Given that the diameter of a circle is to the circumference as 1 : 3·1425; find correctly to the thousandth part of an inch the circumference of a circle whose radius is 2·1 feet.

13. A gig is proceeding at the rate of 8 miles per hour; the diameter of its wheels is 4 feet; find the number of revolutions made by them in the course of one mile, assuming that the circumference of a circle : diameter :: 22 : 7.

14. If the cost of mowing a 3-acre field be 28*s.* „ 6*d.*, what must be paid for mowing 45 acres „ 3 roods „ 20 poles?

15. A field is 121 yards long and 86 yards broad; what is its value at £30 per acre?

16. The removal of a quantity of brick earth 29 square yards in area and of a uniform depth of 4 yards cost £3 „ 17*s.* 4*d.*; what is the cost of the removal of a cubic yard?

17. If 4·4 articles cost £2·86, what is the value of 7·375 such?

18. If 3 cwt. „ 3 qrs. „ 27 lbs. cost £5 „ 16*s.*, what will be the price of 5 cwt. „ 2 qrs. at the same rate?

19. If 2·856 lbs. cost £·2884, find in pounds, florins, cents, and mils the price of 49·47 lbs.

20. If 5 yards „ 7 inches cost £3 „ 15*s.*, what will 23 yards „ 1 foot cost?

21. If 3 cwt. „ 3 qrs. cost £6 „ 16*s.*, what will be the price of 2 cwt. „ 2 lbs.?

22. If 27 sovereigns weigh 3341·25 grains, how many lbs., oz., &c. will 1000 sovereigns weigh?

23. If 17 ells, each containing 5 quarters, cost £6 „ 17*s.*, how much will 18 yards cost?

24. If a pole 10 feet high cast a shadow 12 feet „ 8 inches long, how high is a tower whose shadow at the same time is 57 feet long?

25. If one tower known to be 99 feet high cast a shadow 73 feet „ 3 inches long; what length of shadow will another tower 108 feet high cast at the same time?

26. Find a number which shall bear to 8 the same ratio which 7 does to 4.

27. Find a fourth proportional to 39, 741, and 19.

28. Find a number which shall bear to 15·84 the same ratio which 5·28 bears to 3·71.

29. Find a fourth proportional to $\frac{2}{7}$, $\frac{3}{8}$, and $\frac{5}{21}$.

30. Required the cost of 12 cwt. „ 3 qrs. „ 5·6 lbs., if 39 cwt. 2 qrs. 14 lbs. cost £42 „ 5*s.* „ 4*d.*

31. The estimated rental of a Parish amounts to £1750, and a rate is levied of £32 „ 16*s.* „ 6*d.*; what is the rate in the pound?

32. In a town whose rateable value is estimated at £66640, a rate is levied of 9*d.* in the pound; what amount of money will the rate produce?

33. If I give 1 florin „ 2 cents „ 5 mils for ·0875 of a ton; how much can I buy for £3 „ 10*s.* „ 6*d.*?

34. If a watch gain 3 seconds every 5 hours, how much will it gain in a week?

35. If $\frac{7}{8}$ of $\frac{4}{13}$ of a lottery ticket be worth £$17\frac{10}{13}$, what would $\dfrac{1\frac{1}{9}}{\frac{2}{3}\text{ of }5}$ of such a ticket sell for ?

36. If after paying an income-tax of *5d.* in the pound the remainder of a person's income be £551 „ *4s.* „ *7d.*; what was the gross income ?

37. If a person's gross income of £785. be reduced after paying an income-tax to £762 „ *2s.* „ *1d.*; what was the tax in the pound ?

38. If 7 cwt. 1 qr. „ 17·5 lbs. cost £110 „ 1 florin „ 2 cents „ 5 mils, what is the cost of 10 cwt. „ 3 qrs. 7·672 lbs ?

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39. If 9 men reap a field of 8 acres in 12 hours, how many men will reap a field of 28 acres in 18 hours ?

40. If 5 persons can be kept 4 weeks for £14, how long may 7 persons be kept for £21 ?

41. If 7 men can reap 6 acres in 12 hours, how many men will reap 15 acres in 14 hours ?

42. If 10 men can reap 20 acres of corn in 4 days, how many men can reap 70 acres in 10 days ?

43. If a man can reap $345\frac{5}{7}$ square yards in an hour, how long will 7 men take to reap a field of 6 acres ?

44. If with a capital of £3000 a tradesman gain £300 in 7 months, with what capital would he gain £60 „ *10s.* in 11 months ?

45. If 100 men make 80 yards of a road in 6 days, how many men will be required to make 50 miles of road in 150 days ?

46. If 1100 men make 10 miles of railroad in 3 months, how long will it take 2750 men to make 75 miles ?

47. If 84 men eat 126 lbs. of meat in 9 weeks, what supply of meat will be sufficient for 70 men during 6 weeks and 3 days ?

48. If 7 men earn £9 „ *10s.* „ *6d.* in $10\frac{1}{2}$ days, what sum will 28 men earn in $31\frac{1}{2}$ days ?

49. If 11 cwt. be carried 12 miles for *21s.*, how far can 36 cwt. „ 23 lbs. be carried for £5 „ *5s.* ?

50. If 6 men working 8 hours a day cut a ditch of uniform depth, 4 feet wide and 20 yards long, in 10 days; how many hours a day must 220 men work in order to cut a ditch of the same depth, 5 feet wide and half a mile long, in 18 days ?

51. If 5 men can reap a rectangular field whose length is 800 feet and breadth 700 feet in $3\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each can 7 men reap a field whose length is 1800 feet and breadth 960 feet?

52. If 15 masons working 10 hours a day can build a wall 6 feet high and 200 yards long in 6 days, how long will it take 7 masons working 9 hours a day to build a wall 9 feet high and 140 yards long?

53. A garrison of 4000 has provisions for $5\frac{1}{4}$ months; how long will the provisions last the garrison if reduced to 3000?

54. Find the weight of water in a bath 6 feet long, 3 feet wide, and 1 foot 9 inches deep, the weight of a cubic foot of water being 62 lbs. „ 8 oz.

55. A piece of cloth 5 times as long as broad costs £19; supposing the price of the cloth to be 4*s.* „ 9*d.* a square yard, find the dimensions of the piece.

56. The driving wheel of a locomotive engine 5 feet in diameter turned 2500 times in going 6 miles; supposing the circumference of a circle to be 3.1416 times the diameter, find what distance was lost, owing to the slipping of the wheel on the rail.

57. If 5 men can reap a field the length of which is 1400 feet and the breadth 400 in 3 days of 14 hours each, in how many days of 12 hours each can 7 men reap a field 1600 feet long and 700 feet broad?

58. A garrison of 1000 men was victualled for 30 days. After 10 days it was reinforced, and then the provisions were exhausted in 5 days. Of how many did the reinforcement consist?

59. If 50 men can make a wall 3 miles long in 60 days, working 12 hours a day, how many hours a day must 80 men work to finish a wall 4 miles long in 40 days?

60. If 35 men do a piece of work in 24 days, how long will $2\frac{1}{7}$ of that number do a piece of work $7\frac{1}{2}$ times as great, supposing the second set of men to be twice as quick workmen as the first, but only to work a third as long in the day?

61. "The price of a hundred bricks of which the length, the base, and breadth are respectively sixteen, eight, and ten is settled at 6 *dináras*. We have received a hundred thousand of other bricks, a quarter less in every direction; say what ought we to pay?"

62. "Two elephants which are ten in length, nine in breadth, thirty-six in girt, and seven in height, consume one *drona* of grain; how much will be the rations of ten other elephants, which are a quarter more in height and other dimensions?"

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63. If 72 men dig a trench in 63 days, in how many days will 42 men do the same?

64. If 12 men can reap a field in 4 days, in what time can the same work be done by 32 men?

65. What weight ought to be carried $25\frac{3}{4}$ miles for the same sum for which 3 cwt. are carried 40 miles?

66. How many yards worth 3*s*. „ $7\frac{1}{2}$*d*. per yard must be given in exchange for $935\frac{1}{2}$ yards worth 18*s*. „ $1\frac{1}{2}$*d*. per yard?

67. If a man walking 7 hours a day finish his journey in 9 days, in how many days could he have finished it if he had walked 10 hours in the day?

68. If 6 persons in a tour of 3 months spend £365, how long may 9 persons reckon that the same sum will last them?

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69. If 8 ounces of bread are sold for 6*d*. when wheat is £15 a load, what should be the price of wheat per load when 12 ounces are sold for 4*d*.?

70. A person is able to perform a journey of 142.2 miles in $4\frac{1}{2}$ days when they are 10·164 hours long; how many days will he be in travelling 505·6 miles when the days are 8·4 hours long?

71. The solid content of a sphere being $\frac{4}{3}$ of $\frac{355}{113}$, of a cube, the side of which is the radius of the sphere, and a cubic foot of iron weighing 450 lbs., find the diameter (in inches, tenths of an inch, &c.) of a 68 lb. cannon ball.

72. If gold can be beaten out so thin that a grain will form a leaf of 56 square inches, how many of these leaves will make an inch thick, the weight of a cubic inch of gold being 10 oz.?

73. If either 5 oxen or 7 horses will eat up the grass of a field in 87 days, in what time will 2 oxen and 3 horses eat up the same?

74. As there is always a *constant* expense in making bread, it is not *strictly* correct to make the weight of the loaf larger or smaller in the same proportion that the price of

wheat falls or rises; hence, if the cost for grinding and baking amount to 2*s.* per bushel of wheat, what is the price of wheat when the 4*d.* loaf is twice as large as it would be if wheat were 80*s.* per quarter?

75. Supposing the cost of digging a trench to depend upon the *depth* to which it is sunk *as well* as the *quantity* of earth taken out, and that the cost of digging a trench 3 feet broad by 8 feet deep is 9*d.* per yard; what would be the cost of digging a trench 120 yards long, 5 feet broad, and 10 feet deep? (S.-H., Jan., 1861).

76. At the siege of Sebastopol it was found that a certain length of trench could be dug by the Soldiers and Navvies in 4 days, but that when only half the Navvies were present it required 7 days to dig the same length of trench. Shew that the Navvies did six times as much work as the Soldiers. (S.-H., Jan. 1, 1861).

77. If 25 men can do as much as 40 boys in a day, how many days will it take 64 boys to finish a piece of work which 30 men did half of in 32 days?

78. If the work done by a man, a woman, and a child be in the ratio 3, 2, 1, and there be in a factory 24 men, 20 women, and 16 children, whose weekly wages amount to £20 „ 8*s.*; what will be the yearly wages of 27 men, 40 women, and 15 children?

79. Ten excavators, such as are usually employed in digging iron ore, can dig out 12 loads of earth in 16 hours, while 12 other common excavators, less powerful than the former, dig out only 9 loads of earth in 15 hours; it is required to find in what time they will conjointly dig out 100 loads of earth.

80. Two persons agreed to pay £81 for the use of a certain tract of pasture meadows for 10 months; the first put on 27 oxen for 3 months, the second 270 sheep for 7 months; supposing the feed equally good throughout and that 3 oxen eat as much as 11 sheep, how much of the rent ought each to pay?

81. If in the backwoods 3 waggons with a team of 3 oxen in each cost £195 „ 5*s.*, and 4 waggons without oxen cost £84 „ 6*s.* „ 8*d.*; supposing the emigrant wished to buy the 3 teams of oxen only without the waggons, what ought he to give for them?

82. *A* barters some sugar with *B*, for flour which is worth 2*s*. „ 3*d*. per stone, but in weighing his sugar uses a false stone weight of $13\frac{1}{2}$ lbs.; *B* on discovering this says nothing, but raises the price of his flour; what value should *B* set on his flour that the exchange may be fair?

CHAPTER XIII.

PROPORTIONAL PARTS.

§ 93. As in some sort a sequel to the rule of proportion, we place next the method of finding, by proportional parts, into what portions any quantities should be divided, when the ratio which the several required parts bear to one another is given. Most questions under this head might be solved by making several statements in proportion; but the simpler process by which the result may be arrived at will now be explained.

Ex. 1. *Let it be required to divide the number* 65 *into two parts which shall bear to one another the ratio of* 6 : 7.

We have here to find two numbers which shall together make up 65, and shall be in the ratio of 6 : 7.

Now we may either say that the first of the two numbers in the given ratio is to the sum of those two numbers as the first of the required parts is to the whole number 65; which would give us the statement

$$6 : 13 :: Ans. : 65,$$
$$13 \times Ans. = 6 \times 65,$$
$$Ans. = \frac{6}{13} \times 65$$
$$= 30,$$

or we may more directly apply the following rule: "Form fractions which shall have the numbers composing the given ratio as the respective numerators, and the sum of these numbers as the common denominator; take these fractional parts of the proposed quantity; they will be the parts required." This would give us

$$\frac{6}{13} \text{ of } 65 = 6 \times 5 = 30,$$

$$\frac{7}{13} \text{ of } 65 = 7 \times 5 = 35.$$

The reason of the above rule may be thus explained: $\frac{6}{13}$ and $\frac{7}{13}$ are clearly in the ratio of 6 : 7; as likewise are $\frac{6}{13}$ of 65 and $\frac{7}{13}$ of 65; for we may multiply and divide both the terms of any ratio by the same quantities without thereby altering the value of the ratio. Again $\frac{6}{13}$ added to $\frac{7}{13}$ make $\frac{13}{13}$, or 1; therefore $\frac{6}{13}$ of 65 added to $\frac{7}{13}$ of 65 will make 65. But the conditions of the problem before us only required that we should find two numbers which were in the ratio of 6 : 7, and which when added together would make 65. Hence $\frac{6}{13}$ of 65 and $\frac{7}{13}$ of 65 are the numbers required.

Ex. 2. *Divide* 1065 *into parts which shall be to one another in the ratio of* 3, 5, 7.

The fractions are $\frac{3}{15}$, $\frac{5}{15}$, $\frac{7}{15}$, and

$$\frac{3}{15} \text{ of } 1065 = 3 \times 71 = 213$$

$$\frac{5}{15} \text{ of } 1065 = 5 \times 71 = 355,$$

$$\frac{7}{15} \text{ of } 1065 = 7 \times 71 = 497.$$

Ex. 3. *Divide the sum of* £47 „ 10*s.* „ 1*d. among* 3 *persons in the ratio of* $\frac{1}{2}$, $\frac{1}{3}$ $\frac{1}{4}$.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}.$$

Therefore the fractions are

$$\frac{1}{2} \div \frac{13}{12}, \text{ or } \frac{6}{13},$$

$$\frac{1}{3} \div \frac{13}{12}, \text{ or } \frac{4}{13},$$

$$\frac{1}{4} \div \frac{13}{12}, \text{ or } \frac{3}{13},$$

and

$\frac{6}{13}$ of £47 „ 10*s.* „ 1*d.* $= 6 \times$ (£3 „ 13*s.* „ 1*d.*) $=$ £21 „ 18*s.* „ 6*d.*,

$\frac{4}{13}$ of £47 „ 10*s.* „ 1*d.* $= 4 \times$ (£3 „ 13*s.* „ 1*d.*) $=$ £14 „ 12*s.* „ 4*d.*,

$\frac{3}{13}$ of £47 „ 10*s.* „ 1*d.* $= 3 \times$ (£3 „ 13*s.* „ 1*d.*) $=$ £10 „ 19*s.* „ 3*d.*

Ex. 4. *Gunpowder is made up of nitre, sulphur, and charcoal, which are mixed in this proportion,* 75 *parts of nitre to* 10 *of sulphur and* 15 *of charcoal; in half a ton of powder how many pounds of each ingredient?*

$$75 + 10 + 15 = 100, \text{ and } \frac{1}{2} \text{ ton} = 1120 \text{ lbs.}$$

Hence $\frac{75}{100}$ of $1120 = \frac{3}{4}$ of $1120 = 840$ lbs. nitre,

$\frac{10}{100}$ of $1120 = \frac{1}{10}$ of $1120 = 112$ lbs. sulphur,

$\frac{15}{100}$ of $1120 = \frac{3}{2}$ of $112 = 168$ lbs. charcoal.

Ex. 5. *Divide a legacy of* £1187 „ 12*s.* „ 1*d. among a son, wife, and daughter, so that the son's share shall be thrice the wife's, and the daughter's share a third of the wife's.*

If 1 represent the daughter's share, the wife's share will be represented by 3, and the son's by 9.

Hence the fractions will be

$$\frac{1}{13}, \frac{3}{13}, \frac{9}{13},$$

and $\frac{1}{13}$ of £1187 „ 12*s.* „ 1*d.* $=$ £91 „ 7*s.* „ 1*d.*

Hence 3 times £91 „ 7*s.* „ 1*d.*, or £274 „ 1*s.* „ 3*d.*, is the share of the wife; and 3 times £274 „ 1*s.* „ 3*d.*, or £822 „ 3*s.* „ 9*d.*, the share of the son.

Ex. 6. *A Bankrupt surrenders his property, which is worth* £336, *to* 3 *creditors to whom he owes respectively* £450, £560, *and* £670; *share the property equitably among them.*

$$450 + 560 + 670 = 1680.$$

Hence $\quad \frac{450}{1680}$ of $336 = \frac{45}{168} \times 336 = 45 \times 2 = 90,$

$\frac{560}{1680}$ of $336 = \frac{56}{168} \times 336 = 56 \times 2 = 112,$

$\frac{670}{1680}$ of $336 = \frac{67}{168} \times 336 = 67 \times 2 = 134.$

Ex. 7. *A field of grass is rented by two persons for £27: the former keeps in it* 15 *oxen for* 10 *weeks, the latter* 21 *oxen for* 7 *weeks; find the rent paid by each.*

The rent must be divided in proportion to the number of cattle and the number of weeks; the shares therefore will be in the ratio of 15×10 and 21×7.

The fractions will be $\frac{150}{297}$ and $\frac{147}{297}$;

Hence $\quad \frac{150}{297}$ of $27 = \frac{150}{11} =$ £13 „ 12*s.* „ 8*d.* „ $2\frac{10}{11}$ *far.*,

and $\quad \frac{149}{297}$ of $27 = \frac{147}{11} =$ £13 „ 7*s* „ 3*d.* „ $1\frac{1}{11}$ *far.*

Ex. 8. *At the beginning of the year A embarks in trade a capital of* £3000; *and at the end of* 5 *months takes into partnership B with a capital of* £4000; *at the end of the year the profits are* £594 „ 10*s.* „ 8*d.*; *how should they be shared between them?*

$$\begin{aligned} A\text{'s share} : B\text{'s share} &:: 3000 \times 12 : 4000 \times 7 \\ &:: 3 \times 12 : 4 \times 7 \\ &:: 9 : 7, \end{aligned}$$

and

$\frac{9}{16}$ of £594 „ 10*s.* „ 8*d.* = 9 × (£37 „ 3*s.* „ 2*d.*) = £334 „ 8*s.* „ 6*d.*

$\frac{7}{16}$ of £594 „ 10*s.* „ 8*d.* = 7 × (£37 „ 3*s.* „ 2*d.*) = £260 „ 2*s.* „ 2*d.*

Ex. 9. *If the sum of* £1 „ 13*s.* „ 9*d. be divided among* 13 *men and* 19 *boys, so that the share of each man shall be to the share of each boy as* 2 : 1, *find what a boy's share would be.*

The sum given to all the men would be to the sum given to all the boys as $13 \times 2 : 19 \times 1$, or as $26 : 19$.

The fractions would be $\frac{26}{45}$ and $\frac{19}{45}$; therefore

the share of one boy would be $\frac{1}{19}$ of $\frac{19}{45}$ of £1 „ 13*s.* „ 9*d.*,

or $\frac{1}{45}$ of 405 pence, or 9*d.*

Ex. 10. *The sum of £600 is to be divided among 24 men, 36 women, and 72 children, so that the shares of 2 men shall be equal to those of 3 women, and each woman's share to the shares of 2 children. What will be the share of each?* (S.-H., Jun. 5, 1858.)

The shares of 24 men = those of 36 women,
.... 36 women = ... 36 ....,
.... 72 children = ... 36 ....;

therefore all the shares are equivalent to those of 108 women; therefore

$$\text{each woman gets } £\frac{600}{108} = \frac{50}{9} = £5 \text{ „ } 11s. \text{ „ } 1\tfrac{1}{3}d.$$

$$\text{...... man ... } \frac{3}{2} \text{ of } \frac{50}{9} = \frac{25}{3} = £8 \text{ „ } 6s. \text{ „ } 8d.$$

$$\text{...... child ... } \frac{1}{2} \text{ of } \frac{50}{9} = \frac{25}{9} = £2 \text{ „ } 15s. \text{ „ } 6\tfrac{2}{3}d.$$

Ex. 11. *If 3 men and 4 boys can do as much work as 2 men and 16 girls in the same time, and 4 men and 2 boys as much as 12 boys and 12 girls, how should a man who receives 44s. for a piece of work reward a boy and a girl who have been helping him all the time?*

The work of 3 men + 4 boys = that of 2 men + 16 girls,
therefore .... 1 man + 4 boys = ... 16 girls,
whence .... 4 men + 16 boys = ... 64 girls.

But from the question, we know also that

the work of 4 men + 2 boys = that of 12 boys + 12 girls,
therefore .... 4 men = ... 10 boys + 12 girls,

adding to each side of this equality the work of 16 boys, we get

the work of 4 men + 16 boys = that of 26 boys + 12 girls;

therefore the work of 64 girls = ... 26 boys + 12 girls:
therefore .... 52 girls = ... 26 boys;
therefore .... 2 girls = ... 1 boy.
But .... 1 man + 4 boys = ... 16 girls;
therefore .... 1 ...... = ... 8 girls.

Consequently the work of a man, a boy, and a girl = that of 11 girls; and this would be paid for by 44*s.*;

therefore the girl should get 4*s.*,

the boy 8*s.*,

and the man should keep 32*s.* for himself.

Ex. 12. *A hundred gallons of liquid contains* 70 *per cent. wine and the rest water. How much wine should be added, to raise the strength of the wine to* 80 *per cent.?*

The quantity of water is unchanged, and amounts to 30 gallons after the addition of the wine.

The quantity of wine : the quantity of water :: 80 : 20,
:: 120 : 30.

But the water continues to be 30 gallons, while now the wine is 120 gallons; therefore 50 gallons of wine have been added.

Ex. 13. *A guinea is divided between A, B, and C. The share which A gets is $\frac{3}{5}$ of B's share; but it is likewise $\frac{2}{5}$ of B's and C's shares together. How was the guinea divided?*

A has the guinea $-$ (B's share + C's share).

But B's share + C's share is $\frac{5}{2}$ of A's share;

therefore A's share = the guinea $-\frac{5}{2}$ of A's share;

therefore $\frac{7}{2}$ of A's share = the guinea.

A's share $=\frac{2}{7}$ of the guinea = 6*s.*,

B's share $=\frac{5}{3}$ of A's share $=\frac{5}{3}\times 6 = 10$*s.*;

therefore the remainder, which is C's share, = 5*s.*

Ex. 14. *The price of gold is* £3 „ 17*s.* „ 10½*d. per oz.; a composition of gold and silver weighing* 18 *lbs. is worth* £637 „ 7*s.*: *but if the proportions of gold and silver were interchanged, it would be worth only* £259 „ 1*s.* *Find the proportion of gold and silver in the composition, and the price of silver per oz.* (S.-H., Jan. 1, 1856.)

If the two lumps were added together, there would clearly be 18 lbs. of gold + 18 lbs. of silver, and the value of the two together would be

$$£637 \text{ „ } 7s. + £259 \text{ „ } 1s.;$$

therefore

the worth of 18 lbs. of gold + 18 lbs. of silver $= £896 \text{ „ } 8s.$

But the

worth of 18 lbs. of gold $= 18 \times 12(£3 \text{ „ } 17s. \text{ „ } 10\frac{1}{2}d.) = £841 \text{ „ } 1s.$

therefore the worth of 18 lbs. of silver $= £55 \text{ „ } 7s.$

Hence the value of 1 oz. of silver will be found to be $5s. \text{ „ } 1\frac{1}{2}d.$

Again for the quantity:

The difference in value of the two compositions is

$$(£637 \text{ „ } 7s.) - (£259 \text{ „ } 1s.), \text{ or is } £378 \text{ „ } 6s.$$

The difference in value of 1 oz. of gold and 1 oz. of silver is

$$(£3 \text{ „ } 17s. \text{ „ } 10\tfrac{1}{2}d.) - (5s. \text{ „ } 1\tfrac{1}{2}d.), \text{ or is } £3 \text{ „ } 12s. \text{ „ } 9d.$$

And dividing £378 „ 6s. by £3 „ 12s. „ 9d., we find the former contains the latter just 104 times, the half of which is 52.

In the first composition

the quantity of gold is 9 lbs. + 52 oz., or 13 lbs. „ 4 oz.,

.... silver is 9 lbs. − 52 oz., or 4 lbs. „ 8 oz.,

whence in that composition

the quantity of gold : the quantity of silver

:: 13 lbs. „ 4 oz. : 4 lbs. „ 8 oz.

:: 160 oz. : 56 oz.

:: 20 : 7.

Exercise XV.

1. Divide the number 100 into two parts which shall have to one another the ratio of 2 : 3.

2. Divide the number 45 into three parts which shall be to one another in the ratio of 7, 5, 3.

3. Divide the number 2679 into parts which shall be to one another in the ratio of $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

4. A person bequeathed £21463 „ 8s. to be divided between his two sons in the ratio of 3 : 5. What is the share of each?

5. Divide the number 2848 into two parts which are to one another in the ratio of $\cdot\dot{2} : \cdot\dot{3}\dot{4}$.

6. There is a mixture of brandy, wine, and water: for every gallon of brandy there are $2\frac{1}{2}$ gallons of wine, and for every half-gallon of wine there is $\frac{2}{3}$ of a gallon of water. In 123 gallons of the mixture, how much of each ingredient?

7. A person has four creditors; to the first he owes £624, to the second £546, to the third £492, and to the fourth £368. He fails, and runs away, and the creditors find the whole amount of property he leaves behind him is only £830: how ought it to be divided amongst them?

8. Three persons go into partnership, their several contributions being £235, £430, and £520; at the end of a certain time they find that their capital and gains amount to £1732: what portion belongs to each?

9. Three highwaymen agree to rob in company, and share the plunder; but the first, being an older hand, says he shall claim twice as much as the second; while the second claims half as much again as the third. Out of a booty of £127 „ 1*s.* how much does each get?

10. Divide £31 between 12 men, 20 women, and 33 children, so that the share of each man, woman, and child shall be in the ratio of 5, 3, 2.

11. If I distribute 5 guineas among 3 applicants, for every sixpence that I give to the first, giving ninepence to the second, and fifteenpence to the third, what do I give to each?

12. Three persons form a company, the first of whom contributes 300 florins, the second 600 *canne* of cloth, and the third 1200 *lire* of saffron; they gain 900 florins, of which the first receives 60, the second 360, and the third 480; what was the value of a *canna* of cloth, and of a *lira* of saffron?

13. Three companions are in a ship, one of whom has a butt of *Malvasia* which holds 36 gallons, another one of Greek wine which holds 24, the third one of the wine of Romania, which holds 40. By a violent movement of the ship the butts are upset, and the wine is spilt in the hold. The butts are afterwards replaced and filled with the mixture; what portion of each wine do they severally hold?

14. Four persons, a gentleman, an artisan, a barber, and a friar, make a pilgrimage in company, and spend 60 ducats; the barber agrees to pay 4 times as much as the friar, and 4 soldi besides; the artisan 3 times as much as the barber, and 16 soldi besides; the gentleman twice as much as the artisan, and 10 soldi besides. What sum was paid by each? (One Ducat = 20 Soldi.)

15. *A* at the beginning of the year commences trade with a capital of £2580; at the end of 3 months he takes into partnership *B* with a capital of £4420; at the end of the year the gain they had made was £3537; how was it to be divided between them?

16. The sum of £1000 is to be divided between 10 men, 32 women, and 48 children; if each man's share is to be equal to the shares of two women, and the 32 women are to have twice as much as the 48 children, how much will the several individuals receive? (S.-H., Jan. 4, 1859.)

17. The sum of £177 is to be divided among 15 men, 20 women, and 30 children, in such a manner that a man and a child together may receive as much as two women, and all the women may together receive £60. What will they respectively receive? (S.-H., Jan. 3, 1860.)

18. If 10 men and 6 boys can do as much work as 8 men and 24 girls; and 7 men and 9 boys as much as 60 girls; how many men must help 3 men, 5 boys, and 6 girls to do as much work as 5 men, 3 boys, and 4 girls in the same time?

19. The price of pure gold is £4 „ 2*s.* „ 6*d.* an oz.; the price of a mixture of gold and silver weighing 18 lbs. is £694 „ 13*s.*, but if the weights of silver and gold in the mixture were interchanged, the value would be £255 „ 15*s.* Find the weight and value of the silver in the mixture.

20. The price of gold is £3 „ 17*s.* „ 10½*d.* per oz. The price of a mixture of silver and gold weighing 18 lbs. is £171 „ 15*s.*; but if the weights of the silver and gold in the mixture were interchanged, the price would be £724 „ 13*s.* Find the portions of the silver and gold in the mixture, and the price of silver per oz.

21. Supposing that a cubic inch of gold weighs 20 oz., and an equal bulk of silver weighs 12 oz., and a lump composed of gold and silver weighs 32 oz., less than if it were all gold,

but 56 oz. more than if it were all silver; what is its actual weight?

22. If the water in a mixture of brandy and water be $\frac{1}{3}$ of the whole, and the addition of 40 gallons of water would reduce the strength to half and half; how many gallons were there originally in the mixture?

23. A hundred and twenty gallon cask is partially filled with wine and water, the proportion of wine being 75 per cent. The cask is then filled up with wine, and the wine is now $\frac{5}{6}$ of the mixture. How much wine and how much water was there in the cask originally?

24. 40 per cent. of a mixture of wine and water is wine; but when 10 gallons of water are added, the wine is then only 35 per cent. How many gallons in all were there at the first in the mixture?

25. A sum of money is divided among A, B, and C, so that A has one-third of what B and C together have; and B has one-half of what A and C together have. What portion of the whole has C?

26. Of a sum of money divided between A, B, and C,

$$A\text{'s share} : B\text{'s share} :: 3 : 2,$$
$$B\text{'s share} : C\text{'s share} :: 3 : 4;$$

what portion had each?

27. If a cask contains 3 parts wine and 1 part water, how much of the mixture must be drawn off and water substituted, for the mixture in the cask to become half and half?

CHAPTER XIV.

INTEREST.

§ 94. *Def. Interest* is the consideration paid for the use of money borrowed.

Def. When the Interest of the Principal alone is taken, it is called *Simple Interest;* but if the interest, as soon as it becomes due, be added to the principal, and interest be charged upon the whole, (*i.e.* upon the whole sum compounded of the Principal and the Interest from time to time accruing); it is called *Compound Interest.*

Def. The *Rate* of Interest is the consideration paid for the use of a certain sum for a certain time; as of £100 for one year.

By the "rate per cent." the rate per cent. *per annum* is always understood unless the contrary be expressly stated.

Def. The *Amount* is the whole sum due at the end of any time, Interest and Principal together.

The following abbreviations are commonly used in commerce to save space in writing: 0/0 for *per cent.*; @ for *at*; *a/c* for *account*; *o/a* for *on account.*

§ 95. The solution of questions in Simple Interest depends upon an easy practical rule, deduced from a simple proportion; *e.g.* if it be required to find the simple interest on £885 for 1 year at 4 per cent., we should say, 'If £100 in one year gain £4 as interest, what will £885 gain in the same time?'

$$£100 : £885 :: £4 : £\mathit{Ans.}$$

$$100 \times \mathit{Ans.} = 885 \times 4,$$

$$\mathit{Ans.} = \frac{885 \times 4}{100}.$$

Hence we deduce the Rule, viz. "*Multiply the Principal by the rate per cent. and divide by* 100." It is therefore unnecessary to state each sum as a proportion, because the division by 100 being effected in whole numbers by cutting off with a decimal point the last two figures, and in decimals by shifting the decimal point two places to the *left*, it is the easiest plan generally, instead of reducing the quantities by cancelling, to multiply the principal at once by the rate per cent., and then effect the division by 100.

Ex. 1. *Find the Simple Interest on, and amount of £2275 for* $3\frac{1}{2}$ *years at* 5 *per cent. per annum.*

$$\begin{array}{rl}
2275 & \\
5 & \\
\hline
113{\cdot}75 & \text{interest for 1 year} \\
3\frac{1}{2} & \\
\hline
341{\cdot}25 & \\
56{\cdot}875 & \\
\hline
398{\cdot}125 & \text{interest for } 3\frac{1}{2} \text{ years.}
\end{array}$$

Hence £398 „ 2*s.* „ 6*d.* is interest required; and

$$\begin{array}{r} 2275 \\ 398 \text{ „ } 2s. \text{ „ } 6d. \\ \hline £2673 \text{ „ } 2s. \text{ „ } 6d. \text{ amount.} \end{array}$$

Ex. 2. *What will* £480 *amount to in* 3 *years and* 3 *months at* £4 „ 3*s.* „ 4*d. per cent. per annum Simple Interest?*

$$\begin{array}{r} 480 \\ 4\frac{1}{6} \\ \hline 1920 \\ 80 \\ \hline 20{,}00 \end{array}$$

therefore £20 is the interest for 1 year; and £20 × $3\frac{1}{4}$ = £65, whence £545 is the amount required.

Ex. 3. *Find Simple Interest on* £600 *for* $2\frac{1}{3}$ *years at* $4\frac{1}{2}$ *per cent. per annum.*

Fractions may be sometimes used with advantage. Thus in this case, multiply the principal by the rate per cent. and the number of years, and divide by 100, simultaneously, using fractions.

$$\begin{aligned} &600 \times \frac{9}{2} \times \frac{7}{3} \times \frac{1}{100} \\ &= 100 \times 9 \times 7 \times \frac{1}{100} \\ &= 63. \end{aligned}$$

Ex. 4. *Find Simple Interest on* £2666 „ 13*s.* „ 4*d. for* 73 *days at* 4 *per cent.*

$$2666\tfrac{2}{3} \times 73 \; : \; 100 \times 365 \; :: \; Ans. \; : \; 4,$$

$$100 \times 365 \times Ans. = \frac{8000}{3} \times 73 \times 4,$$

$$Ans. = \frac{\overset{16}{\overset{\cancel{80}}{\cancel{8000}}} \times \cancel{73} \times 4}{3 \times \cancel{100} \times \underset{\cancel{5}}{\cancel{365}}}$$

$$= \frac{64}{3}$$

$$= £21 \text{ „ } 6s. \text{ „ } 8d.$$

Ex. 5. *Find the Simple Interest upon £1277 „ 10s. from the* 31*st of March to the* 4*th of July, at* 4 *per cent.*

When interest is calculated from one date to another, leave out of the reckoning the *first* day named, but count the *last.*

Thus, *omitting* the 31st of March, there will be in April 30 days, in May 31 days, in June 30 days, in July 4 days; in all 95 days.

First calculate interest on £1277·5 for a year:

$$\begin{array}{r} 1277{\cdot}5 \\ 4 \\ \hline 5110{\cdot}0 \end{array}$$

therefore 51·1 is interest for 1 year,

and interest for 95 days will be $\frac{95}{365}$, or $\frac{19}{73}$ of this; therefore

$$\frac{19}{73} \text{ of } 51{\cdot}1 = 19 \times {\cdot}7$$
$$= 13{\cdot}3$$
$$= £13 \text{ „ } 6s.,$$

the interest required for the given time of 95 days.

Obs. Since to multiply by 5 and divide by 100 is to multiply by $\frac{5}{100}$, or $\frac{1}{20}$, it is sufficient in finding the Simple Interest on any sum at 5 per cent. to take $\frac{1}{20}$th part, or to divide by 20. Hence 5 per cent. interest is often reckoned at sight by calling it "one shilling in the pound."

Again, since $\frac{4}{100}$ is $\frac{1}{25}$, to find the Simple Interest *at* 4 *per cent.,* divide by 25.

e.g. If it be required to find the Simple Interest on £5050 for 3 years at 4 per cent., we may perform the operation thus:

$$\begin{array}{rl} 25\overline{)\,5050} & \\ \hline 202 & \text{interest for 1 year} \\ 3 & \\ \hline 606 & \text{interest for 3 years.} \end{array}$$

N.B. Do *not* multiply the Principal *first* by the number years, *then* by the rate of Interest; but rather find the

Interest for one year first, and then multiply that by the given number of years.

Ex. 6. *What is the Simple Interest on £825 for 7 years and 97 days at $2\frac{1}{2}$ per cent.?*

To multiply by $2\frac{1}{2}$ and to divide by 100 is to multiply by $\frac{1}{40}$; hence, writing 97 days as a decimal of a year,

$$825 \times \frac{1}{40} \times 7{\cdot}2657506, \text{ \&c.}$$
$$= 20{\cdot}625 \times 7{\cdot}2657506, \text{ \&c.}$$

and by contracted multiplication we have

$$\begin{array}{r} {\cdot}0072657506 \\ 52602 \\ \hline 1453150 \\ 43595 \\ 1453 \\ 363 \\ \hline 149{\cdot}8561 \end{array}$$

whence, *by inspection*, £149 „ 17*s.* „ $1\frac{1}{2}$*d.* is the interest required.

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§ 96. When it is required to find what *principal* at any given rate will amount to a certain sum in a certain time; or *at what rate*, or *in what time* a given principal will amount to some given sum, the questions may be worked by proportion. For the *principal* which gains certain *interest* in a certain time may be treated just as a number of *men* who earn certain *wages* in a certain time; thus the *principal* and the *time* may be looked upon as the *causes*, and the interest as the *effect* produced.

The following examples will serve to explain the method here suggested:

Ex. 7. *What sum of money must be put out at Simple Interest for 4 years at $4\frac{1}{2}$ per cent. to gain £24 „ 9s. „ $7\frac{1}{2}$d.?*

In other words, if £100 in 1 year gain £$4\frac{1}{2}$, what sum in 4 years will gain £24 „ 9*s.* „ $7\frac{1}{2}$*d.*?

$$\overset{\pounds.}{100} \times \overset{\text{year}}{1} : \overset{\pounds.}{Ans.} \times \overset{\text{years}}{4} :: \overset{\text{int.}}{4\tfrac{1}{2}} : \overset{\text{int.}}{24\tfrac{12}{25}},$$

$$Ans. \times \overset{2}{\cancel{4}} \times \frac{9}{\cancel{2}} = \overset{4}{\cancel{100}} \times \frac{612}{\cancel{25}}$$

$$Ans. = \frac{4 \times \overset{68}{\cancel{612}}}{2 \times \cancel{9}}$$

$$= \pounds 136.$$

Ex. 8. *What is the Principal that must be put out at $3\frac{1}{2}$ per cent. Simple Interest for 5 years, to amount to £173 ,, 18s.?*

In 5 years at $3\frac{1}{2}$ per cent. £100 will gain £$17\frac{1}{2}$, and will amount to £$117\frac{1}{2}$. Hence if, in the given time at the given rate, £100 amount to £$117\frac{1}{2}$, what sum will amount to £173 ,, 18s. in the same time at the same rate?

$$\overset{\text{principal}}{100} : \overset{\text{principal}}{Ans.} :: \overset{\text{amount}}{117\tfrac{1}{2}} : \overset{\text{amount}}{173\tfrac{9}{10}},$$

$$Ans. \times \frac{235}{2} = 100 \times \frac{1739}{10},$$

$$Ans. = \overset{2}{\cancel{10}} \times \overset{37}{\cancel{1739}} \times \frac{2}{\underset{\cancel{47}}{\cancel{235}}}$$

$$= 148.$$

Ex. 9. *In what time will £1360 amount to £1802 at 5 per cent. Simple Interest?*

	1802	amount
subtract	1360	principal
	442	interest.

Hence, if £100 gain £5 in 1 year, in how many years will £1360 gain £442?

$$100 \times 1 : 1360 \times Ans. :: 5 : 442,$$

$$Ans. \times 1360 \times 5 = 100 \times 442,$$

$$Ans. = \frac{\overset{\cancel{20}}{\cancel{100}} \times 442}{\underset{68}{\cancel{1360}} \times \cancel{5}}$$

$$= \frac{221}{34}$$

$$= 6\tfrac{1}{2} \text{ years.}$$

Ex. 10. *At what rate of Simple Interest will £776 „ 15s. amount to £978 „ 14s. „ $1\frac{1}{8}$d. in $6\frac{1}{2}$ years?*

	£. s. d.	
	978 „ 14 „ $1\frac{1}{8}$	amount
subtract	776 „ 15 „ 0	principal
	201 „ 19 „ $1\frac{1}{8}$	interest.

Hence the question is, "If £776 „ 15s. gain £201 „ 19s. „ $1\frac{1}{8}$d. in $6\frac{1}{2}$ years, what will £100 gain in 1 year?"

$$\overset{\text{principal}}{776\tfrac{3}{4}} \times \overset{\text{yrs.}}{6\tfrac{1}{2}} : \overset{\text{principal}}{100} \times \overset{\text{yr.}}{1} :: \overset{\text{interest}}{201\cdot955} : \overset{\text{interest}}{Ans.},$$

$$Ans. \times 776\cdot75 \times 6\cdot5 = 100 \times 201\cdot955,$$

$$Ans. = \frac{20195\cdot5}{5048\cdot875}$$
$$= 4 \text{ per cent.}$$

Ex. 11. *In what time will a sum of money double itself, if put out at Simple Interest at any given rate per cent. per annum?*

In other words, in what number of years will the interest on £a become £a at any given rate, say r per cent. per annum?

The interest on £a for 1 year at r per cent. is $a \times \frac{r}{100}$;

and in x years, $a \times \frac{r}{100}$ will be gained x times over.

By the hypothesis

$$x \times a \times \frac{r}{100} = a;$$

multiplying each side by 100, $x \times a \times r = 100 \times a$,

and dividing each by a, $\quad x \times r = 100$,

therefore, dividing each by r, $\quad x = \frac{100}{r}$.

Hence by dividing 100 by the given rate per cent., we find the number of years required. Thus a sum of money will double itself at 5 per cent. in $\frac{100}{5}$, or 20 years; at 4 per cent. in $\frac{100}{4}$, or 25 years; and so on.

§ 97. *Compound Interest* consists of a series of Simple Interest sums, where the amount at the end of the first year becomes the principal for the second year, and so on.

Ex. 12. *Required the Compound Interest and amount of £457 „ 10s. at 4 per cent. for 3 years.*

Such examples are most easily worked by writing the shillings, &c. as decimals of a pound; and remembering that the division by 100 will be effected by shifting the decimal point two places to the left. Thus

$$\begin{array}{r} 457{\cdot}5 \\ 4 \\ \hline 1830{\cdot}0 \end{array}$$

therefore £18·3 is interest for first year.

$$\begin{array}{r} 457{\cdot}5 \\ 18{\cdot}3 \\ \hline 475{\cdot}8 \\ 4 \\ \hline 1903{\cdot}2 \end{array}$$

therefore £19·032 is interest for second year.

$$\begin{array}{rl} 475{\cdot}8 & \\ 19{\cdot}032 & \\ \hline 494{\cdot}832 & \text{principal for third year} \\ 4 & \\ \hline 1979{\cdot}328 & \end{array}$$

therefore £19·79328 is the interest for third year.

Hence

$$\begin{array}{rl} 494{\cdot}832 & \\ 19{\cdot}79328 & \\ \hline 514{\cdot}62528 & \text{amount.} \end{array}$$

£.	s.	d.	
514 „	12 „	6	amount
457 „	10 „	0	principal
57 „	2 „	6	compound interest.

Ex. 13. *Required the amount of £750 at Compound Interest for $1\frac{1}{2}$ years at 4 per cent., the interest being payable half-yearly.*

Instead of finding the interest for a year and halving it, we ·ıld halve the rate of interest; because 2 per cent. per half- ˈis the same thing as 4 per cent. per annum.

```
            750
              2
           ─────
           15·00
750                     765
 15                     15·3
────                    ─────
765 principal for 2nd half-year   780·3 principal for 3rd half-year
  2                       2
────                    ──────
15·30                   1560·6
```

therefore £15·606 is interest for the third half-year.

```
 780·3
 15·606
───────
795·906
```

therefore £795 „ 18*s*. „ 1½*d*. amount.

Ex. 14. *A and B lend each £248 for 3 years at 3½ per cent., one at Simple, the other at Compound Interest: find the difference of the amount of interest which they respectively receive.*

```
248
 3½
────
744          8·68
124             3
────         ─────
8·68         26·04 Simple Interest for 3 years.

      248
      8·68
     ──────
     256·68  principal for second year
         3½
     ──────
     770·04
     128·34
     ──────
     898·38
```

therefore £8·9837 is interest for second year.

```
256·68
  8·9838
────────
265·6638
      3½
────────
796·9914
132·8319
────────
929·8233
```

therefore £9·298233 is interest for third year.

Hence

$$\begin{array}{rl} 8{\cdot}68 & \\ 8{\cdot}9838 & \\ 9{\cdot}298233 & \\ \hline 26{\cdot}962033 & \text{Compound Interest} \\ 26{\cdot}04 & \text{Simple Interest} \\ \hline {\cdot}922033 & \text{difference} \end{array}$$

therefore 18*s.* „ $5\frac{1}{4}d.$ is the difference.

Ex. 15. ***What is the Compound Interest on £32000 for 2 years at 5 per cent., interest being payable half-yearly?***

We must find the interest for each of the four half-years at $2\frac{1}{2}$ per cent. But to multiply by $2\frac{1}{2}$ and divide by 100 is to multiply by $\frac{1}{40}$; hence

$$\begin{array}{rl} 40)\ \underline{32000} & \\ 800 & \text{interest for first half-year.} \\ 40)\ \underline{32800} & \\ 820 & \text{interest for second half-year.} \\ 40)\ \underline{33620} & \\ 840\frac{1}{2} & \text{interest for third half-year.} \\ \text{£.} \quad s. \quad d. & \\ 40)\ \underline{34460 \text{ „ } 10 \text{ „ } 0} & \\ 861 \text{ „ } 12 \text{ „ } 6 & \text{interest for fourth half-year.} \end{array}$$

	£.	s.	d.	
Hence	800 „	0 „	0	
	820 „	0 „	0	
	840 „	10 „	0	
	861 „	12 „	6	
	3322 „	2 „	6	Compound Interest for 2 years.

Ex. 16. ***What sum at 4 per cent. Compound Interest will amount in $2\frac{1}{2}$ years to £4247 „ 8s. „ 10·368d.?***

First we find that £100 in $2\frac{1}{2}$ years at 4 per cent. will amount to £110·3232.

Next £4247 „ 8*s.* „ 10·368*d.* = £4247·4432.

$$\begin{array}{lcccc} & \text{principal} & \text{principal} & \text{amount} & \text{amount} \\ \text{Hence} & 100 & : Ans. :: & 110{\cdot}3232 & : 4247{\cdot}4432, \end{array}$$

$$Ans. \times 110{\cdot}3232 = 4247{\cdot}4432 \times 100,$$

$$Ans. = \frac{42474432}{1103232} \times 100$$
$$= 38{\cdot}5 \times 100$$
$$= 3850.$$

Ex. 17. *Find the amount of* £842 „ 8*s*. *for* 4 *years at* 5 *per cent. Compound Interest.*

The following method of performing the operation of Compound Interest deserves attention:

In one year £100 amounts to £105; what will 842·4 amount to?

$$100 : 842{\cdot}4 :: 105 : \overset{\text{first year's amount}}{x},$$

$$x \times 100 = 842{\cdot}4 \times 105,$$

$$x = (842{\cdot}4)\left(\frac{105}{100}\right)$$

$$= (842{\cdot}4)\,(1{\cdot}05).$$

Now suppose the first year's amount, viz. (842·4)(1·05) to be put out to interest for the second year.

$$100 : (842{\cdot}4)\,(1{\cdot}05) :: 105 : \overset{\text{second year's amount}}{x},$$

$$x \times 100 = (842{\cdot}4)\,(1{\cdot}05)\,(105),$$

$$x = (842{\cdot}4)\,(1{\cdot}05)\,(1{\cdot}05)$$

$$= (842{\cdot}4)\,(1{\cdot}05)^2.$$

Similarly the amount at the end of the *third* year would be $(842{\cdot}8)(1{\cdot}05)^3$, and at the end of the *fourth* year the amount would be $(842{\cdot}4)\,(1{\cdot}05)^4$.

Now $\qquad (1{\cdot}05)^4 = 1{\cdot}21550625,$

and this we multiply by the given principal, using the contracted form;

$$\begin{array}{r}
{\cdot}121550625 \\
4248 \\
\hline
972405 \\
48620 \\
2431 \\
486 \\
\hline
1023{\cdot}942
\end{array}$$

whence the amount required is £1023 „ 18*s*. „ 10*d*.

§ 98. A few examples are subjoined of *Insurance*, *Commission*, *&c.*, which are commonly placed as a separate rule, although they are only instances of interest in a peculiar form.

Ex. 1. *What is the premium of a policy of assurance for £3500 upon the life of a person aged 44 last birthday, when in the Tables it is found that for every £100 assured a person aged 44 must pay £3 „ 12s. „ 6d. annually?*

It is here requisite to find the Simple Interest on £3500 at $3\frac{5}{8}$ per cent.

Hence $\frac{3500 \times 3\frac{5}{8}}{100} = 35 \times 3\frac{5}{8}$

$= 126\frac{7}{8}$ premium required.

Ex. 2. *What would be the cost of insuring a vessel and cargo valued at £3562 „ 15s. at $6\frac{1}{4}$ per cent.?*

It is only necessary to find the Simple Interest on £3562 „ 15s. at $6\frac{1}{4}$ per cent.

$$\begin{array}{r} 3562{\cdot}75 \\ 6\frac{1}{4} \\ \hline 21376{\cdot}50 \\ 890{\cdot}6875 \\ \hline 222{\cdot}671875 \end{array}$$

(shifting the decimal point two places to divide by 100), whence, by inspection, £222 „ 13s. „ 5d. is the sum required.

Ex. 3. *What would be the Commission paid to an Agent for selling a cargo which realized £4853 „ 13s. „ 4d. at $2\frac{3}{4}$ per cent.?*

Multiplying 4853·6625 by 2·75, by the contracted method, we have

$$\begin{array}{r} 48{\cdot}536625 \\ 572 \\ \hline 9707325 \\ 3397594 \\ 242683 \\ \hline 13347{\cdot}572 \end{array}$$

Hence, shifting the decimal point two places, and finding, by inspection, the value of the decimal, £133 „ 9s. „ 6d. is the sum required.

Ex. 4. *A vessel with its cargo was valued at £45387. The owner insured in such a manner, that although the vessel was lost, he received the full value of vessel and cargo, likewise got back the premium he had paid. Supposing*

that the underwriters insured at the rate of $7\frac{3}{4}$ per cent., at how much above its real value did he insure the vessel and cargo?

If every £$92\frac{1}{4}$ worth, were insured for £100, this would cover both the loss of goods worth £$92\frac{1}{4}$, and likewise a premium of $7\frac{3}{4}$.

Therefore he insured every $92\frac{1}{4}$ worth as if worth £100.

Hence $$92\tfrac{1}{4} : 45387 :: 100 : Ans.,$$

$$\frac{369}{4} \times Ans. = 45387 \times 100,$$

$$Ans. = \frac{\overset{123}{\cancel{45387}} \times 100 \times 4}{\cancel{369}}$$

$$= 49200;$$

therefore the vessel and cargo, worth £45387, was insured for £49200, or for £3813 more than its real value.

Exercise XVI.

1. Find the Interest on £325 „ 10*s.* for $\frac{1}{4}$ of a year at $3\frac{1}{2}$ per cent. per annum.

2. Required the amount of £400 in 3 years and 35 days at $3\frac{3}{4}$ per cent. per annum, Simple Interest.

3. What is the amount of £380 in 3 years and 45 days @ $4\frac{3}{4}$ 0/0 Simple Interest?

4. What is the Interest on £357 „ 10*s.* for 49 days @ $3\frac{1}{2}$ 0/0 per annum?

5. What is the Simple Interest for 2 years on £120 „ 5*s.* at $3\frac{1}{2}$ per cent. per annum?

6. Find the Simple Interest on £172 „ 18*s.* „ 9*d.* for 3 years at 4 per cent.

7. Find the Simple Interest on £1618 „ 1*s.* for 20 years at $3\frac{3}{4}$ per cent.

8. Two persons invest respectively £579 „ 3*s.* „ 4*d.* and £2895 „ 16*s.* „ 8*d.* in a business which returns 12 per cent. per annum on the capital invested; find the annual share of each.

9. Find the Simple Interest on £2833 „ 6*s.* „ 8*d.* for $2\frac{1}{2}$ years, @ 3 0/0 per annum.

10. Find the Simple Interest on £5555 for $5\frac{1}{4}$ years at $5\frac{1}{8}$ per cent.

11. What is the Simple Interest on £3715 „ 10*s*. for 2 years and 131 days at $4\frac{1}{2}$ per cent. ?

12. Find the Simple Interest on £312 „ 19*s*. „ 6*d*. for 1 year and 27 days @ 3 0/0.

13. What is the Simple Interest on £474 „ 15*s*. for 2 years and 4 months at 4 per cent. per annum ?

14. Find the Simple Interest on £63 „ 15*s*. „ 7*d*. for $1\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.

15. An annuity of £50 is put out to interest immediately after payment; what will it amount to at the end of the seventh year after the first payment, allowing 5 per cent. Simple Interest ?

16. What would be the interest on £150 from the 10th of September to the 2nd of November at $4\frac{3}{4}$ per cent. ?

17. An estate of 750 acres, which pays an average rent of £1 „ 12*s*. „ 6*d*. per acre, is burdened with a mortgage of £2500, for which interest is paid at the rate of 4 per cent. per annum ; what is the clear rental of the estate ?

18. What amount of capital put out to Simple Interest @ $3\frac{1}{2}$ 0/0 will produce £14 interest in $2\frac{1}{3}$ years ?

19. What would be the interest on £425 from 1st January to the 4th of May, in Leap Year, at $2\frac{1}{2}$ per cent. ?

20. What sum at $3\frac{1}{4}$ per cent. will produce an income of £445 per annum ?

21. Suppose a man receive interest on £65 „ 4 florins „ 3 cents „ 2 mils at the rate of 1 cent for each florin per annum, what sum (in pounds, shillings, &c.) does he receive at the end of the year ?

22. Find the annual interest on the following sums: £25 „ 1 florin „ 7 cents „ 5 mils at 30 per cent. ; and £368 „ 7 florins „ 5 cents at 5 per cent., expressing the interest as pounds, shillings, and pence.

23. If an exchequer bill for £1000 bear interest at the rate of 2*s*. per diem, what is the rate of interest per cent. per annum ?

---

24. What principal must be put out for $2\frac{1}{2}$ years at 4 per cent. Simple Interest to amount to £132 „ 11*s*. ?

25. Find in what time £963 „ 10*s*. „ 6*d*. will amount to £988 „ 16*s*. „ $4\frac{83}{400}$*d*. at $3\frac{1}{2}$ per cent. Simple Interest.

26. At what rate of Simple Interest will £225 „ 6*s*. „ 8*d*. gain £3 „ 13*s*. „ $2\frac{4}{5}$*d*. in 6 months?

27. In what time will £450 amount to £516 „ 18*s*. „ 9*d*. at $3\frac{1}{2}$ per cent. Simple Interest?

28. What is the capital which, put out at $3\frac{1}{4}$ per cent. Simple Interest for 5 years and 4 months, will amount to £9973 „ 6*s*. „ 8*d*.?

29. When in 2 years and 63 days the Simple Interest on £325 is £24 „ 14*s*. „ $3\frac{1}{7}\frac{5}{8}$*d*., what is the rate per cent. per annum?

30. In what time will the Interest upon £320 „ 12*s*. „ 6*d*. be £70 „ 10*s*. „ 9*d*. at 4 per cent. Simple Interest?

31. What principal, put out for $6\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. Simple Interest, will amount to £1002 „ 19*s*. „ $7\frac{1}{5}$*d*.?

32. What is the rate of Simple Interest when £315 „ 6*s*. „ 8*d*. will amount to £359 „ 9*s*. „ $7\frac{1}{5}$*d*. in 4 years?

33. In what time will £150 „ 15*s*. amount to £175 „ 1*s*. „ $2\frac{1}{40}$*d*. at $4\frac{1}{2}$ per cent. Simple Interest?

34. What capital would be required to gain £12 „ 1*s*. „ $0\frac{3}{4}$*d*. in 2 years and 5 months at $3\frac{1}{2}$ per cent.?

35. What must be the capital employed to gain £95 „ 6*s*. „ 4*d*. as interest in $3\frac{1}{2}$ years at $4\frac{3}{4}$ per cent. Simple Interest?

36. If 12*s*. „ 4*d*., when left in a Savings Bank for $5\frac{1}{2}$ years, gained 3*s*. „ $0\frac{63}{100}$*d*., what was the rate of interest allowed?

Exercise XVII.

COMPOUND INTEREST.

1. Find the amount of £250 in 2 years at $3\frac{1}{2}$ per cent. Compound Interest.

2. What is the amount of £690 for 3 years at $4\frac{1}{2}$ per cent. Compound Interest?

3. Find the amount of £1000 for 4 years at 5 per cent. Compound Interest?

4. Find the amount at Compound Interest of £363 „ 10*s*. for 4 years @ 5 0/0.

5. What is the Compound Interest on £300 at 4 per cent. per annum for 2 years, if the interest be paid half-yearly?

6. Find the interest on £20000 for 4 years at 3 per cent. Compound Interest.

7. Find the Compound Interest on £750 for 2 years at 4 per cent.: also the amount of the same sum in 2 years if the interest be payable half-yearly.

8. If the sum of £1200 be put out at 10 per cent. per annum Compound Interest, and the interest be paid half-yearly, to what will it amount in a year and a half?

9. Find the difference between the Simple and Compound Interest of £2475 „ 13*s*. „ 4*d*. for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.

10. Find the amount of £540 in 3 years at 4 per cent. Compound Interest.

11. Find the amount of £130 in 3 years at 5 per cent. Compound Interest.

12. Find the amount of £769 in 4 years @ 4 0/0 Compound Interest.

13. Find the difference between the Simple and Compound Interest on £150 „ 15*s*. for $3\frac{1}{2}$ years at 4 per cent.

14. What sum must be put out to Compound Interest for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. to amount to £174·39543?

15. What is the difference between the amount of £250 accumulating during 3 years at 3 per cent. Compound Interest, and the amount of the same sum for the same period at 4 per cent. Simple Interest?

16. Find the Compound Interest on £1563 „ 19*s*. „ 8*d*. for $2\frac{3}{4}$ years at $3\frac{1}{2}$ per cent.

17. Find the difference between the amount at Simple and Compound Interest of £895 „ 16*s*. for 2 years at $3\frac{1}{2}$ per cent.

18. At $3\frac{1}{2}$ per cent. Compound Interest, what capital will amount to £289 „ 4*s*. „ 7·38*d*. in 2 years?

19. Find the amount of £415 „ 10*s*. in $2\frac{1}{2}$ years at 4 per cent. Compound Interest.

20. What is the amount of £230 „ 10*s*. for 3 years at $3\frac{1}{2}$ per cent. Compound Interest.

21. What would £25 „ 12*s*. amount to in 3 years at $4\frac{1}{2}$ per cent. Compound Interest?

22. In 3 years at 4 per cent. Compound Interest what would £1080 amount to?

23. What is the Compound Interest on £3350 at 4 per cent. per annum for 2 years, if the interest be paid half-yearly?

24. What sum must be put out at 10 per cent. Compound Interest to amount in 3 years to £1597 „ 4*s.*?

25. Find the sum of money which in 4 years at 5 per cent. Compound Interest will amount to £881 „ 4*s.* „ $10\frac{7}{80}$*d.*

CHAPTER XV.

DISCOUNT.

§ 99. The principle upon which the Rule of Discount depends, being frequently misunderstood, the following explanation should be read attentively:

When a debt due at some future time,—not a Tradesman's Account, but a Bill, or a Promissory Note, or the Rent of a House, or any debt which cannot be claimed at present, but which will fall due some time hence—when such a debt is paid *before it is due*, a sum *smaller* than the actual debt may be paid down by the Debtor, and will be accepted by the Creditor as payment in full.

The reason why the Creditor accepts a sum smaller than his full due is because he would at once put out to interest the money he receives from the Debtor; thus, whatever he will gain as interest he can afford to remit from the debt; and this principle will be manifestly fair to both payer and receiver.

Now call the sum accepted as the present payment, the *Present Worth:* and call the money that is thrown off, the *Discount.* The *Present Worth* must be such a sum as would, if put out to interest for the given time at some agreed on rate, amount to the debt; and the interest it would gain in

that time must be the sum remitted, or the *Discount.* Hence we deduce the following:

Def. The Present Worth of any debt due at some future time is the smaller sum accepted at the present time in lieu of the entire debt at the future time; and is such that, if put out to interest at a given rate for the time during which the debt had to run, it would at the end of that time amount to the debt itself.

Def. Discount is the abatement made in consideration of the payment of a debt before it is due; and is the simple interest of the present worth of the debt.

Hence debt − discount = present worth,

and debt − present worth = discount.

From these definitions we can shew that the discount of a debt must always be *less* than the interest upon it for the same time. For (1) the discount is the interest upon the present worth; but the present worth is always less than the debt; and therefore the interest on the present worth will always be less than the interest on the debt; or the discount on any sum will always be less than the interest of the same sum for the same time.

Again, (2) since present worth + discount = debt, it follows that if the debt in a certain time would give certain interest, we may say that in the same time present worth + discount, if put out to interest, would amount to debt + interest. But by the definition, the present worth would amount to the debt; therefore the discount would amount to the interest; that is, the discount is the *present worth* of the interest.

§ 100. We see from both these considerations that interest is really greater than discount: yet the two are commonly confused, discount being frequently supposed to be the same thing as interest. This is perhaps to be accounted for by these two circumstances; first, tradesmen as a rule deduct interest from an account, and call it discount; an element of confusion which will be more fully explained below, in § 103; secondly, the terms in which the questions are expressed sometimes lead to a mistaken notion; for instance, if it be required to find the present worth of any sum "allowing discount at 5 per cent.," it is taken for granted that this means "throwing off £5 from every £100;" whereas in reality the

first thing in allowing discount is for the payer and receiver to agree upon the rate at which the interest on the present worth is to be calculated; and then "allowing discount at 5 per cent." will not mean throwing off £5 from every £100; but will mean "allowing discount when the rate of interest agreed on is 5 per cent."

§ 101. We now proceed to explain the practical rule for finding the discount of any sum.

If £100 were due a year hence, and if £95 were accepted as the present worth of this debt, the £95 being put out to interest at 5 per cent. would *not* gain £5, and would *not* amount to £100 at the end of the year: and £5 would be too large a sum to allow as the discount on £100 for a year.

But if £105 were due a year hence, and £100 were accepted as the present worth, the £100 being put out to interest at 5 per cent. *would* gain £5, and *would* amount to £105 at the end of the year.

Hence, we observe that £5, the *interest* on £100 for a year, is the *discount* on £105 for the same period: and generally, *the same sum that is the interest of £100 for any time, will, for the same time, be the discount of £100 increased by that interest.*

The rule therefore for finding the discount on any sum will be this: "First find the interest upon £100 for the given time at the given rate, and add it to the £100: the sum found as *interest* on £100 will be the *discount* on the £100 increased by its interest: then the discount on any other sum for the same time at the same rate can be found by proportion."

This will now be illustrated by examples:

Ex. 1. *Find the Discount on £770 due 8 months hence, allowing interest at 4 per cent. per annum.*

$$8 \text{ months} = \frac{8}{12} = \frac{2}{3} \text{ of a year};$$

therefore interest at 4 per cent. on £100 for 8 months

$$= \frac{2}{3} \text{ of } £4 = £\frac{8}{3} = £2\tfrac{2}{3};$$

therefore £$2\frac{2}{3}$ is *discount* on £$102\frac{2}{3}$ for 8 months.

$$\text{Hence}\qquad \overset{\text{Debt}}{102\tfrac{2}{3}} : \overset{\text{Debt}}{770} :: \overset{\text{Disct.}}{2\tfrac{2}{3}} : \overset{\text{Disct.}}{Ans.},$$

$$\frac{308}{3}\times Ans. = 770\times\frac{8}{3}$$

$$Ans. = 770\times\frac{\overset{2}{\cancel{8}}}{\cancel{3}}\times\frac{\cancel{3}}{\underset{77}{\cancel{308}}}$$

$$= 770\times\frac{2}{77}$$

$$= 20.$$

Ex. 2. *Find the exact Discount which should be allowed upon £100 due a year hence, reckoning interest at 5 per cent.*

£5, which is the interest upon £100, is the discount upon £105 for a year; therefore

$$105 : 100 :: 5 : Ans.,$$

$$105\times Ans. = 100\times 5,$$

$$Ans. = \frac{100\times\cancel{5}}{\underset{21}{\cancel{105}}}$$

$$= \frac{100}{21}$$

$$= £4 ,, 15s. ,, 2\tfrac{6}{7}d.$$

Hence £100 − £4 „ 15*s.* „ $2\frac{6}{7}$*d.* = £95 „ 4*s.* „ $9\frac{1}{7}$*d.* is the exact present worth of £100 due a year hence, when interest is at 5 per cent.

§ 102. In mercantile transactions, if a bill for £100 due a year hence were to be discounted, interest being at 5 per cent., the merchant or banker would give to the holder of the bill only £95 as the present worth; deducting the interest, *i.e.* £5, instead of £4 „ 15*s.* „ $2\frac{6}{7}$*d.*, the exact mathematical discount. The difference between £5 and £4 „ 15*s.* „ $2\frac{6}{7}$*d.* viz. 4*s.* „ $9\frac{1}{7}$*d.*, is the discounter's profit, and the sum which the holder of the bill pays for the accommodation. This 4*s.* „ $9\frac{1}{7}$*d.* is the interest at 5 per cent. on £4 „ 15*s.* „ $2\frac{6}{7}$*d.* for 12 months (for discount is the present worth of interest, § 99); and therefore the loss incurred by the holder of the bill by being charged interest instead of discount, is a sum which is the simple interest upon the exact discount, for the time during which the bill has to run.

From this we see, generally, that the *difference* between interest and discount on any given sum for a given time is equal to the interest *on* the discount for the same time.

§ 103. When a tradesman lowers the price of any article in consideration of payment in ready money, this is neither mathematical discount, nor mercantile discount, nor discount in any proper sense of the word. For, strictly speaking, payment cannot be said to be made *before due*, when once the article has passed into the possession of the purchaser. But the tradesman marks his goods at such prices above what he gave for them, that he may be enabled to make a profit by retailing them, and give credit besides for, say, 12 months. Suppose he marks the credit price of his goods at 35 per cent. above the cost price; this rate of interest he will lower to 30 per cent. if he be paid in ready money: and this he calls allowing discount at 5 per cent. The so-called discount of trade is therefore only lowering the rate of interest charged by the retail dealer.

§ 104. Since £100 is the present worth of £105 due a year hence, interest being at 5 per cent., we can find the *present worth* of any given sum by proportion, without first finding the discount and then subtracting it from the debt. The method is as follows:

Ex. 3. *Find the Present Worth of £10500 due 15 months hence at 4 per cent. Simple Interest.*

$$15 \text{ months is } \frac{15}{12} \text{ or } \frac{5}{4} \text{ of a year;}$$

therefore the interest of £100 for 15 months is $\frac{5}{4}$ of 4, or £5; therefore

$$\overset{\text{Debt}}{105} : \overset{\text{Debt}}{10500} :: \overset{\text{Prt.Wth.}}{100} : \overset{\text{Prt.Wth.}}{\mathit{Ans.}},$$

$$105 \times \mathit{Ans.} = 10500 \times 100,$$

$$\mathit{Ans.} = 100 \times 100$$

$$= 10000.$$

If it be agreed to reckon *compound* interest on the present worth, we must find the compound interest on £100 for the given time at the given rate, add it to £100, and proceed as before.

Ex. 4. *Find the Discount of £1035 „ 17s. „ 6d. for two years at 4 per cent. Compound Interest.*

At 4 per cent. Compound Interest, £100 amounts to £108·16. Hence

$$108\cdot16 : 1035\cdot875 :: \overset{\text{Disct.}}{8\cdot16} : \overset{\text{Disct.}}{Ans.},$$

$$Ans. \times 108\cdot16 = 1035\cdot875 \times 8\cdot16,$$

$$Ans. = \frac{1035\cdot875 \times \overset{51}{\cancel{8\cdot16}}}{\underset{676}{\cancel{108\cdot16}}}$$

$$= \frac{52829\cdot625}{676}$$

$$= 78\cdot15033, \text{ \&c.}$$

$$= £78 \text{ „ } 3s. \text{ „ } 0\cdot0792d.$$

We may be asked to find the *principal*, where the discount is given; or the *rate of interest*; or the *time* for which the debt has to run: and to illustrate such cases, the following examples are subjoined.

Ex. 5. *What was the Debt of which the Discount for 3 years at 4 per cent. Simple Interest, was £36?*

£12 is the interest of £100 for given time at given rate. Hence

$$112 : Ans. :: 12 : 36,$$

$$12 \times Ans. = 112 \times 36,$$

$$Ans. = \frac{112 \times 36}{12}$$

$$= 112 \times 3$$

$$= £336.$$

Ex. 6. *When the Discount on £256 „ 10s. paid half a year before it is due is £5 „ 0s. „ $7\frac{3}{51}d.$, at what rate is Simple Interest calculated?*

From £256 „ 10s. deduct £5 „ 0s. „ $7\frac{3}{51}d.$, and the result, £251 „ 9s. „ $4\frac{48}{51}d.$, is the present worth. But the *discount* of the debt is the *interest* on the present worth. Hence £5 „ 0s. „ $7\frac{3}{51}d.$ is the interest on £251 „ 9s. „ $4\frac{48}{51}d.$ for 6 months; and we have to find the interest on £100 for 12

months at the same rate; therefore

$$251\tfrac{8}{17}\times 6 \ :\ 100\times 12 \ ::\ 5\tfrac{1}{34} \ :\ Ans.,$$

$$\frac{4275}{17}\times 6\times Ans.=100\times 12\times\frac{171}{34},$$

$$Ans.=\overset{4}{\cancel{100}}\times 12\times\frac{171}{34}\times\frac{17}{\underset{171}{\cancel{4275}}}\times\frac{1}{6}$$

$$=4 \text{ per cent.}$$

Ex. 7. *If* £82 „ 18*s.* „ $9\frac{15}{17}$*d. be the Discount of a debt of* £1410, *Simple Interest being at the rate of* $3\frac{3}{4}$ *per cent., how many months before due was the debt paid?*

£	s.	d.
1410 „	0 „	0
82 „	18 „	$9\frac{15}{17}$
1327 „	1 „	$2\frac{2}{17}$

Hence £82 „ 18*s.* „ $9\frac{15}{17}$*d.* is the interest on the present worth of £1327 „ 1*s.* „ $2\frac{2}{17}$*d.* for an unknown number of months; while $3\frac{3}{4}$ is the interest on £100 for 12 months. Therefore

$$100\times 12 \ :\ 1327\tfrac{1}{17}\times Ans. \ ::\ 3\tfrac{3}{4} \ :\ 82\tfrac{16}{17},$$

$$Ans.\times\frac{22560}{17}\times\frac{15}{4}=100\times 12\times\frac{1410}{17},$$

$$Ans.=\overset{20}{\cancel{100}}\times 12\times\frac{\cancel{1410}}{17}\times\frac{17}{\underset{16}{\cancel{22560}}}\times\frac{4}{\underset{3}{\cancel{15}}}$$

$$=20 \text{ months.}$$

Ex. 8. *The interest on a certain sum of money for two years is* £71 „ 16*s.* „ $7\frac{1}{2}$*d., and the Discount for the same time is* £63 „ 17*s., Simple Interest being reckoned in both cases. Find the rate per cent. per annum, and the sum.* (S.-H., 6 Jan., 1863.)

Since the discount is the present worth of interest (§ 99), it follows that

	£ s.		£ s. d.
	63 „ 17	is the present worth of	71 „ 16 „ $7\frac{1}{2}$
i.e.	63 „ 17	would in 2 years amount to	71 „ 16 „ $7\frac{1}{2}$
i.e.	63 „ 17	would in 2 years gain as interest	7 „ 19 „ $7\frac{1}{2}$
∴	....	1 year	3 „ 19 „ $9\frac{3}{4}$

From this we can find what £100 would gain in 1 year,

$$63\tfrac{17}{20} : 100 :: 3\tfrac{317}{320} : Ans.,$$

$$\frac{1277}{20} \times Ans. = 100 \times \frac{1277}{320},$$

$$Ans. = 100 \times \frac{1277}{320} \times \frac{20}{1277}$$

$$= 6\tfrac{1}{4} \text{ rate per cent.}$$

Next to find the sum in question, we know that £71 „ 16*s.* „ 7½*d.* is the interest on it for 2 years at 6¼ per cent.; hence

$$100 \times 1 : Ans. \times 2 :: 6\tfrac{1}{4} : £71 \text{ „ } 16s. \text{ „ } 7\tfrac{1}{2}d.,$$

$$Ans. \times 2 \times \frac{25}{4} = 100 \times (£71 \text{ „ } 16s. \text{ „ } 7\tfrac{1}{2}d.),$$

$$Ans. = \frac{2}{25} \times 100 \times (£71 \text{ „ } 16s. \text{ „ } 7\tfrac{1}{2}d.)$$

$$= 8 \times (£71 \text{ „ } 16s. \text{ „ } 7\tfrac{1}{2}d.)$$

$$= £574 \text{ „ } 13s.$$

Ex. 9. *If £4 be allowed as Discount off a bill of £40 due 6 months hence, how much should be allowed off a bill of the same amount due 13½ months hence?*

	£ 4 is 6 months' discount off	£ 40;
therefore	4 .... interest on	36;
therefore	9 is 13½ months' interest on	36;
therefore	9 .... discount off	45.

From this we can find what will be the discount off £40 for the same time and rate; for

$$45 : 40 :: 9 : Ans.,$$

$$45 \times Ans. = 40 \times 9,$$

$$Ans. = \frac{40 \times 9}{45}$$

$$= 8, \text{ discount required.}$$

Ex. 10. *If £3 be allowed as Discount off a bill of £33 due 6 months hence, what should be the bill from which the same sum is allowed as 3 months' discount?*

	£ 3 is 6 months' discount off	£ 33;
therefore	3 .... interest on	30;
therefore	3 is 3 months' interest on	60;
therefore	3 .... discount off	63.

Hence £63 is the amount required.

Ex. 11. *If £3 be allowed as 6 months' Discount off £33, and at the same rate of interest £10 be allowed off a bill of £60, for how long a period had the latter to run?*

	£ 3 is 6 months' discount off £ 33;
therefore	3 .... interest on 30;
therefore	5 .... 50;
therefore	10 is 12 months' interest on 50;
therefore	10 .... discount off 60.

Hence 12 months is the time required.

Ex. 12. *If £10 be the Interest upon £110 for a given time, what should be the Discount off £110 for the same time?*

	£ 10 is the interest upon £ 110;
therefore	10 is the discount off 120.

From this we can find the discount to be allowed off £110 for the same time and rate; for

$$120 : 110 :: 10 : Ans.,$$

$$120 \times Ans. = 110 \times 10,$$

$$Ans. = 9\tfrac{1}{6}$$

$$= £9 \text{ ,, } 3s. \text{ ,, } 4d.$$

Exercise XVIII.

DISCOUNT.

1. What discount should be allowed on £420 paid 9 months before due, simple interest being calculated at 5 per cent.?

2. Find the discount on £243 ,, 10s. due 5 months hence, at 3½ per cent. interest.

3. What is the discount on £1120, due in 16 months, at 5 per cent. per annum?

4. A bill of £46 ,, 0s. ,, 6¾d. is due 10 months hence; what is the discount for ready money, when interest is 3¾ per cent. per annum?

5. Find the discount on £600, due in 6 months, interest being at the rate of 5½ per cent.

6. State the difference between Interest and Discount; and find the discount on £150, due 8 months hence, at 5 per cent.

7. What is the difference between the interest on a bill of £138 „ 13*s.* „ 4*d.* for 3 months, at 4 per cent. per annum, and the discount on the same for a quarter of a year, at the same rate?

8. What is the present value of £875 „ 9*s.* „ 6*d.*, due $5\frac{1}{2}$ years hence, at $3\frac{1}{2}$ per cent. simple interest?

9. Shew that the interest on £266 „ 13*s.* „ 4*d.* for 3 months, at $4\frac{1}{2}$ per cent., is equal to the discount on £83 for 15 months, at 3 per cent.

10. What is the discount of £430, paid 8 months before it is due, interest being at 4 per cent.?

11. Find the discount of £125 „ 10*s.*, paid 3 months before it is due, the interest of money being 4 per cent. per annum.

12. What is the discount of £1250, due 9 months hence, at $5\frac{1}{2}$ per cent.?

13. Required the present worth of £572, due 8 months hence, at $3\frac{3}{4}$ per cent. interest.

14. What is the present value of £120, due 10 months hence, at 4 per cent.?

15. In consideration of immediate payment, what sum ought a tradesman who gives 2 years' credit to abate in a bill of 14 guineas, allowing interest at $7\frac{1}{2}$ per cent.?

16. What is the discount on £485 „ 2*s.*, due 2 years hence, at 5 per cent. compound interest?

17. What is the difference between the interest and discount of £125 „ 8*s.* „ 6*d.* for half a year, at $3\frac{1}{4}$ per cent.?

18. Required the discount and present worth of £151 „ 17*s.* „ 6*d.* due at the end of 4 years, at $5\frac{5}{8}$ per cent.

19. Required the present worth of a debt of £1242 „ 6*s.* „ 8*d.* due 245 days hence, at $3\frac{1}{4}$ per cent.

20. What present payment will discharge a debt of £75, due 17 months hence, interest being at 4 per cent.?

21. What ready money will discharge a debt of £170, due 5 months hence, allowing 8 per cent. interest?

22. Find the discount on £150 for 55 days, at $4\frac{3}{4}$ per cent. per annum.

23. Find the present worth of £694 „ 15*s.* due in 9 months, allowing $3\frac{1}{2}$ per cent. interest.

24. What is the rate of Simple Interest, when £578 „ 13*s.* „ 4*d.* paid down is considered equivalent to £593 „ 2*s.* „ 8*d.* at the end of 8 months?

25. What is the discount of £964 „ 19*s.* „ 6*d.* due in 3 years hence, at 10 per cent. Compound Interest?

26. What ready money will discharge a debt of £85, due 5 months hence, allowing interest at the rate of 13*s.* „ 4*d.* per cent. per month?

27. What is the present worth of £101236 „ 7*s.* „ 2*d.* due 3 years hence, at 6 per cent. Compound Interest?

28. What is the present worth of £120 payable as follows: viz., £50 at 3 months, £50 at 5 months, and the remainder at 8 months; interest being at 5 per cent.?

29. Required the present worth of £868 „ 4*s.* „ $3\frac{1}{8}$*d.* due 3 years hence, at 5 per cent. Compound Interest?

30. Bought a quantity of goods for ready money for £150, and sold them for a bill for £200 payable $\frac{3}{4}$ of a year hence. If this bill were at once fairly discounted, at $4\frac{1}{2}$ per cent. interest, what would be the ready money gain on the transaction?

31. Find the present worth of £562 „ 8*s.* „ $7\frac{1}{2}\frac{7}{8}$*d.* due 3 years hence, at 4 per cent. Compound Interest.

32. If on a debt of £252 „ 19*s.* „ 3*d.* due a year hence, the discount allowed be £7 „ 19*s.* „ 3*d.*, at what rate was interest calculated?

33. If £1137 „ 10*s.* be the present worth of a debt of £1336 „ 11*s.* „ 3*d.* when Simple Interest is calculated at 5 per cent., how long before due was the debt paid?

34. If on a debt of £16992 „ 1*s.* „ 9*d.* due 4 years hence, the present worth were £14648 „ 8*s.* „ 9*d.*, at what rate was the Simple Interest calculated?

35. If £666 „ 13*s.* „ 4*d.* be the present worth of a debt due six months hence, money being worth 3 per cent. per annum, find what was the debt.

36. When it is reckoned that £262 „ 4*s.* „ $5\frac{1}{3}$*d.* is the exact present worth of a debt of £275 „ 6*s.* „ 8*d.*, interest being at 4 per cent. per annum, how long had the debt to run?

37. If the discount on £678 „ 8*s.*, which is due at the end

of a year and a half, be £38 „ 8*s*., what is the rate per cent. of Simple Interest? (S.-H., 2 Jan. 1855).

38. A certain sum of money ought to have £20 „ 16*s*. allowed as 8 months' interest on it: but a bill for the same sum due 8 months hence at the same rate of interest, should have £20 only allowed off as discount in consideration of present payment. Required the sum and the rate per cent. per annum.

39. Prove that the difference between the interest and the discount on a given sum for a given time is equal to the interest on the discount for the same time. (Ch. Coll., Dec. 1863).

40. If interest be reckoned at $4\frac{1}{2}$ per cent. per annum, and I accept £40 as present payment for £42 „ 8*s*., for how long a period was this discounted?

41. If £5 be allowed as discount off a bill of £125, due a certain time hence, what would be the discount allowed off, if the bill had twice as long to run?

Supposing *Compound* Interest to be allowed, what then would be the answer to the above question?

42. If £98 were accepted in present payment of £128, due some time hence, what should be the proper discount off a bill of £128 which has only half the time to run?

Solve the above question, allowing *Compound* Interest.

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§ 105. As a supplement to the rule of discount and present worth, we may add a few examples in a rule called the *Equation of payments*. In the method of calculation which is adopted in Arithmetic to find the *Equated time*, as it is called, that is, the exact time at which several debts due at different times should be paid in one sum, it is usual to reckon interest as equivalent to discount. By this method a rough approximation only is obtained, and the rule is accordingly of little practical utility. In Algebra more exact methods are given. The Arithmetical process will be understood from the following examples:

Ex. 1. *Find the equated time of payment of* £700 *due* 15 *months hence, and* £500 *due* 9 *months hence.*

The rule in Arithmetic is as follows:

"Multiply each debt by the time hence it is due; add "these results; and divide them by the sum of the various

"debts; the quotient is the time hence at which the whole "sum is due."

Therefore
$$\frac{700 \times 15 + 500 \times 9}{700 + 500}$$
$$= \frac{10500 + 4500}{1200}$$
$$= \frac{15000}{1200}$$
$$= \frac{150}{12}$$
$$= 12\tfrac{1}{2} \text{ months.}$$

Ex. 2. *What would be the present value of the following bills, supposed to be due at the equated time, viz. £250 due 5 months hence, £490 due* 15 *months hence, and £1860 due* $1\frac{2}{3}$ *months hence; it being agreed that interest shall be calculated at* 4 *per cent.?*

The equated time will be

$$\frac{250 \times 5 + 490 \times 15 + 1860 \times \frac{5}{3}}{250 + 490 + 1860}$$
$$= \frac{1250 + 7350 + 3100}{2600}$$
$$= \frac{11700}{2600}$$
$$= \frac{117}{26}$$
$$= 4\tfrac{1}{2} \text{ months.}$$

We have now to determine the present worth of £2600, due $4\frac{1}{2}$ months hence at 4 per cent.

Hence
$$\frac{9}{2} \times \frac{1}{12} \times 4 = \frac{3}{2} = 1\tfrac{1}{2},$$
$$101\tfrac{1}{2} : 2600 :: 100 : Ans.,$$
$$\frac{203}{2} \times Ans. = 2600 \times 100,$$
$$Ans. = 2600 \times 100 \times \frac{2}{203}$$
$$= \frac{520000}{203}$$
$$= £2561 \text{ ,, } 12s. \text{ ,, } 6\tfrac{30}{203}d.$$

EXERCISE XIX.

1. If £200 be due one year hence, and £100 be due two years hence, find the equated time of one payment, allowing interest at the rate of 5 per cent. per annum.

2. If £760 be due 13 months hence, and £440 be due 8 months hence; what is the equated time of payment?

3. What is the equated time of payment of the following bills: £400 due in $2\frac{1}{2}$ years, £500 due in $1\frac{3}{4}$ years, and £300 due in 9 months?

4. If £450 be due 16 months hence, and £250 be due $13\frac{1}{2}$ months hence; find the present worth of the whole sum supposed to be due at the equated time, interest being at 4 per cent.

CHAPTER XVI.

STOCKS.

§ 106. It is often necessary for the Government of a country to borrow money, in order to carry on expensive wars, supply any sudden deficiency, &c. A loan is then contracted, and the Government borrowing pledges the credit of the country to pay a certain fixed rate of Interest on the entire sum borrowed, until such time as the debt may be paid off. Now all such loans, contracted by any Government whether English or Foreign, are called generically "Funds:" and we speak of the holders of such loans as having money in the "Funds," or in the "Public Funds." But to any consolidated fund, whether a Government loan or a joint-stock company's capital, the word "Stock" is applied, and the purchase of "Stock" means simply buying some portion of the total loan or capital.

Railway Companies, and others, as soon as their "shares" are fully paid up, may, and generally do, convert them into "Stock," so as to admit of the transfer of any amount, from 20*s.* upwards. As long as the capital remains in "shares" it

is only possible to deal in multiples of the amount at which each share stands, inasmuch as these cannot be divided: hence the convenience of conversion into Stock.

In explaining a transaction in the Public Funds, let us suppose that Government, being in want of money, proposes to give 4 per cent. per annum for the money they borrow. If *A* should lend £100 to the Government, he would receive £2 every half-year; these half-yearly *dividends*, as they are called, forming together a perpetual annuity of £4 paid to him out of the public revenues. But if *A* wanted to be repaid his principal, he could not demand from Government £100; because they only agreed to pay *interest*, but named no time when the *principal* would be paid off. *A* however is at liberty to transfer his claim to any other person: he would therefore sell his *stock* to the highest bidder in the money market: but if money at this time should be more scarce, and so valuable as generally to fetch 5 per cent. in other investments, it is clear that nobody would give him £100 cash for the right of getting £4 per annum: no one would consent to receive 4 per cent. for his money if he could get 5 per cent. *A* must therefore lower his price, and his £100 stock would sell for somewhat less than £100 cash. It is therefore necessary to remember the difference between *money in the funds*, and *ready cash;* indeed £100 *in the funds* is usually a very different thing from £100 *ready cash.*

When the stocks are said to be selling *at* a certain rate, (*e.g.* at 95,) this means that £100 *stock* is selling for £95 *cash.*

Different loans are called the 3 per cents., the $3\frac{1}{2}$ per cents., the 4 per cents., &c. according to the rate of interest agreed on at the time of borrowing.

When a person is said to *invest* so much money in the funds, this means that he takes so much *cash*, and buys with it as much *stock* as he can at the current market price. On the contrary, *selling out* is selling his *stock* for as much *cash* as it will produce at the market price of the day.

The *income* that a man possesses by holding so much stock can be computed at once by simple interest: *i. e.* by multiplying the stock (which is the principal) by the rate per cent., and dividing by 100. The income however derived from *investing* so much cash in any stock will depend upon the price of the stock at the time of purchase; for government pays interest on the *stock* held, and therefore the more stock that can be

bought for any sum, (or the cheaper the stock), the greater in proportion is the income produced.

When the state of the money market is such that £100 of any stock is worth £100 cash, then that kind of stock is said to be at par. If the rate of interest be high, and money plentiful, it is possible that £100 stock may be worth *more* than £100 cash, and the stock is said to be above par. The fluctuation in the price of stock is not caused by any variation in *the rate of interest* which is paid; that is fixed, once for all, at the time the money is borrowed; and Government continues to pay this settled rate of interest on every £100 stock, by whomsoever it may be held. Commercial and political changes at home and abroad, the state of trade, the prospect of the harvest, investments in railway shares, and other speculations, all affect the value of money and the price of the Funds.

All questions in the rule depend upon the principles of Proportion; the subjoined examples contain the commonest forms in which questions occur; it must however be premised that, unless expressly required in the question, no account is taken of brokerage or commission, stamps or transfer fees. The ordinary practice of brokerage is explained below, at example 12.

Ex. 1. *What quantity of stock can be bought at* 92 *for* £27600?

In other words, if £92 cash will buy 100 stock, what stock will £27600 cash purchase?

$$\overset{\text{cash}}{92} : \overset{\text{cash}}{27600} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{Ans.},$$

$$92 \times Ans. = 27600 \times 100,$$

$$Ans. = \frac{27600 \times 100}{92}$$

$$= 30000 \text{ stock}.$$

Ex. 2. *What money will be obtained by the sale of* 7800, 3 *per cent. stock, at* 89?

That is, if 100 stock obtain £89 cash, what will 7800 stock obtain?

$$\overset{\text{cash}}{89} : \overset{\text{cash}}{Ans.} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{7800},$$

$$100 \times Ans. = 89 \times 7800,$$

$$Ans. = 89 \times 78$$

$$= £6942 \text{ cash}.$$

Ex. 3. *If £11040 be invested in the 3 per cents. at 92, what quantity of stock will be obtained by the investment? And what annual income will be derived?*

$$\overset{\text{cash}}{92} : \overset{\text{cash}}{11040} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{\textit{Ans.}},$$

$$\textit{Ans.} \times 92 = 11040 \times 100,$$

$$\textit{Ans.} = \frac{11040 \times 100}{92}$$

$$= 120 \times 100$$

$$= 12000 \text{ stock.}$$

Next, for the income, find the Simple Interest on £12000 at 3½ per cent., *i.e.* multiply 12000 by 3½ and divide by 100.

Hence $$12000 \times \frac{7}{2} \times \frac{1}{100} = £420,$$

the interest required.

Ex. 4. *What income will be derived from the investment of £29400 in the 4 per cents. at 98?*

It is not necessary to find the quantity of stock held, unless it is expressly asked for. But since every £98 we invest buys £100 stock, and every £100 stock pays £4 annually as interest, we may say that every £98 invested yields an annual income of £4. And of course £29400 invested yields an income proportionably larger. Hence

$$\overset{\text{cash}}{98} : \overset{\text{cash}}{29400} :: \overset{\text{income}}{4} : \overset{\text{income}}{\textit{Ans.}},$$

$$\textit{Ans.} \times 98 = 29400 \times 4,$$

$$\textit{Ans.} = \frac{29400 \times 4}{98}$$

$$= 300 \times 4$$

$$= £1200.$$

Ex. 5. *Bought stock in the 3½ per cents. at 87½; what was the real rate per cent. obtained by the investment?*

In other words, if every £87½ cash invested produce £3½ per annum, what would £100 cash so invested produce?

$$87\tfrac{1}{2} : 100 :: 3\tfrac{1}{2} : \textit{Ans.},$$

$$\frac{175}{2} \times Ans. = 100 \times \frac{7}{2},$$

$$Ans. = 100 \times \frac{7}{2} \times \frac{2}{175}$$

$$= \frac{100}{25}$$

$$= 4 \text{ per cent.}$$

Ex. 6. *When the funds are at 75, how much stock must be sold out to realise £125?*

That is, if the sale of £100 stock realises £75 cash, how much stock must be sold to realise £125 cash?

$$\overset{\text{cash}}{75} : \overset{\text{cash}}{125} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{Ans.},$$

$$Ans. \times 75 = 125 \times 100,$$

$$Ans. = \frac{125 \times 100}{75}$$

$$= \frac{5 \times 100}{3}$$

$$= \frac{500}{3}$$

$$= 166\tfrac{2}{3} \text{ stock.}$$

Ex. 7. *What price are the funds at when a person buys £500 stock for £401 „ 13s. „ 4d.?*

$$\overset{\text{cash}}{Ans.} : \overset{\text{cash}}{401\tfrac{2}{3}} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{500},$$

$$Ans. \times 500 = \frac{1205}{3} \times 100,$$

$$Ans. = \frac{1205}{3} \times \frac{1}{5}$$

$$= \frac{241}{3}$$

$$= 80\tfrac{1}{3} \text{ price of stock.}$$

Ex. 8. *The interest on a certain sum in the 4 per cents. was allowed to accumulate for 14 years, simple interest only being reckoned. At the end of that time the principal and interest amounted to £1326; what was the original sum in the funds?*

In 14 years £100 would amount, at 4 per cent. simple interest, to £156. Hence

$$100 : Ans. :: 156 : 1326,$$

$$Ans. \times 156 = 1326 \times 100,$$

$$Ans. = \frac{1326 \times 100}{156}$$

$$= \frac{221 \times 100}{26}$$

$$= \frac{221 \times 50}{13}$$

$$= 17 \times 50$$

$$= 850 \text{ stock.}$$

Ex. 9. *Which is the more advantageous stock to invest in, the 3 per cents. at 80, or the $3\frac{1}{2}$ per cents. at 98?*

We must find in each case the real rate of interest per cent., and compare the results.

First then, if £80 invested yields £3, what will £100 so invested yield?

$$80 : 100 :: 3 : Ans.,$$

$$8 \times Ans. = 10 \times 3$$

$$Ans. = \frac{30}{8}$$

$$= 3\frac{3}{4} \text{ per cent.}$$

Next, if 98 yields $3\frac{1}{2}$, what will £100 yield?

$$98 : 100 :: 3\frac{1}{2} : Ans.,$$

$$Ans. \times 98 = 100 \times \frac{7}{2},$$

$$Ans. = 50 \times 7 \times \frac{1}{98}$$

$$= \frac{50}{14}$$

$$= \frac{25}{7}$$

$$= 3\frac{4}{7} \text{ per cent.}$$

The first investment yields interest at $3\frac{3}{4}$ per cent., the second at $3\frac{4}{7}$ per cent., and *comparing* these fractions, *i.e.* reducing them to equivalent fractions having a common denomi-

nator, we see that the rates become $3\frac{2}{2}\frac{1}{8}$ and $3\frac{1}{2}\frac{2}{8}$; and therefore the first investment would be the more advantageous.

Ex. 10. *A person transferred £16500 stock from the 3 per cents. at 90, to the $3\frac{3}{4}$ per cents. at 99. Required what quantity of the latter stock he held, and what alteration was made in his income.*

What is meant by a *transfer* of stock is this; that a person possessed of a certain quantity of one kind of stock, being either dissatisfied with the security, or in hopes of improving his income, sells that stock at the current market price, and invests all the cash so obtained in the purchase of another kind of stock; he may thus improve his annual income, or may be content with a smaller income if he considers he has better security.

In the case supposed the person held at first £16500 stock in the 3 per cents.; from which his income was

$$16500 \times 3 \times \frac{1}{100}, \text{ or } £495.$$

He then sold this stock at 90, obtaining for it cash;

$$\overset{\text{cash}}{90} : \overset{\text{cash}}{Ans.} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{16500},$$

$$Ans. \times 100 = 90 \times 16500,$$

$$Ans. = 14850 \text{ cash}.$$

He now invests this 14850 cash at 99, purchasing with it stock;

$$\overset{\text{cash}}{99} : \overset{\text{cash}}{14850} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{Ans.},$$

$$\begin{aligned} Ans. \times 99 &= 14850 \times 100 \\ &= 150 \times 100 \\ &= 15000 \text{ stock}. \end{aligned}$$

But the income from this stock is

$$15000 \times \frac{15}{4} \times \frac{1}{100}, \text{ or } 75 \times \frac{15}{2},$$

$$\text{or } £562\tfrac{1}{2}.$$

Hence he held £15000 of the latter kind of stock; and increased his annual income by £67 „ 10*s*.

Ex. 11. *A person holding £4400 of the 6 per cents. Turkish loan, distrusting the security, sold out at $91\frac{3}{4}$, and invested in*

the English 3 per cents. at 88. By how much did this diminish his income?

In the last example we exhibited the entire process of selling out of one kind of stock, and buying into another. But the process may be shortened by considering that the quantity of stock held will be greater or less according as the price is lower or higher; thus in the given case clearly *more* of the English stock will be held than of the Turkish; also the quantity of English stock will be greater than that of Turkish, in the same ratio that the price of the Turkish is greater than the price of the English. Hence, making one statement, we may say

$$\overset{\text{Eng. stock}}{Ans.} : \overset{\text{Turk. stock}}{4400} :: \overset{\text{price of Turk.}}{91\tfrac{3}{4}} : \overset{\text{price of Eng.}}{88},$$

$$Ans \times 88 = 4400 \times \frac{367}{4},$$

$$Ans. = 1100 \times 367 \times \frac{1}{88}$$

$$= \frac{36700}{8}$$

$$= 4587\tfrac{1}{2} \text{ English stock.}$$

But income derived from the Turkish 6 per cents. was

$$4400 \times 6 \times \frac{1}{100}, \text{ or } £264;$$

while income derived from the English 3 per cents. is

$$4587\tfrac{1}{2} \times 3 \times \frac{1}{100}, \text{ or is } £137 \text{ ,, } 0s. \text{ ,, } 6d.$$

Hence his income will be diminished by £126 ,, 19*s.* ,, 6*d.*

Ex. 12. *What will be the cost of purchasing £720 Russian 5 per cents. at 91⅛, commission at ⅛ per cent. being charged? also if I sell out again when the price has risen to 93⅞, (brokerage being also charged in this case,) what do I gain by the transaction?*

The purchase or sale of stock is generally effected by means of a Stock-Broker, who is paid a certain per centage on all the *stock* that passes through his hands [1].

[1] This is the usual practice in all transactions in Government Funds, whether British or Foreign. But upon other kinds of Stock, such as the Guaranteed Indian Railway Stocks, &c. the brokerage is charged upon the proceeds, and not upon the amount bought or sold.

This *commission* of *brokerage*, as it is called, is generally 2*s*. „ 6*d*., or $\frac{1}{8}$ of £1, on every 100 stock which is bought or sold.

Hence in *buying* stock when commission is charged, the current price of every 100 stock will be *increased* by $\frac{1}{8}$. On the contrary, in *selling* stock, the current price will be *diminished* by $\frac{1}{8}$ when brokerage is charged.

In the example given, when brokerage is charged, every £100 stock will cost the purchaser the market price of $91\frac{1}{2}$ *plus* $\frac{1}{8}$ for the broker: hence the cost will be altogether $91\frac{5}{8}$. Therefore

$$\overset{\text{cash}}{91\tfrac{5}{8}} : \overset{\text{cash}}{Ans.} :: \overset{\text{stock}}{100} : \overset{\text{stock}}{720},$$

$$Ans. \times 100 = \frac{733}{8} \times 720,$$

$$Ans. = 733 \times 90 \times \frac{1}{100}$$

$$= \frac{6597}{10}$$

$$= £659 \text{ „ } 14s.$$

Next, the £720 stock was sold at $93\frac{7}{8}$ *minus* $\frac{1}{8}$ for the broker; *i.e.* it was sold at $93\frac{3}{4}$. Therefore

$$\overset{\text{cash}}{93\tfrac{3}{4}} : \overset{\text{cash}}{Ans.} :: 100 : 720,$$

$$Ans. \times 100 = \frac{375}{4} \times 720,$$

$$Ans. = 375 \times 180 \times \frac{1}{100}$$

$$= £675.$$

Hence £675 − £659 „ 14*s*. = £15 „ 6*s*. = the gain on the transaction.

Ex. 13. *A person having to pay £1045 two years hence, invested a certain sum in the 3 per cent. consols to accumulate interest until the debt be paid, and also an equal sum the next year; supposing the investments to be made when consols are at 73, and the price to remain the same, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?* (S.-H., Jan. 1, 1861).

On the hypothesis that the first year's interest will be invested in stock, and no allowance be made for brokerage, let

us suppose that S represents the sum first invested, then

every £73 invested will give £3 interest;

therefore ...... £1.......................... $\frac{3}{73}$ interest;

therefore £S.......................... $\frac{3}{73}S$ interest;

and the interest, or £$\frac{3}{73}S$........................ $\frac{3}{73} \times \frac{3}{73}S$ interest.

So that at the end of 2 years there was in hand the first investment and its 2 years' interest, or $S + 2 \times \frac{3}{73}S$; there was the interest on the first year's interest which had been invested, or $\frac{9}{73^2}S$; there was a second investment of S and one year's interest on it, or $S + \frac{3}{73}S$. Altogether there was $2S + \frac{9}{73}S + \frac{9}{73^2}S$ to meet the debt of 1045; therefore

$$\frac{11324}{5329}S = 1045,$$

$$S = \frac{1045}{11324} \times 5329$$

$$= \frac{55}{596} \times 5329$$

$$= 491\tfrac{459}{596}.$$

Ex. 14. *If the* 3 *per cents. be at* 95 *and the Government offer to receive tenders for a loan of* £5000000; *the lender to receive five millions in the* 3 *per cents. together with a certain sum in the* $3\frac{1}{2}$ *per cents., what sum in the* $3\frac{1}{2}$ *per cents. ought the lender to accept?* (S.-H., Jan. 4, 1853).

First determine what is the money value of five millions, 3 per cent. stock, which the lender is to take.

Now 100 : 5000000 :: 95 : *Ans.*,

$$Ans. = 95 \times 50000$$

$$= 4750000.$$

The lender will therefore still want stock to represent a money value of 250000; and he is to take it in $3\frac{1}{2}$ per cents.

So if the 3 per cents. are at 95, we must find the price of the $3\frac{1}{4}$ per cents. at the same rate of interest.

$$3 : 3\tfrac{1}{4} :: 95 : Ans.,$$

$$3 \times Ans. = \frac{13}{4} \times 95,$$

$$Ans. = \frac{13}{4} \times 95 \times \frac{1}{3}$$

$$= \frac{1235}{12}$$

$$= 102\tfrac{11}{12}$$

The question therefore is, how much stock, at $102\frac{11}{12}$, is an equivalent for £250000 cash?

$$100 : Ans. :: 102\tfrac{11}{12} : 250000,$$

$$Ans. \times \frac{1235}{12} = 25000000,$$

$$Ans. = 25000000 \times \frac{12}{1235}$$

$$= 24291\tfrac{123}{247} \text{ stock,}$$

$$\text{or} \quad 24291\tfrac{1}{2} \text{ nearly.}$$

§ 107. From these examples it will be seen that all government securities may be looked upon as stock, divided, for the convenience of transfer, into shares of £100 each; although if needful any such share might be subdivided, and any portion of it bought and sold. The Stock of Railway and other Companies is usually divided into Debenture, or Preference, or Guaranteed Stock, which pays (like government securities,) a *fixed* rate of interest; and ordinary or open stock, or shares, in which the rate of interest fluctuates according to the profits. Hence we generally speak of Railway "Stock;" but we may speak of "Shares" in relation to Banks, Mines, and the like; since these, being seldom paid up in full, cannot be consolidated into Stock.

Hence any question about Railway Stock, or Mining Shares, would properly fall under the rule we are considering.

And it is upon the principles which have been illustrated in the foregoing examples that all the statements made daily in the Newspapers concerning the Money Market, the Railway, Mining, and other shares, are to be explained: for instance, to

explain at length such a quotation as the following from the *Times* of November 29th, 1865:

"Consols opened this morning at a fresh decline of an eighth, and ultimately experienced a further fall. The first bargains were at $89\frac{3}{8}$ to $\frac{1}{2}$, and the last at $89\frac{1}{4}$ to $\frac{3}{8}$. For the 7th of December the final quotation was $87\frac{7}{8}$ to 88 ex dividend." This means that on the morning of the day in question a £100 share in the English funds, or consolidated debt of the Nation, was selling for 2*s*. 6*d*. or $\frac{1}{8}$ of a pound *less* than the day before. That the first actual sales effected were at prices ranging from $89\frac{3}{8}$ to $89\frac{4}{8}$, (*i.e.* $89\frac{1}{2}$); but before the business of the day closed, the price fell again to $89\frac{2}{8}$ (or $89\frac{1}{4}$), or ranged from that to $89\frac{3}{8}$. Next, as of course shortly *before* a dividend is paid, a share would be *more* valuable, and immediately after one has been paid, would be *less* valuable, the market price of a share for the 7th of December, *i.e.* what a person would give now for a share to be transferred to him on that day, was quoted at $87\frac{7}{8}$, or in some cases at 88; a share purchased for that date being "ex dividend," or not entitling the purchaser to receive the half year's dividend of £1 „ 10*s*. payable on 5th January, 1866.

Again, in the same article we read, "Bank Stock left off at 248 to 250; India 5 per cents 105 to $\frac{1}{4}$; Exchequer bills, March, 6*s*. to 2*s*. discount." This means that Bank of England Stock was looked on as so good a security and paid so high a rate of interest, that a share was selling for from £248 to £250; and that a £100 share of the debt of the Indian Government, which paid 5 per cent., was selling for from £105 to £$105\frac{1}{4}$. Exchequer bills are bills issued under the authority of Parliament for sums varying from £100 to £1000; and they form the principal part of the unfunded or floating debt of the country. They bear interest at so much *per diem* for each £100. In the war at the commencement of the present century the interest was $3\frac{1}{2}$*d*. per cent. per diem, which was £5 „ 6*s*. „ $5\frac{1}{2}$*d*. per cent. per annum. The rate of interest was afterwards reduced to 2*d*. per diem, or £3 „ 0*s*. „ 10*d*. per cent. per annum. Besides the fact of bearing interest, they are convenient to purchasers because they pass from hand to hand without any formal transfer, and because option is periodically given to the holders to be paid their amount at par, or to exchange them for new bills to which the same advantage is extended. When they bear in the market a price *above* their actual value, say 9*s*. per

cent., they are said to be at 9*s.* *premium;* when they bear a price in the market *below* their actual value, they are said to be at so much *discount;* as in the passage quoted, Exchequer bills due in March, were selling at from 6*s.* to 2*s.* *less* per cent. than the sum which the Government was pledged to pay; *i.e.* for a bill of £100 the sum paid in the market would be from £99 „ 14*s.* to £99 „ 18*s.*

The quotations made in the Newspapers concerning shares in Railways and Mines are all to be explained on the foregoing principles.

Ex. 15. *A person has* 500 *ordinary Stock in the Great Eastern Railway, and* 800 *Preference Stock in the South Western: the first of which paid a dividend at the rate of* $1\frac{3}{4}$, *and the latter of* 4 *per cent. Having sold these at* 48 *and* 95 *respectively, he invested half the money in the Cape Copper Mine, where the* £24 *share, paying interest at* 6 *per cent., was at* £6 *premium; and the other half in a Joint Stock Bank; what rate of interest ought he to receive from the Bank, in order to increase his income by* £9 „ 10*s.* *yearly?*

His original income from 500 Stock paying $1\frac{3}{4}$ per cent., and 800 Stock paying 4 per cent., was £40 „ 15*s.*

By selling the 500 Stock at 48, and the 800 Stock at 95, he realized £1000. Of this he invests £500 in the Cape Copper Mine, being entitled, at 6 per cent., to receive £$1\frac{11}{25}$ on each £24 share, but giving £30 for every such share. Hence

$$£30 : £500 :: 1\tfrac{11}{25} : Ans.,$$

$$30 \times Ans. = 500 \times \frac{36}{25},$$

$$Ans. = 20 \times 36 \times \frac{1}{30}$$

$$= 24, \text{ income.}$$

He has now £500 to invest in the Joint Stock Bank; and from the interest derived from that investment, together with the £24 produced by the copper mine, he is to make up his income from £40 „ 15*s.* to £50 „ 5*s.* He must therefore obtain £26 „ 5*s.* from the investment in the Bank; what is the rate per cent.?

$$500 : 100 :: 26\tfrac{1}{4} : Ans.,$$

$$5 \times Ans. = \frac{105}{4},$$

$$Ans. = \frac{21}{4}$$

$$= 5\tfrac{1}{4} \text{ per cent.}$$

Exercise XX.

1. How much stock at 93 can be bought for £1581 ?

2. By the sale of 1600 stock at $88\frac{1}{8}$, what money was produced ?

3. By investing £1000 in the 3 per cents. at $92\frac{3}{8}$, what annual income is produced ?

4. A person invested £1500 in the 3 per cent. stock at $88\frac{3}{8}$; what was the amount of his half-yearly dividends ?

5. When the 3 per cents. are at 75, what amount must be invested to produce an income of £120 per annum ?

6. At what rate per cent. will a person receive interest, who invests his capital in the 3 per cents. at 91 ?

7. A person transfers £3000 from the 4 per cents. at 90 to the 3 per cents. at 72 ; find how much of the latter stock he will hold, and the alteration made in his income.

8. If a person be left a third share in a legacy of £3195, and invest his share in the 3 per cents. at $88\frac{3}{4}$, what would be the amount of his dividend each half-year ?

9. *A* invests £1000 in the 3 per cents. at 84, *B* the same sum in the 4 per cents. at 110 ; find the difference between their respective incomes.

10. How much money must be invested in the 3 per cents. at 84 to produce an annual income of £150 ?

11. How much must a person invest in the 3 per cents. at $90\frac{3}{4}$, in order to secure a half-yearly dividend of £50 ?

12. What is the real rate of interest obtained by investing in the 3 per cents. at 93 ?

13. How much stock in the 3 per cents. must be bought when the funds are at $88\frac{1}{2}$, in order that by selling out when they are at $90\frac{1}{4}$, twenty guineas may be gained ?

14. What is the actual value of a bequest of £2000 in the 3 per cents., if sold out when the funds are at $92\frac{1}{8}$, supposing

that a legacy duty of 10 per cent. has to be paid on the money realised?

15. When the 3 per cents. are at $90\frac{5}{8}$, and the $3\frac{1}{2}$ per cents. at $97\frac{7}{8}$, in which may capital be invested to the greater advantage?

16. The 3 per cent. stock is at $98\frac{3}{8}$; what then ought to be the price given for the $3\frac{1}{2}$ per cents., so as to produce exactly the same rate of interest?

17. What income can be derived from the sum of £1000, by investing it in the 3 per cent. consols, when the price of £100 stock is $88\frac{1}{2}$?

18. If £3714 „ 19*s*. be laid out in purchasing 3 per cent. stock at $95\frac{1}{2}$, what annnal income would be derived from this? and would it be more or less advantageous to invest in the $3\frac{1}{4}$ per cent. at $97\frac{1}{4}$?

19. If a man invested £1000 in the 3 per cent. stock at 90, and sold out when it rose to £100, and then invested the sum he received in $3\frac{1}{4}$ per cent. stock at 105; find the income he will receive at last.

20. A person sells out £1250 stock of 3 per cent. consols when the funds are at 96, and invests the proceeds in Railway stock at 75, paying an annual dividend of $2\frac{1}{2}$ per cent.; what is his increase of income?

21. A Banker invests money in the 3 per cents. when they are at $93\frac{5}{8}$, and at the end of 5 months, after receiving half-a-year's dividend, sells out at $94\frac{1}{8}$; how much per cent. per annum does he make by the transaction?

22. A person finds that if he invests a certain sum in the shares of a Mine, paying a dividend of £6 per share, when the £100 share is at £132, he will obtain £10 „ 16*s*. a year more for his money than if he invest in 3 per cent. consols at 93. What sum had he to invest?

23. A person having £10000 in the 3 per cents., sells out when they are at 65, and invests the produce in the 4 per cents. at $82\frac{1}{2}$. Find the change in his income.

24. How much money must a man invest in the 3 per cent. consols when they are 10 per cent. below par, that he may enjoy a dividend of a thousand a year?

25. If I buy £1000 stock in the new Italian loan at $84\frac{3}{4}$, and sell out when the price has fallen to $78\frac{7}{8}$, how much do I lose by the transaction?

26. Calculate the difference in income produced by the investment of £1580 in the 3 per cents. at $87\frac{1}{2}$, and in the $3\frac{1}{2}$ per cents. at 98.

27. How much stock at $99\frac{3}{4}$ can be purchased by selling out £1400 of a different stock which is at 95?

28. A person having £5600 in the $3\frac{1}{4}$ per cents., sells out at 93, and invests the proceeds in a stock which pays 5 per cent. and is at $110\frac{1}{2}$; required the alteration in his income.

29. A person investing in the $3\frac{1}{4}$ per cents. pays $\frac{1}{8}$ for brokerage and obtains 4 per cent. on his money. At what price does he buy in?

30. On a certain quantity of stock the unclaimed dividends amounted in 7 years to £616: if $2\frac{3}{4}$ were the rate of interest which the stock paid, and if when claimed the stock were sold at $81\frac{5}{8}$; find what sum it realised.

31. A person invests £4470 in the 3 per cent. consols at 93; what amount of stock does he receive, allowing for commission 2*s*. „ 6*d*. per cent. on the stock purchased? And what income will be derived from the investment, after deducting an income-tax of 16 pence in the pound?

32. How much would a person increase or diminish his income by selling £1157 stock in the 3 per cents. at $83\frac{1}{4}$, and purchasing into the $3\frac{1}{4}$ per cents. at $90\frac{3}{10}$?

33. A person invests £1500 in the 3 per cents. when they are at $96\frac{1}{4}$, what is his annual income therefrom? If he sell out at 94, what will be his loss, the broker's commission being in each transaction 10*s*. per cent.?

34. What sum must a person invest in the 3 per cents. at 90, in order that by selling out £1000 stock, when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to $84\frac{1}{4}$, he may gain £6 „ 5*s*. by the transaction? If he invest the produce in 4 per cents. at par, what will be the difference in his income?

35. A person sells out of the 3 per cent. consols at $91\frac{1}{8}$, and buys in again when they have fallen $2\frac{1}{2}$ per cent. What difference will this make in his income, if he now possesses £800 stock?

36. A person buys 800 stock in the 3 per cents. at 85, and 500 more at 97; how much per cent. will he realise on his outlay, after paying an income-tax at 4*d*. in the pound?

37. If the 3 per cents. give 3 per cent. clear, after paying an income-tax at 9*d.* in the pound, what must be the price of the 3 per cents.?

38. Shew that the rate of interest obtained by investing in the Dutch $2\frac{1}{2}$ per cents. at $87\frac{1}{2}$ is to that obtained by investing in the Russian $3\frac{1}{2}$ per cents. at $94\frac{1}{2}$, in the ratio of 18 : 25.

39. How much stock at 88 must be sold out, in order to pay immediately a bill of £913, due 9 months hence, allowing 5 per cent. simple interest?

40. One company guarantees to pay 5 per cent. on shares of £100 each; another guarantees at the rate of $4\frac{5}{8}$ per cent. on shares of £7 „ 10*s.* each; the price of the former is £124 „ 10*s.*, and of the latter £8 „ 10*s.*; compare the rates of interest which the shares return to purchasers.

41. When the French 3 per cents. are at 69 francs „ 45 centimes, and the English 3 per cents. are at $87\frac{3}{8}$, compare the rates of interest obtained by investments made in France and in England.

42. How much stock in the French 3 per cents will £351 „ 10*s.* purchase, £1 being reckoned equivalent to 25·2 francs, and the French stock being at 70·3?

CHAPTER XVII.

EXCHANGE.

§ 108. By the rule of Exchange we are to find what amount of the money of one country will pay a debt in the money of another country.

International trade is carried on by exporting from one country the articles produced in it, and importing from other countries the articles of commerce produced by them; but in order to facilitate the transmission of money to pay for the goods imported, Merchants have devised a scheme of drawing upon one another by *bills of exchange;* which may be ex-

plained to be written orders, addressed by one person to another, directing the latter to pay on account of the former to some third person a certain sum of money at a specified time. These bills are negotiable, and are transferred from hand to hand.

Now as different countries make use of different coins, containing different weights of pure metal, and consequently differing in value, the first thing the merchant would require to know would be the actual amount of pure gold and pure silver contained in the several coins of the various countries with which he traded; he then could reckon how many of the coins of any foreign country would be an exact equivalent for a certain number of the coins of his own country. If for instance it be known that the English sovereign is exactly equivalent to 4·87 American gold dollars, or, what is the same thing, that 100 sovereigns are equivalent to 487 gold dollars, this establishes what is called the *par of exchange* between England and America. But if one country makes use of *gold* for its standard, as England does, and another country makes use of *silver*, it is impossible to fix an *invariable* par; because as the market price of silver varies, the value of foreign silver coins fluctuates, while the silver coin of England will possess a conventional value, independent of the market price of silver. Notwithstanding this, it is usual among Merchants to assume a *par of exchange* as actually existing between their own and each of the countries with which they trade; and this is arrived at by taking the *average* value of the currency of these various countries.

Whenever one country takes an excess of imports over exports, the balance of trade is *against* that country; and as it can only pay for part of the goods taken by goods exported, it must pay for the remaining part by bills of exchange obtained from some other country, and for which a premium must be paid. Thus an excess of importation causes exchange to advance against the importing country. When this occurs, the *real* exchange, or the *course of exchange*, as it is called, rises *above par*. There is however a limit beyond which this rise cannot advance; for as a debt to a foreign country can be liquidated by the transmission of bullion as well as by a bill of exchange, whenever the exportation of bullion becomes the cheaper method, the demand for bills of exchange will cease.

From this it will be seen that while by the *par of exchange* we know what sum of money of one country is *actually* an equivalent to a given amount of the money of another country, by the *course of exchange* we find what sum would, in point of fact, be allowed for it at the current market price.

After a bill falls due, it is customary to allow a short time for the requisite sum to be provided, and a certain number of *days of grace* are granted; thus, in England a period of three days is allowed to elapse after a bill is actually due, before it is legally due.

§ 109. In the following examples, the method of performing operations by the rule of exchange will be explained:

Ex. 1. *If the course of exchange between Paris and London be at* 25·5 *francs per pound sterling, what is the value in British money of a debt of* 2703 *francs,* 51 *centimes?*

$$25{\cdot}5 : 2703{\cdot}51 :: \overset{\pounds}{1} : \overset{\pounds}{Ans.},$$

$$Ans. \times 25{\cdot}5 = 2703{\cdot}51,$$

$$Ans. = 106{\cdot}02$$

$$= \pounds 106 \text{ ,, } 0s. \text{ ,, } 4\tfrac{4}{5}d.$$

Ex. 2. *Find the value in Portuguese money of £226 ,, 2s., exchange being 5s. ,, 9d. English per milree.*

[*In Portugal* 1000 *rees* = 1 *milree.*]

$$(5\tfrac{3}{4} \div 20) : 226\tfrac{1}{10} :: 1 : Ans.,$$

$$\frac{23}{80} \times Ans. = \frac{2261}{10},$$

$$Ans. = \frac{2261}{10} \times \frac{80}{23}$$

$$= \frac{18088}{23}$$

$$= 786{\cdot}4347, \text{ \&c.}$$

$$= 786 \text{ milrees ,, } 434{\cdot}7, \text{ \&c. rees.}$$

§ 110. Sometimes the course of exchange may be such, that it is more advantageous to the Merchant to draw not directly, but indirectly through one or more intermediate places. The following questions will illustrate the nature of such transactions.

Ex. 3. *When the course of exchange between London and Paris is $9\frac{1}{2}d.$ per franc, and 3·63 francs are equivalent to 1 Prussian thaler, and 24·5 thalers to 34 Austrian florins, and 25 Austrian florins to 12·6 Venetian ducats,—if a London Merchant owe to one in Venice 1000 ducats, will it be more advantageous to remit by way of Paris, Berlin, and Vienna, or direct to Venice, supposing a ducat to be equivalent to 4s. ,, 2d.?*

$$4s.\ ,,\ 2d. = 50 \text{ pence.}$$

Therefore if he remitted directly, he would remit 50000 pence.

But remitting circuitously through Paris, Berlin, and Vienna, we have the following proportions: writing for the unknown number of Austrian florins AF, of Prussian thalers PT, of francs Fr, and x for the required number of pence.

Since 12·6 ducats $= 25\ AF$;

therefore $\quad 1 \text{ ducat} = \frac{25}{12 \cdot 6} AF.$

Similarly, since

$$34\ AF = 24 \cdot 5\ PT;$$

therefore $\quad 1\ AF = \frac{24 \cdot 5}{34} PT.$

Also $\quad 1\ PT = 3 \cdot 63\ Fr,$

and $\quad 1\ Fr = 9 \cdot 5$ pence.

Now as $1 \text{ ducat} = \frac{25}{12 \cdot 6} AF,$

$$\begin{aligned}
1000 \text{ ducats} &= \frac{1000 \times 25}{12 \cdot 6} AF \\
&= \frac{1000 \times 25 \times 24 \cdot 5}{12 \cdot 6 \times 34} PT \\
&= \frac{1000 \times 25 \times 24 \cdot 5 \times 3 \cdot 63}{12 \cdot 6 \times 34} Fr \\
&= \frac{1000 \times 25 \times 24 \cdot 5 \times 3 \cdot 63 \times 9 \cdot 5}{12 \cdot 6 \times 34} \text{ pence} \\
&= \frac{1000 \times 25 \times 24 \cdot 5 \times 1 \cdot 21 \times 9 \cdot 5}{4 \cdot 2 \times 34} \text{ pence} \\
&= \frac{7040687 \cdot 5}{142 \cdot 8} \text{ pence} \\
&= 49304 \cdot 5, \text{ \&c. pence.}
\end{aligned}$$

Hence x, the required number of pence, is the difference between 50000 pence and 49304·5 pence, *i.e.* $x = 695{\cdot}5$ pence, or £2 „ 17*s.* „ $11\frac{1}{2}$*d.*, which is the sum that he gains by remitting circuitously.

Ex. 4. "*Twenty braccia of Brescia are equal to* 26 *braccia of Mantua, and* 28 *of Mantua to* 30 *of Rimini; what number of braccia of Brescia correspond to* 39 *of Rimini?*"

Writing B for the braccia of Brescia,
M .... Mantua,
R .... Rimini,

we have $30\,R = 28\,M$;

therefore $$1\,R = \frac{28}{30}\,M.$$

Also $26\,M = 20\,B$;

therefore $$1\,M = \frac{20}{26}\,B.$$

Now since $$1\,R = \frac{28}{30}\,M;$$

therefore
$$39R = \frac{39 \times 28}{30}\,M$$
$$= \frac{39 \times 28 \times 20}{30 \times 26}\,B$$
$$= \frac{3 \times 28 \times 2}{3 \times 2}\,B$$
$$= 28\,B,$$

or, the required number of braccia of Brescia is 28.

Ex. 5. *If the par of exchange be* 4*s.* „ 2*d. English for the American dollar, but if an American bill of exchange for* 540 *dollars be negotiated in London for* £104; *how much per cent. is the course of exchange below the par of exchange?*

At par 540 dollars would be equivalent to $540 \times 4\frac{1}{6}$ shillings, or to 2250 shillings, or to £$112\frac{1}{2}$.

But at the current course of exchange only £104 was paid; hence

$$104 : 112{\cdot}5 :: x : 100,$$

$$112{\cdot}5 \times x = 10400,$$

$$x = 92{\cdot}\dot{4},$$

and $$100 - 92{\cdot}\dot{4} = 7{\cdot}\dot{5} = 7\tfrac{5}{9} \text{ per cent.}$$

§ 111. In some countries where the coin in circulation is much clipped, Merchants transact their dealings with other nations and keep their bank accounts in what they call *banco*, which may be defined to be *money as it ought to be;* and the difference between banco, or money as it ought to be, and the current coinage, or *money as it is*, is called *agio;* which is in fact the per centage by which the clipped coin is depreciated.

Ex. 6. *Convert* 1886 *florins* „ 5 *stivers* „ 12 *pennings current coinage into banco, agio being* $3\frac{1}{2}$ *per cent.*

[16 *pennings* = 1 *stiver*, 20 *stivers* = 1 *florin.*]

$$103{\cdot}5 : 100 :: 1886{\cdot}2875 : Ans.,$$

$$103{\cdot}5 \times Ans. = 188628{\cdot}75,$$

$$Ans. = \frac{1886287{\cdot}5}{1035}$$

$$= 1822{\cdot}5$$

$$= 1822 \text{ florins „ } 10 \text{ stivers.}$$

EXERCISE XXI.

1. How many florins, &c., will be paid in Amsterdam for a bill of £1718 „ 2*s.* received from London, when the course of exchange is 11 florins „ 10 stivers for £1 English?

2. What sum in London will be paid for a bill of 17694 francs „ 22 centimes, when the exchange is at 24 francs „ 10 centimes per pound

3. What is the course of exchange per milree between Lisbon and London, when 4536 milrees are drawn at Lisbon for an English bill of £945?

4. What is the agio when 861 florins „ 18 kreutzers currency are equivalent to 805 florins banco?

[60 kreutzers = 1 florin.]

5. "Eight *soldi* of Venice are equal to 13 of Ferrara, and 15 of Ferrara are equal to 9 of Bologna, and 12 of Bologna are equal to 16 of Pisa, and 24 of Pisa are equal to 32 of Genoa; it is required to find what number of Venetian *soldi* correspond to 300 of Genoa."

6. "Six eggs are worth 10 *danari*, and 12 *danari* are worth 4 thrushes, and 5 thrushes are worth 3 quails, and 8 quails are worth 4 pigeons, and 9 pigeons are worth 2 capons, and 6 capons are worth a *staro* of wheat; how many eggs are worth 4 *stara* of wheat?"

7. When the direct exchange between London and Lisbon is 44 pence per milree, and 340 milrees are due to an English merchant, how much would he lose or gain if, instead of being remitted directly, it was remitted as follows: from Lisbon to Venice at 96 milrees per 100 ducats; from Venice to Cadiz at 1 ducat per 320 maravedies; from Cadiz to Paris at 80 maravedies per franc; and from Paris to London 25 francs per £1 sterling?

CHAPTER XVIII.

PROFIT AND LOSS.

§ 112. The point to be considered in all questions in this rule is *not* the *actual gain* on each article sold, but the *gain which every* £100 outlaid upon such articles would bring in. For this is what a tradesman requires to know, viz. *the rate per cent.* at which he is employing his money; he can then determine whether it would be more profitable to lower the price of his goods in order to obtain larger custom and quicker returns; or whether he should raise the price of everything he sells. But in either case he will lower or raise the price in proportion to the *cost price*, and will not add the *same* sum to the price of every article he has to sell, irrespectively of what he gave for it.

It is a common mistake to suppose that in trade the *gain per cent.* means the gain made by selling *an hundred articles;* whereas, we repeat, it is the gain which £100, outlaid upon such articles, would bring in.

Again, it is a fallacy to suppose, because the actual gain on each article sold may be higher in one case than another, that this must necessarily be the most profitable trade. When

one man buys clay pipes for a farthing each, and sells them for a half-penny each, although he gains only a farthing on each pipe sold, yet he doubles the capital he employs, or gains *cent. per cent.;* whereas should another buy meerschaum pipes for a pound apiece and sell them for a guinea, he gains a shilling on each pipe; and yet as he only gains one shilling on every twenty outlaid, this is but *five per cent.*, and his trade is not nearly so good as the other man's.

The things necessary to be considered by the trader are the *prime cost* or *cost price*, that is, the sum originally given by him for the article; the *retail price*, or *selling price*, that is, the sum at which he determines to sell it; and the gain or loss *per cent.* which he will make or incur by this.

Besides these points, the trader will have in practice to consider the loss of interest on capital lying idle in the interval between the wholesale purchase, and the retail sale; this, to a trader working with borrowed capital, would be a very important consideration; and so in some cases would the *Depreciation* and the *Waste* which must be allowed for. But these are details not often introduced into the theoretical questions given; and in general it will be only necessary to remember the following proportions, viz.:

$$\textit{cost price} : \textit{retail price} :: 100 : 100 + \textit{gain per cent.},$$

or $$\textit{cost price} : \textit{retail price} :: 100 : 100 - \textit{loss per cent.},$$

for from these we shall be able to determine the term which is unknown, as will be illustrated in the following examples:

Ex. 1. *Bought an article for £15; what must I sell it at to gain 40 per cent.?*

$$\overset{\text{cost price}}{15} : \overset{\text{retail price}}{Ans.} :: 100 : 140,$$

$$Ans. \times 100 = 15 \times 140,$$

$$Ans. = \frac{15 \times 140}{100}$$

$$= £21.$$

Ex. 2. *What was the cost price of an article which when sold for £90 realised a gain of 20 per cent.?*

$$\overset{\text{cost price}}{Ans.} : \overset{\text{retail price}}{90} :: 100 : 120,$$

$$120 \times Ans. = 90 \times 100,$$

$$Ans. = \frac{90 \times 10}{12}$$
$$= £75.$$

Ex. 3. *When I sell for £25 „ 4s. goods for which I give £15 „ 15s., what is the gain per cent.?*

In this case the unknown term is not the answer, but is 100 + the gain, where the gain is the answer. We state therefore

$$15\tfrac{3}{4} : 25\tfrac{1}{5} :: 100 : x,$$
$$\frac{63}{4} \times x = \frac{126}{5} \times 100,$$
$$x = \frac{126}{5} \times 100 \times \frac{4}{63}$$
$$= 160.$$

Hence $160 - 100 = 60 =$ the required gain per cent.

We might have arrived at the same result thus: from £25 „ 4s. deduct the prime cost £15 „ 15s. The difference, £9 „ 9s., is the gain made upon the outlay of £15 „ 15s. What would £100 outlaid on the same terms bring in?

$$15\tfrac{3}{4} : 100 :: 9\tfrac{9}{20} : Ans.,$$
$$Ans. \times \frac{63}{4} = 100 \times \frac{189}{20},$$
$$Ans. = 100 \times \frac{189}{20} \times \frac{4}{63}$$
$$= 60, \text{ the gain per cent.}$$

Ex. 4. *If by selling goods at £13 „ 6s. „ 8d. per cwt. a loss of 20 per cent. was sustained; what was the prime cost?*

$$Ans. : 13\tfrac{1}{3} :: 100 : 80,$$
$$80 \times Ans. = \frac{40}{3} \times 100,$$
$$Ans. = \frac{40}{3} \times 100 \times \frac{1}{80}$$
$$= \frac{50}{3}$$
$$= 16\tfrac{2}{3};$$

therefore £16 „ 13s. „ 4d. the prime cost.

Ex. 5. *If a tradesman gain* 5*s*. „ 6*d*. *on an article which he sells for* 22*s*., *what does he gain per cent. on his outlay?*

By selling for 22*s*. he gains 5*s*. „ 6*d*.; therefore he gave 22*s*. − (5*s*. „ 6*d*.), or 16*s*. „ 6*d*.

$$16\tfrac{1}{2} : 22 :: 100 : x,$$

$$\frac{33}{2} \times x = 22 \times 100,$$

$$x = 22 \times 100 \times \frac{2}{33}$$

$$= \frac{2 \times 100 \times 2}{3}$$

$$= \frac{400}{3}$$

$$= 133\tfrac{1}{3};$$

therefore $33\frac{1}{3}$ is the gain per cent.

Ex. 6. *The cost price of a cask of wine containing* 36 *gallons was* £42; *to this a merchant added* 3 *gallons of water; at what price per gallon must he sell the mixture in order to gain* 30 *per cent.?*

He sold 39 gallons, which cost him £42, at a gain of 30 per cent.; therefore

$$42 : x :: 100 : 130,$$

$$10 \times x = 13 \times 42,$$

$$x = \frac{13 \times 21}{5},$$

and each gallon cost $\frac{1}{39}$ of this sum; therefore

$\frac{1}{39} \times \frac{13 \times 21}{5} = \frac{7}{5} = 1\frac{2}{5} =$ £1 „ 8*s*. the price per gallon.

Ex. 7. *By selling flour at the rate of* 3*s*. „ $5\frac{3}{5}$*d*. *per stone, a dealer gained* 4 *per cent. on his outlay. How much per cent. would he lose if he sold it at* 3*s*. $1\frac{1}{2}$*d*. *per stone?*

We might in this case find first the prime cost; and when we had found this (which is 3*s*. „ 4*d*.) we might then by a second statement find the loss by selling at 3*s*. „ $1\frac{1}{2}$*d*. But it will be sufficient to make one statement only, if we bear in mind the following proportion:

Price when he gains : price when he loses :: 100 + the gain per cent. : 100 − the loss per cent.

$$3s. \,„\, 5\tfrac{3}{5}d. : 3s. \,„\, 1\tfrac{1}{2}d. :: 104 : x,$$

$$3\tfrac{7}{15} \times x = 3\tfrac{1}{8} \times 104,$$

$$\frac{52}{15}x = \frac{25}{8} \times 104,$$

$$x = \frac{25}{8} \times 104 \times \frac{15}{52}$$

$$= 93\tfrac{3}{4};$$

therefore $6\frac{1}{4}$ is the loss per cent.

Ex. 8. *A merchant sells* 72 *quarters of corn at a profit of* 8 *per cent., and* 37 *quarters at a profit of* 12 *per cent.; if he had sold the whole at a uniform profit of* 10 *per cent., he would have received* £2 „ 14*s.* „ 3*d. more than he actually did; what was the price he paid for the corn?*

The *Ans*. will be the prime cost in shillings of a quarter of corn. Every £100 outlaid in purchasing the 72 quarters would bring in 108; every £100 outlaid in purchasing the 37 quarters would bring in 112. So that the sum realised by the sale was

$$72 \times Ans. \times \frac{108}{100} + 37 \times Ans. \times \frac{112}{100}.$$

Had he sold the 109 quarters at 10 per cent. profit, he would in that case have received

$$109 \times Ans. \times \frac{110}{100}.$$

But this was greater than the sum actually received by $54\frac{1}{4}s.$ Hence

$$109 \times Ans. \times \frac{110}{100} - \left(72 \times Ans. \times \frac{108}{100} + 37 \times Ans. \times \frac{112}{100}\right) = 54\tfrac{1}{4},$$

$$\left(\frac{109 \times 11}{10} - \frac{72 \times 27}{25} - \frac{37 \times 28}{25}\right) Ans. = \frac{217}{4},$$

$$\frac{7}{10} Ans. = \frac{217}{4},$$

$$Ans. = \frac{217}{4} \times \frac{10}{7}$$

$$= \frac{310}{4}$$

$$= 77s. \,„\, 6d.$$

Ex. 9. *If a grocer retails his sugar so that he charges for every* 8 *lbs. the exact sum which he paid for every* 9 *lbs., what will be his gain per cent.?*

The buying price of 1 lb. : the selling price of 1 lb. :: 8 : 9;

therefore $$8 : 9 :: 100 : x,$$
$$8x = 900,$$
$$x = 112\tfrac{1}{2};$$

therefore $12\frac{1}{2}$ is the gain per cent.

Ex. 10. *A merchant buys tea at* 2*s.* „ 3*d. a lb, and some at* 3*s.* „ 6*d. a lb.; in what proportion must he mix them, so that by selling the mixture at* 4*s. a lb. he may gain* 20 *per cent.?*

As he gains 20 per cent. by selling a lb. of the mixture for 4*s.*, suppose x to be the prime cost of a lb. of the mixture; then
$$x : 4 :: 100 : 120,$$
$$12x = 40,$$
$$x = 3\tfrac{1}{3} = 3s. \text{ „ } 4d.$$

Now on every lb. of the 2*s.* „ 3*d.* tea that is raised in value by mixing to 3*s.* „ 4*d.* he gains 13*d.*

And on every lb. of the 3*s.* „ 6*d.* that is depreciated in value by mixing to 3*s.* „ 4*d.* he loses 2*d.*;

Therefore on 2 lbs. of the cheaper tea his gain is equivalent to his loss on 13 lbs. of the dearer tea.

Or he must mix them in the ratio of 2 : 13.

Ex. 11. *By selling* 4 *dozen cigars for* 13*s. it was found that* $\frac{3}{10}$ *ths of the money outlaid was gained; what ought the retail price per cigar to have been, in order to have gained* 60 *per cent.?*

First, we can find the prime cost of the 48 cigars, which, when sold for 13*s.*, brought in the prime cost *plus* $\frac{3}{10}$ of the prime cost, *i.e.* brought in $\frac{13}{10}$ of the prime cost; for if
$$\frac{13}{10} \text{ of prime cost} = 13s.,$$
$$\text{prime cost} = 10s.$$

Next, $10s. : x :: 100 : 160,$

$$100 \times x = 10 \times 160,$$

$$x = 16s.;$$

therefore price per cigar is $\frac{17 \times 12}{48}$ or $4d.$

Exercise XXII.

1. Bought articles for 15*s.* each; what must I sell them at to gain 60 per cent.?

2. What was the retail price of an article which cost 10*s.*, and when sold realised a profit of 10 per cent.?

3. Bought a horse for £72, and sold it for £84; what was the gain per cent.?

4. I bought a pipe for 17*s.* 6*d.*, coloured it, and sold it to a friend for 28*s.*; he tired of it and sold it again for a guinea; how much per cent. did I gain and he lose?

5. If a grocer buys tea at 4*s.* per lb., and sells it at 4*s.* „ 8*d.*, what is his gain per cent.?

6. If 25 yards of butter cost 30*s.*, what is the gain per cent. by retailing it at $5\frac{1}{2}d.$ per foot?

7. A grocer mixes equal quantities of teas which cost him 3*s.* „ 8*d.* and 4*s.* „ 4*d.* respectively; what must be the selling price of the mixture, in order that he may gain 15 per cent. on his outlay?

8. A tradesman finds that if he asks for his goods 15 per cent. above the prime cost, he can sell his whole stock in 4 months; whereas, if he asks 20 per cent., he requires 6 months to sell the same amount; which will he find the more profitable system at the year's end?

9. A merchant has teas worth 5*s.* and 3*s.* „ 6*d.* per lb. respectively, which he mixes in the proportion of 2 lbs. of the latter to one of the former; how much per cent. will he gain or lose by selling the mixture at 4*s.* „ 6*d.* per lb.?

10. If a 38-gallon cask of wine cost a merchant £25, and he loses 8 gallons by leakage, how must he sell the remainder per gallon in order to gain 10 per cent. upon his outlay?

11. Bought 2 tons „ 3 cwt. „ 3 qrs. of sugar for £95; freight and other expenses amounted to £4 „ 3*s.* „ 4*d.*; what must be the retail price per lb. to gain 50 per cent.?

12. If by selling tracts at 7*s*. per thousand, $\frac{2}{5}$ths of the money outlaid in purchasing them was cleared, find, when afterwards the price was raised to 8*s*. „ 6*d*. per thousand, what was the gain per cent. at the increased price.

13. A grocer buys some tea at 4*s*. a pound, and some at 5*s*. „ 6*d*.; in what proportion must he mix them, that when he sells the tea at 6*s*. per pound he may be making a profit of 20 per cent.?

14. A corn factor buys 2 quarters of wheat at 49*s*. per quarter, and 7 bushels at 7*s*. per bushel; at what rate per bushel must he sell the mixture so as to gain 5 per cent. by the transaction?

15. A person buys some tea at 6*s*. per lb., and also some at 4*s*. per lb.; in what proportion must he mix them so that by selling the mixture at 5*s*. „ 3*d*. per lb., he may be gaining at the rate of 20 per cent.?

16. A merchant buys 1260 quarters of corn; one-fifth of which he sells at a gain of 5 per cent., one-third at a gain of 8 per cent., and the remainder at a gain of 12 per cent. If he had sold the whole at a gain of 10 per cent., he would have obtained £22 „ 13*s*. more; what was the prime cost per quarter? (S.-H., Jan. 1856).

17. If 100 lbs. of tea be bought for 4*s*. „ 4*d*. a pound and sold at 5*s*., and 100 lbs. of sugar be bought at 6*d*. and sold at 7*d*., what profit per cent. will be realized on the outlay? (S.-H., Jan. 6, 1852).

18. A wine merchant buys 12 dozen port at 84*s*. per dozen, and 60 dozen more at 48*s*. per dozen; he mixes them and sells at 72*s*. per dozen; what profit per cent. does he realize on his outlay? (S.-H., Jan. 3, 1860).

19. A tradesman bought rice at £2 „ 2*s*. „ 6*d*. per cwt., and finding it damaged sold it at a loss of 7 per cent.; what did he sell it at per lb.?

20. An article is sold for £1 „ 8*s*. „ 10¾*d*. at a loss of 5 per cent.; at what price should it have been sold to gain 5 per cent.?

21. If by selling sugar at 6*d*. per lb. 10 per cent. be gained; what would be the gain or loss per cent. by selling it at 5½*d*.?

22. If a corn dealer by selling 52 quarters of oats for £69 „ 6s. „ 8d. lose 10 per cent., what ought to have been the price per bushel in order to have gained 8 per cent.?

23. A merchant bought wines at 30s., 40s., and 50s. per dozen; these he mixes in the ratio of 5, 4, 3; and sold the mixture at 57s. „ 6d. per dozen; what did he gain per cent.?

24. If wine which in Germany cost a thaler (3s.) the bottle, after paying a duty that is 40 per cent. on its prime cost be sold in England for 72s. the dozen, what is the merchant's gain per cent.

25. If the price of goods be 35 florins „ 5 cents „ 4 mils per cwt., and they be retailed at $8\frac{1}{2}d.$ per lb., find (in pounds) the gain per cent.

26. Bought 1000 cigars abroad for £3 „ 15s.; paid an ad valorem duty of 120 per cent.; what must be the retail price per cigar to clear a profit of $48\frac{1}{3}\frac{8}{3}$ per cent. on the entire outlay?

27. A man having bought a lot of goods for £150, sells one-third at a loss of 4 per cent.; if he then sold the remainder so as not only to cover his loss, but also to clear £6 on the whole transaction, what was the gain per cent. on the money originally outlaid?

28. A person buys 400 yards of silk for £80, and sells 300 yards at 5s. „ 6d. a yard, and the rest, which is damaged, at 2s. a yard; find how much per cent. he gains or loses.

29. A person sells a piano at a loss of 4 per cent. on the cost price; had he sold it for £4 „ 10s. more he would have gained 5 per cent.; what was the prime cost of the piano?

30. If a person by selling an article for 8s. „ 3d. lose $17\frac{1}{2}$ per cent., what should he have sold it for, to gain 40 per cent.?

31. A merchant sells 49 quarters of corn at a profit of 7 per cent., and 84 quarters at a profit of 11 per cent.; and if he had sold it all at a profit of 9 per cent., he would have received £2 „ 10s. „ 9d. less than he actually did; what was the price he paid for the corn?

32. A person sold 72 yards of cloth for £8 „ 14s.; his profit being the cost of 11·52 yards, how much did he gain per cent.?

33. A market-woman in the morning sells her butter at

15 per cent. profit: in the afternoon the price rises a penny per lb., and she makes 20 per cent. profit; what did her butter cost her?

34. If a costermonger sells his cabbages so as to get for four what he paid for five, what is his gain per cent.?

35. Supposing a tradesman wishes to make 20 per cent. profit on his outlay, how many lbs. of tea must he have bought for the price which he charges for 5 lbs.?

36. The cost of the labour on a farm in a certain year is £610: the rent and the other expenses amount to £950, and in that year the return is only just equal to the expenditure: find the amount which must be paid for labour in the next year in order that if the return be better in the ratio of 5 : 3, and the rent be lowered £50, the farmer may gain $38\frac{2}{3}$ per cent. on his whole outlay for the two years.

37. If 7lbs. of tea bought with 6 months' credit cost as much as 8lbs. paid for with ready money, and if, when no credit was given, the tradesman was gaining $21\frac{3}{7}$ per cent., at what rate per cent. above the prime cost was he charging when he gave credit?

38. A tradesman marks his goods with two prices, one for ready money, the other for one year's credit, allowing, as he says, discount. Now if the credit price of a certain article be £13, and the cash price £12 „ 10*s.*, and if the tradesman has marked the credit price at 24 per cent. above what he gave for it, what is his gain per cent. at the cash price?

CHAPTER XIX.

DUODECIMALS, OR CROSS MULTIPLICATION.

§ 113. Cross Multiplication is the method of computing how many superficial or square feet there may be in any surface; or how many solid or cubic feet there may be in any solid body.

Here each foot is divided into 12 equal parts called *primes*, each prime into 12 equal parts called *seconds*, each second into 12 equal parts called *thirds*, and so on. And it must be remembered that a *prime* is the twelfth part of a foot, whether the foot be linear, square, or cubic: in *linear* measure, since

an inch is $\frac{1}{12}$ of a foot, a *prime* and an *inch* will mean the same thing: but in *square* measure an inch is $\frac{1}{144}$ of a square foot, and in *cubic* measure an inch is $\frac{1}{1728}$ of a cubic foot: and therefore a *prime* (which is *always* $\frac{1}{12}$ of a foot) will be a very different thing from an inch in *square* and *cubic* measure.

[*Obs.* In the actual use of duodecimals the Builder never distinguishes between "primes," "seconds," &c. All his quantities are entered as "feet," "inches," and "parts," whether they be linear, superficial, or cubic. And though he knows that this is not strictly correct, he adopts it as convenient; for he understands by the word "inch," as he uses it, only the twelfth part of the integral dimension, and by "part," the 144th part. Besides, in squaring up he never goes beyond the "part," or 144th of the foot; as it would be needless to be more exact than this; indeed the "part" is usually thrown out, except in the addition of a large number of small items in such expensive work as gilding, or the like.]

§ 114. When any line is divided into feet, primes, seconds, &c. we observe that *all* the denominations are connected by the same number, *viz.* 12; or, that they decrease in a *twelve-fold* ratio, from the place of feet towards the right hand.

Hence the process is often called duodecimal multiplication; but this name cannot be properly applied to it, because the different digits of the various denominations are not connected with each other by the number 12, though the denominations themselves are.

Thus the system is often found to be confusing; and it is further obscured in practice by the incorrect names given, as above mentioned, to the denominations of "inches" and "parts." Indeed it may be questioned whether "cross multiplication" ought not to be banished from treatises which profess to treat Arithmetic on scientific principles, and regular "duodecimal multiplication" be introduced in its place. However, as the ordinary process is that almost universally adopted in the computations of artificers, it may be the most practical course first to explain the principles upon which it depends; and afterwards to investigate the more scientific method. We will accordingly proceed to explain the process

commonly employed to calculate superficial areas, and solid contents.

§ 115. A *square yard*, a *square foot*, a *square inch*, mean respectively *a square each side of which measures a yard, a foot*, or *an inch*. Similarly a *cubic* yard, foot, or inch is a solid contained by six equal squares, each side of which measures a yard, foot, or inch.

§ 116. We can compute the number of square feet, &c. contained in any rectangular parallelogram, if we multiply the feet, inches, &c. in one of its sides by the feet, inches, &c. in the adjacent side. Similarly the content of a solid is obtained by multiplying together its length, breadth, and thickness.

Although the rule for finding the area of a rectangle is commonly given in some such short and compendious form as this, it is nevertheless important to observe that, when so stated, it is in a very abbreviated form; we should be more correct if we said "multiply together the number of linear units in the two sides; the result is the number of square units (that is, squares *on* the linear unit) in the rectangle." When without any explanation it is nakedly stated that "feet into feet give square feet," or, what is the same thing, that "the product of the adjacent sides of a rectangle is the area," the misapprehension is very often produced on the learner's mind that it is possible for two *lines* to be multiplied together, and that a *rectangular figure* is the product; as well might he imagine that it would be possible to multiply together ten shillings and seven yards of silk, and obtain as a product seventy shillings. It is true that, at ten shillings a yard, seven yards of silk would cost as many shillings as there are units in 10×7; and a rectangle, whose sides are ten and seven feet, contains as many square feet as there are units in 10×7; but ten feet can no more be multiplied by seven feet than ten shillings by seven yards.

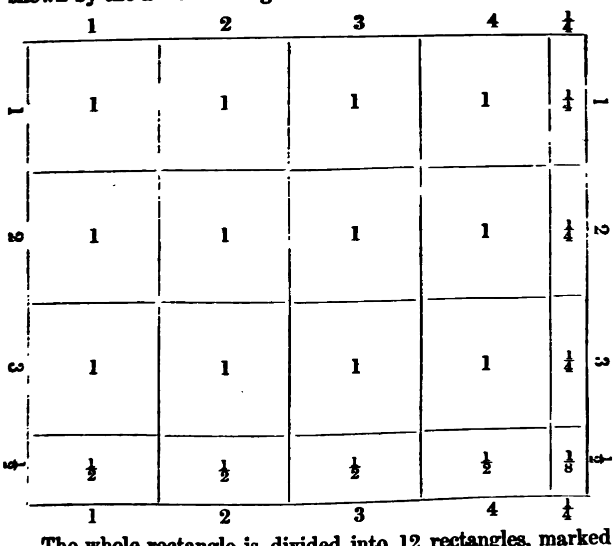
A good deal of this confusion arises from the two-fold sense in which the word *square* is used; in Geometry, a square is a four-sided figure having all its sides equal and all its angles right angles; in Arithmetic, it signifies the number produced by multiplying a number by itself. A square therefore described upon a line 5 inches long will be a rectangle 5 inches in length and 5 inches in width; and if this contain 25 square inches, the operation of multiplying 5 into 5 will be the

Arithmetic of finding the content of a square 5 inches long and 5 inches wide.

We may now go on to explain how the area of *any* rectangle may be obtained if we know the length of its two sides. If as the superficial unit we choose a square whose side is equal to the unit used in measuring length (say an inch), we may immediately tell how many times and parts of a time the given rectangle contains the assumed unit. Measure one side and see how often it contains the unit of length (the inch); suppose it contains it $3\frac{1}{2}$ times; measure the other side, and suppose it contains it $4\frac{1}{4}$ times; then the product of these two, viz.

$$\frac{7}{2}\times\frac{17}{4}, \text{ or } \frac{119}{8}, \text{ or } 14\tfrac{7}{8},$$

is *the number of times which the rectangle to be measured contains the square assumed as the unit.* This is what is meant when we say that the area of any rectangle may be found by multiplying together the units of length in two of its adjacent sides; and the correctness of the result may be shown by the annexed diagram:



The whole rectangle is divided into 12 rectangles, marked each equal to the unit of superficial measurement, *i.e.* each

of which is a square inch; four rectangles at the bottom marked $\frac{1}{2}$, each of which is *half* of a square inch; three rectangles at the side, marked $\frac{1}{4}$, each of which is a *quarter* of a square inch; and one rectangle, marked $\frac{1}{8}$, which is *one-eighth* of a square inch; hence on the whole we have, in the large rectangle, the square inch repeated

$$12+\frac{4}{2}+\frac{3}{4}+\frac{1}{8} \text{ times, or } 14\tfrac{7}{8} \text{ times.}$$

After this explanation we may adopt the practical rule for measuring any surface, and multiplying the two sides of any rectangle together may call the product the area. But it would help us to avoid misconceptions if while we spoke of the square *of* a number, we always spoke of the square *on* a line.

The observations regarding square measure apply, mutatis mutandis, to cubic measure.

§ 117. Having premised thus much, when practically applying the rule of cross-multiplication, remember that

feet × feet give square feet; *e.g.*

$$2 \text{ feet} \times 2 \text{ feet} = 4 \text{ square feet},$$

feet × primes give primes; *e.g.*

$$2 \text{ feet} \times \frac{2}{12} \text{ feet} = \frac{4}{12} = 4',$$

feet × seconds give seconds; *e.g.*

$$2 \text{ feet} \times \frac{2}{144} \text{ feet} = \frac{4}{144} = 4'',$$

primes × primes give seconds; *e.g.*

$$\frac{2}{12} \text{ feet} \times \frac{2}{12} \text{ feet} = \frac{4}{144} = 4'',$$

primes × seconds give thirds; *e.g.*

$$\frac{2}{12} \text{ feet} \times \frac{2}{144} \text{ feet} = \frac{4}{1728} = 4''',$$

seconds × seconds give fourths; *e.g.*

$$\frac{2}{144} \text{ feet} \times \frac{2}{144} \text{ feet} = \frac{4}{20736} = 4'''',$$

seconds × thirds give fifths; *e.g.*

$$\frac{2}{144}\text{ feet}\times\frac{2}{1728}\text{ feet}=\frac{4}{248832}=4'''''.$$

It may be a help to the memory here to observe that, as in Algebra when we multiply together powers of like quantities, we *add the indices*, so in this cross multiplication, by adding together the denominations of the factors, primes and seconds, seconds and thirds, &c., we obtain the correct denomination of the product: thus $3''\times 2'=6'''$, (multiply and add the *indices*).

Ex. 1. *Find the area of a parallelogram whose sides measure 5 feet „ 3 inches and 4 feet „ 9 inches.*

$$\begin{array}{r}
5\text{ „ }3' \phantom{\text{ „ }3''}\\
4\text{ „ }9' \phantom{\text{ „ }3''}\\
\hline
21\text{ „ }0' \phantom{\text{ „ }3''}\\
3\text{ „ }11'\text{ „ }3''\\
\hline
24\text{ „ }11'\text{ „ }3''
\end{array}$$

Commencing with 4 feet, the *highest* denomination in the multiplier, we multiply by it 3 primes, the *lowest* denomination in the multiplicand: now 4 feet × 3 primes = 12 primes; but 12 primes = 1 foot, therefore set down 0 in the place of primes, and carry one, to the place of feet; 4 feet × 5 feet = 20 feet, and adding the one carried, we get 21 feet. Next multiplying by the other term in the multiplier, we have 9 primes × 3 primes = 27 seconds: but these, on dividing by 12, are found to equal 2 primes and 3 seconds: write 3″ *one place to the right hand*, which is the place of seconds, and carry 2 to the place of primes: then 9 primes × 5 feet = 45 primes; and adding the 2 carried, we get 47 primes, which, on dividing by 12, equals 3 feet and 11 primes. Add these results, and we obtain, as the area, 24 square feet, 11 primes, 3 seconds.

If it be required to turn the primes and seconds into *square inches*, we have

$$\frac{11}{12}+\frac{3}{144}=\frac{132}{144}+\frac{3}{144}=\frac{135}{144}=135\text{ square inches};$$

(a result which may be practically obtained by multiplying the *primes* by 12, and adding in the *seconds*); therefore the area is 24 square feet 135 square inches.

Ex. 2. *Required the solid content of a cube, each side of which measures 2 feet „ 9 inches.*

$$\begin{array}{l}
\quad 2\ „\ 9' \\
\quad \underline{2\ „\ 9'} \\
\quad 5\ „\ 6' \\
\quad \underline{2\ „\ 0'\ „\ 9''} \\
\quad 7\ „\ 6'\ „\ 9'' \\
\quad \underline{2\ „\ 9'} \\
15\ „\ 1'\ „\ 6'' \\
\quad \underline{5\ „\ 8'\ „\ 0''\ „\ 9'''} \\
20\ „\ 9'\ „\ 6''\ „\ 9'''
\end{array}$$

Therefore the required solid contains 20 cubic feet „ 9 primes „ 6 seconds „ 9 thirds.

To convert this result into cubic feet and cubic inches, we have

$$\frac{9}{12}+\frac{6}{144}+\frac{9}{1728}=\frac{1296}{1728}+\frac{72}{1728}+\frac{9}{1728}=\frac{1377}{1728};$$

(a result which practically may be obtained by multiplying the *primes* by 144, the *seconds* by 12, and adding to the sum of these the *thirds*); therefore 20 cubic feet „ 1377 cubic inches, would be the accurate result.

[This result a Builder would probably enter as 20 cubic feet „ $9\frac{1}{2}$ inches, meaning by that $\frac{9}{12}+\frac{1}{24}$ of a cubic foot, and intentionally omitting the fraction signified by 9‴. He would however most likely obtain his result by writing 2 ft. „ 9 in. as 2·75, finding from a table of cubes that the cube of 275 was 20786875; and then taking, as sufficiently accurate for practical purposes, 20·78, &c. as equivalent to $20\frac{3}{4}$.]

Ex. 3. *Required area of a square, whose sides measure 7 feet „ $8\frac{3}{4}$ ins.*

$$\begin{array}{l}
\quad 7\ „\ 8'\ „\ \ 9'' \\
\quad \underline{7\ „\ 8'\ „\ \ 9''} \\
54\ „\ 1'\ „\ \ 3'' \\
\quad 5\ „\ 1'\ „\ 10''\ „\ 0''' \\
\qquad \underline{5'\ „\ \ 9''\ „\ 6'''\ „\ 9''''} \\
59\ „\ 8'\ „\ 10''\ „\ 6'''\ „\ 9''''
\end{array}$$

Here 8′ „ 10″ equal 106 square inches, and 6‴ „ 9⁗ are $\frac{6}{12}$ and $\frac{9}{144}$ of a square inch; or are $\frac{81}{144}$, or $\frac{9}{16}$ of a square inch.

Hence 59 square feet $106\frac{9}{16}$ square inches. *Ans.*

§ 118. We will now proceed to explain the duodecimal scale of notation, and multiplication in "duodecimals" proper.

We have already stated (Chap. I. § 10, 12) that it was only by a perfectly arbitrary arrangement that 10 was fixed upon as the base or radix of the ordinary scale of notation; and that the duodecimal scale, with 12 for the base, would present some peculiar advantages. Among other advantages it would considerably simplify all calculations connected with superficial areas. Before however we advance further, we must first explain how numbers may be written in *any* scale; and then how they may be transferred from one scale to another.

It will be easy to trace that in the common scale, where the radix is 10, the number 54321 might be written as $1 + 2 \times 10 + 3 \times 10^2 + 4 \times 10^3 + 5 \times 10^4$; but if 12 were the radix, then the same figures 54321 would mean a number which might be written $1+2\times12+3\times12^2+4\times12^3+5\times12^4$.

Now assuming that 54321 were such a number in the duodecimal scale, it might be converted into the denary or decimal scale as follows:

$$\begin{array}{rcrcr}
5\times12^4 = 5\times & 20736 & = & 103680 \\
4\times12^3 = 4\times & 1728 & = & 6912 \\
3\times12^2 = 3\times & 144 & = & 432 \\
2\times & 12 & = & 24 \\
 & 1 & = & 1 \\
\hline
 & & & 111049
\end{array}$$

Next conversely, if the number 111049 written in the denary scale be divided by 12, and that quotient again divided by 12, and so on till the quotient 0 is arrived at, we shall exactly *reverse* the above operation, and shall obtain *remainders* which will represent the digits of the number in the duodecimal scale: thus

$$\begin{array}{r|l}
12 & 111049 \\
\hline
12 & 9254 - 1 \\
\hline
12 & 771 - 2 \\
\hline
12 & 64 - 3 \\
\hline
12 & 5 - 4 \\
\hline
 & 0 - 5
\end{array}$$

Where the true values of the remainders (cf. § 29, pages 29, 30) are $5\times12^4+4\times12^3+3\times12^2+2\times12+1$; and from them we

accordingly obtain the number 54321 expressed in the duodecimal scale.

In a similar manner we could transform a number written in any one scale to any other scale; for instance, let it be required to transform the number 4532 from a scale whose base is 7 to one whose base is 9.

$$\begin{aligned}\text{4532 in the septenary scale} &= 4\times 7^3+5\times 7^2+3\times 7+2\\ &= 4\times 343+5\times 49+3\times 7+2\\ &= 1372+245+21+2\\ &= 1640, \text{ in the denary scale;}\end{aligned}$$

and dividing the number 1640 by 9, and the result by 9, and so on, to transform it into a number in the scale whose radix is 9, we have

$$\begin{array}{rl} 9\,| & 1640 \\ 9\,| & 182 - 2 \\ 9\,| & 20 - 2 \\ 9\,| & 2 - 2 \\ & 0 - 2 \end{array}$$

whence 2222 is the number in the scale whose base is 9.

It would have been quite possible to have transferred the given number *at once* from one scale to the other[1]; but to avoid the operation of division in scales to which we are not accustomed, it will generally be the easiest plan first to transform the given number into the ordinary decimal scale, and then to transfer the result into the required scale.

---

[1] The *direct* operation would stand as follows:

$$\begin{array}{rl} 9\,| & 4532 \\ 9\,| & 350 - 2 \\ 9\,| & 26 - 2 \\ 9\,| & 2 - 2 \\ & 0 - 2 \end{array}$$

[9 is contained in $(4\times 7+5)$, or in 33, 3 times, and 6 *sevens* over;
9 is contained in $(6\times 7+3)$, or in 45, 5 times, and 0 over;
9 is contained in $(0+2)$, 0 times, and 2 over.
Second line:
9 is contained in $(3\times 7+5)$, or in 26, 2 times, and 8 sevens over;
9 is contained in $(8\times 7+0)$, or in 56, 6 times, and 2 over.
Third line: 9 is contained in $(2\times 7+6)$, or in 20, twice and 2 over.
Fourth line: 9 is contained in 2 *units*, 0 times, and 2 over.]

Having so far explained the general principle by means of which we may transform a number from any one scale to another, we shall, in what follows, limit ourselves to the decimal and duodecimal scales. In working in the duodecimal scale, however, it is obvious that we shall require two new symbols for the numbers *ten* and *eleven;* and for these it is customary to use the letters *t* and *e*.

Let us now explain the method of strict duodecimal multiplication; and in doing so, we will at the same time compare the process with that of ordinary "cross multiplication."

Ex. 4. Let it be required to find the area of a rectangle 217 feet „ $3\frac{1}{2}$ inches long, and 153 feet „ $7\frac{1}{4}$ inches broad.

First we transform the numbers 217 and 153 into the duodecimal scale.

$$\begin{array}{l} 12\,\underline{|\,217} \\ \;\;12\,\underline{|\,18} - 1 \\ \;\;\;\;12\,\underline{|\,1} - 6 \\ \;\;\;\;\;\;\;\;0 - 1 \end{array} \qquad\qquad \begin{array}{l} 12\,\underline{|\,153} \\ \;\;12\,\underline{|\,12} - 9 \\ \;\;\;\;12\,\underline{|\,1} - 0 \\ \;\;\;\;\;\;\;\;0 - 1 \end{array}$$

Hence the numbers are 161, 109; and to these we append the 3 inches and 6-twelfths, and the 7 inches and 3-twelfths, with a *duo*decimal point before them; we have therefore to multiply 161·36 by 109·73: the operation is performed by saying "3 times 6, eighteen; twelves in 18, one, and 6 over;" set down six, and carry 1: "3 times 3, nine; and 1 carried, ten;" set down *t*, and carry nothing: and so on, dividing each time by 12, instead of by 10:

$$\begin{array}{r} 161{\cdot}36 \\ 109{\cdot}73 \\ \hline 463\,t6 \\ t690\,6\;\; \\ 116e76\;\;\;\; \\ 161360\;\;\;\;\;\; \\ \hline 17394{\cdot}tt46 \end{array}$$

And in order to re-transfer 17394·*tt*46 into square feet and square inches expressed in the denary scale, in practice we write (multiplying by 12, and adding in the next figure),

17394·t t 46
12 12 12
19 130 54
12
231
12
2781
12
33376

and $\frac{54}{144} = \frac{3}{8}$.

Hence 33376 sq. feet „ 130$\frac{3}{8}$ sq. inches is the required area.

Performing the operation by cross-multiplication, and setting down all the figures used in the same exact manner, we have

217 „ 3′ „ 6″
153 „ 7′ „ 3″
33245 „ 7′ „ 6″
126 „ 9′ „ 0″ „ 6‴
4 „ 6′ „ 3″ „ 10‴ „ 6⁗
33376 „ 10′ „ 10″ „ 4‴ „ 6⁗

12 | 918
76 − 6
459
12 | 535
44 − 7
651
1085
217
33245

12 | 1521
126 − 9

12 | 651
12 | 54 − 3
4 − 4

Where the result represents 33376 sq. feet + 10 rectangles 1 ft. long and 1 inch wide + 10 sq. inches + 4 rectangles 1 inch long and $\frac{1}{12}$ inch wide + 6 squares $\frac{1}{12}$ inch long and $\frac{1}{12}$ inch wide; and this result, consisting of alternate squares and oblongs, has to be modified as explained in Exs. 1 and 3, before it can be expressed as square feet and square inches.

The subjoined examples will further illustrate multiplication and division in the duodecimal scale.

Ex. 5. *Find the superficial area of a rectangle whose sides are respectively* 34 *feet* „ $7\frac{2}{3}$ *inches, and* 27 *feet* „ $11\frac{5}{6}$ *inches.*

$$12\ \underline{|\ 34\quad}\qquad\qquad 12\ \underline{|\ 27\quad}$$
$$2-t.\qquad\qquad\qquad 2-3.$$

```
        2t·78
        23·et
      ---------
      2 4t 48
     27 90 4
     87 e0
    593 4
   -----------
    689·4t 88
   12  12  12
   --  --  ---
   80  58  104
   12
   --
  969
```

and $\frac{104}{144}=\frac{13}{18}$.

Hence the area is 969 sq. feet „ $58\frac{13}{18}$ sq. inches.

Ex. 6. *Divide* 1532 *sq. feet* „ 9′ „ 9″, *the area of a floor, by* 81 *feet* „ 9 *inches, the length of the side, in order to find the width.*

```
12 | 1532            12 | 81
   --------              ----
12 | 127—8                6—9
   --------
     10—7
```

We have therefore to divide $t78·99$ by 69·9.

```
69·9) t78·99 ( 16·9
      39e 9    12
               --
       51 39   18
       00 00
```

Whence, expressing the result in the decimal scale, the width is 18 ft. „ 9 in.

§ 119. If notwithstanding the explanations now given, the novelty of duodecimal notation should still deter some, as the complications of cross multiplication will probably continue to bewilder others, we may yet suggest a method for ordinary calculations, where parts smaller than the 144th would be rejected, which is based upon the principle of taking "aliquot parts," as in the rule of "Practice;" the method will be understood from the following examples:

Ex. 7. *Multiply* 17*ft.* „ 3 *in.* „ 6 *pts. by* 12*ft.* „ 9 *in.*

$$\begin{array}{c|c|r}
\text{in.} & & \\
6 & \frac{1}{2} & 17 \text{ „ } 3\frac{1}{2} \\
 & & 12 \\
\cline{3-3}
 & & 207 \text{ „ } 6 \\
3 & \frac{1}{2} & 8 \text{ „ } 7\frac{3}{4} \\
 & & 4 \text{ „ } 3\frac{7}{8} \\
\cline{3-3}
 & & 220 \text{ „ } 5\frac{5}{8}
\end{array}$$

This would probably be written by an artificer as "220 sq. ft. „ 5 in. „ 7½ pts.;" being in reality 220 sq. ft. „ 67½ sq. in.

Ex. 8. *Multiply* 7*ft.* „ 4 *in.* „ 9 *pts. by* 9*ft.* „ 3 *in.*

$$\begin{array}{c|c|r}
\text{in.} & & \\
3 & \frac{1}{4} & 7 \text{ „ } 4\frac{3}{4} \\
 & & 9 \\
\cline{3-3}
 & & 66 \text{ „ } 6\frac{3}{4} \\
 & & 1 \text{ „ } 10\frac{3}{16} \\
\cline{3-3}
 & & 68 \text{ „ } 4\frac{15}{16}
\end{array}$$

which result is 68 sq. feet „ 59¼ sq. inches.

§ 120. We will now add a few general examples, where the cost of certain superficial work is required, besides the area.

Ex. 9. *What is the cost of a carpet for a room measuring* 16 *feet* „ 5 *inches in breadth, and* 20 *feet* „ 9 *inches in length, at* 5*s.* „ 6*d. per yard?*

Writing the numbers in the duodecimal scale, we have

$$\begin{array}{r}
14\cdot5 \\
18\cdot9 \\
\hline
10\ 3\ 9 \\
te\ 4 \\
145 \\
\hline
244\cdot79
\end{array}$$

$$\begin{array}{rr}
12 & 12 \\
\hline
28 & 93 \\
12 & \\
\hline
340 &
\end{array}$$

Therefore 9 square feet : $340\frac{21}{48}$ :: $5\frac{1}{2}$: x shillings,

$$9x = \frac{16351}{48} \times \frac{11}{2},$$

$$x = \frac{16351}{48} \times \frac{11}{2} \times \frac{1}{9} = \frac{16351 \times 11}{48 \times 18 \times 20} £$$

$$= £10 \text{ ,, } 8s. \text{ ,, } 1\tfrac{1}{4}\tfrac{27}{32}d.$$

N.B. Carpets are sold by the linear yard, but are not always a yard wide, as here supposed to be.

Ex. 10. *What would be the cost of papering a room of which the following are the dimensions: length* 24 *feet* ,, 7 *inches; width* 20 *feet* ,, 5 *inches; height* 15 *feet; allow for three windows, each* 11 *feet* ,, 9 *inches by* 2 *feet* ,, 10 *inches, and for a door* 6 *feet* ,, 6 *inches by* 3 *feet; and suppose the paper to be* 30 *inches wide and to cost* 6*d. per yard?*

To obtain the area of the side wall we should multiply the *length* of the room by the *height;* for the area of the end wall we should multiply the *width* by the *height;* by doubling each of these products we should get the area of the two sides and the two end walls; and by adding together these two results we should obtain the whole superficial area of the four walls. But in order to shorten this process, we observe that, if we first add together the length and the breadth, and multiply the sum by the height, we should obtain the area of a space equal to one side and one end wall; and by doubling this we should get the area of the four walls; hence in practice the rule is, *add* together the length and the breadth, *multiply* the sum by the height, *double* this result, and we have the area of the four walls of the room. Hence, exhibiting the process in both methods, we have

24 ,, 7′ length	20·7 length
20 ,, 5′ breadth	18·5 breadth
45 ,, 0	39·0
15 (multiply by height)	13 (multiply by height)
675	e3
2 (double it)	39
1350 area of the 4 walls.	483
	2 (double it)
	946 area of the 4 walls.

For the windows

11 „ 9′	$e \cdot 9$
2 „ 10′	$2 \cdot t$
23 „ 6′	996
9 „ 9′ „ 6″	$1e6$
33 „ 3′ „ 6″	$29 \cdot 36$
3	3
99 „ 10′ „ 6″	$83 \cdot t6$ area of 3 windows
19 „ 6′	$17 \cdot 6$ area of door to be added
119 „ 4′ „ 6″	$9e \cdot 46$ total to be subtracted.
1350	946
119 „ 4′ „ 6″	$9e \cdot 46$
1230 „ 7′ „ 6″	$866 \cdot 76$ area to be papered;
$\frac{12}{90}$	and 866·76 becomes in the denary scale 1230 square feet, with
or $1230\frac{5}{8}$ sq. feet.	$\frac{7}{21} + \frac{6}{144}$; or $1230\frac{5}{8}$ sq. feet.

Now a strip of paper 30 inches, *i.e.* $2\frac{1}{2}$ feet wide, and 3 feet long, although not a *square* yard, is called a yard, and costs 6*d*.

Therefore $\quad 2\frac{1}{2} \times 3 : 1230\frac{5}{8} :: 6d. : Ans.,$

$$\frac{5}{2} \times 3 \times Ans. = \frac{9845}{8} \times 6,$$

$$Ans. = \frac{9845}{4} \times 3 \times \frac{2}{5} \times \frac{1}{3}$$

$$= \frac{1979}{2} \text{ pence}$$

$$= £4 \text{ „ } 2s. \text{ „ } 0\tfrac{1}{2}d.$$

N.B. It should be noticed that the custom is to sell wall papers by the "piece" containing 12 yards. Thus the "piece" at 6*d*. per yard would have cost 6*s*.; and as something over 13 pieces would have been required, the sum paid would have been 14 × 6*s*., or £4 „ 4*s*.

Ex. 11. *The area of a rectangular field is* 1 *acre* „ 4 *poles* „ 12 *yards* „ 7 *feet, and one side is* 50 *yards* „ 2 *feet; find the length of the other side.*

Since $\qquad$ length × breadth = area,

$$\text{length} = \frac{\text{area}}{\text{breadth}}.$$

Now 1 acre „ 4 poles „ 12 yards „ 7 feet $= \frac{44764}{9}$ square yds.,

and $\qquad 50 \text{ yards} \;,,\; 2 \text{ feet} = \frac{152}{3}$ linear yards;

therefore $\qquad \frac{44764}{9} \times \frac{3}{152} = 99 \text{ yards} \;,,\; 0 \text{ feet} \;,,\; 3\tfrac{1}{3} \text{ inches},$

the length required.

Ex. 12. *Gunter's chain consists of* 100 *links, and* 10 *square chains make an acre. What is the area of a rectangular field, whose sides are* 78 *chains* „ 23 *links, and* 42 *chains* „ 70 *links respectively?*

Writing the dimensions in links, we obtain the area by multiplying 7823 by 4270, pointing off in the result 4 decimal places to reduce it to square chains, and shifting the decimal point one place further to the left, to bring the square chain into the denomination of acres. Thus

$$\begin{array}{r}
7823\\
\underline{4270}\\
547610\\
15646\\
\underline{31292}\\
334{\cdot}04210\\
\underline{4}\\
{\cdot}1684\\
\underline{40}\\
6{\cdot}736
\end{array}$$

whence we obtain 334 ac. „ 0 r. „ $6\frac{3}{4}$ p. (nearly).

Ex. 13. *What would be the price of a piece of timber, which measures in length* 10 *feet* „ 4 *inches, in width* 7 *feet* „ 3 *inches, in thickness* 7 *inches, at* £6 „ 5*s. per load of* 50 *cubic feet?*

Again performing the operation by both methods, for the sake of comparison, we have

$$\begin{array}{l|r}
\begin{array}{r}
10 \;,,\; 4'\\
\underline{7 \;,,\; 3'}\\
72 \;,,\; 4'\\
\underline{2 \;,,\; 7' \;,,\; 0''}\\
74 \;,,\; 11'\\
\underline{7'}\\
43 \;,,\; 8' \;,,\; 5''\\
\underline{12}\\
101''
\end{array}
&
\begin{array}{r}
t\cdot4\\
\underline{7\cdot3}\\
2\,7\,0\\
\underline{6\,0\,4}\\
6\,2\cdot e\\
\underline{\cdot7}\\
3\,7\cdot8\,5\\
\underline{12}\;\;\underline{12}\\
43\;\;101
\end{array}
\end{array}$$

therefore $43\frac{101}{144}$ cubic feet is the solid content of the piece of timber.

$$\overset{\text{cub. ft.}}{50} : \overset{\text{cub. ft.}}{43\tfrac{101}{144}} :: £6\tfrac{1}{4} : Ans.,$$

$$50 \times Ans. = \frac{6293}{144} \times \frac{25}{4},$$

$$Ans. = \frac{6293}{144} \times \frac{25}{4} \times \frac{1}{50}$$

$$= \frac{6293}{1152}$$

$$= £5 \text{ ,, } 9s. \text{ ,, } 3\tfrac{12}{576}d.$$

Exercise XXIII.

1. Find, by duodecimal multiplication[1], the area of a rectangle measuring 28 feet „ 9 inches, by 10 feet „ 10 inches.

2. What is the area of a floor measuring 22 feet „ 6 in. „ 4 pts. in length, and 10 feet „ 5 in. „ 8 pts. in width?

3. What is the content in cubic feet and inches of a regular solid, whose dimensions are in length 23 feet „ 10 inches, in width 18 feet „ 4 inches, and in thickness 11 feet „ 3 inches?

4. What is the superficial area of a rectangle 15 feet „ 3 primes „ 5 seconds long, and 8 feet „ 4 primes „ 8 seconds wide?

5. The length of a rectangular area is 3 feet „ $7\frac{1}{2}$ inches, and the width is 2 feet „ $5\frac{1}{2}$ inches; find the square feet and inches it contains, and its value at 15*s.* a square foot.

6. The length of a room is 15 feet, breadth 10 feet, and height 9 feet „ 9 inches; find the expense of painting the walls and ceiling at 1*s.* „ 9*d.* per square yard.

7. Find the difference of the areas of the floors of two rooms, one of which is 12 feet „ 6 inches long, by 10 feet „ 3 inches, and the other 15 feet „ 8 inches, by 11 feet „ 4 inches.

8. What length of paper $\frac{3}{4}$ of a yard wide, will be required to cover a wall 15 feet „ 8 inches long, 11 feet „ 3 inches high?

9. How many cubic feet are there in a solid, whose breadth is 9 feet „ 3 inches, length 11 feet „ 3 inches, and height 3 feet „ 2 inches?

[1] The examples given can be worked by duodecimal multiplication, or by cross multiplication.

10. Find the thickness of a solid, whose length is 2 yards, breadth a yard and a half, and solid content 1 cubic yard „ 6 cubic feet „ 1296 cubic inches.

11. Find the breadth of a room, the length of which is $17\frac{1}{2}$ feet, and the area $250\frac{5}{8}$ feet.

12. What will be the price of carpeting a room 13 feet „ 4 inches long, and 12 feet „ 6 inches broad, at 4*s.* „ 6*d.* a yard, the carpet being a yard wide?

13. What will the flooring of a room $16\frac{1}{2}$ feet square amount to, at 4*s.* $10\frac{1}{2}$*d.* a square yard?

14. How much will remain out of 393 square feet of carpeting after covering a floor 23 feet „ 8 inches long, and 16 feet „ 7 inches broad?

15. Find the number of square feet in a floor whose length is $10\frac{2}{3}$ yards, and breadth $5\frac{1}{4}$ yards; and the price of paving it at 2*s.* per square yard.

16. The dimensions of a grass plot are 23 feet „ 8 inches in length, and 16 feet „ 7 inches in breadth; round it a walk 10 feet wide is constructed, and paved at 1*s.* $10\frac{1}{2}$*d.* per square yard; what is the cost of the paving?

17. There is a court which is 120 feet „ 9 inches square, containing a grass-plot in each corner 50 feet square; the rest of the space consists of walks crossing each other at right angles, which are paved with flag-stones down the middle of them for a width of 6 feet „ 9 inches, and for the remainder with pebbles. The flag-stones cost 3*s.* per square yard and the pebbles 1*s.* „ 6*d.*; what was the entire cost of the paving?

18. If 69 yards of carpet, $\frac{3}{4}$ of a yard wide, will cover a room which is $10\frac{1}{2}$ yards long, what is the width of the room?

19. What length of carpet that is 3-quarters of a yard wide, will cover a room that is 19 feet „ 6 inches long, and 15 feet „ 9 inches wide?

20. Find the number of "pieces" of paper, (12 yards to the piece,) the paper being 3 feet „ 6 inches wide, required to paper a room 9 yards long, 6 yards wide, and 14 feet high.

21. What is the cost of flooring a passage 14 feet „ 6 inches long, 5 feet „ 7 inches broad, at 2*s.* „ 3*d.* per yard?

22. What would be the cost of carpeting a room $31\frac{1}{2}$ feet long, and $23\frac{3}{4}$ feet wide, at 2*s.* „ 9*d.* a yard, the carpet being 18 inches wide?

23. How many cubical feet of air does a room contain which is 19 feet „ 6 inches long, 16 feet „ 9 inches wide, and 10 feet „ 6 inches high? Also, how much would it cost to paper the 4 walls with paper $\frac{7}{8}$ of a yard wide, costing $3\frac{1}{2}d.$ a yard?

24. What is the difference between two areas, one of which is 15 yards square, the other 15 square yards?

25. A school should contain, according to the Government regulations, 80 cubic feet of air for each child. If there are 150 children in a school 60 feet „ 4 inches long, 18 feet „ 5 inches wide, and 10 feet „ 6 inches high; how many would there be above the regulation number?

26. How much yard-wide cloth will cover a passage whose length is 12 feet „ 6 inches, and breadth 2 feet „ 9 inches? and what will it cost at 5*s*. „ 6*d*. per yard?

27. A surface, measuring 120 feet long by 40 feet wide, is covered with iron plating, weighing 21 ton „ $18\frac{389}{448}$cwt.; if a cubic foot of iron weigh 490 lbs. „ 4 oz., what is the thickness of the plating?

28. How many square feet of paper will cover the walls of a room whose dimensions are 20 feet „ 10 inches by 16 feet in breadth, and 10 feet „ 8 inches in height?

29. What will the painting of the walls of a room cost which is $20\frac{1}{4}$ feet long, $18\frac{1}{2}$ broad, and 10 feet high, containing 2 windows, whose dimensions are 7 feet by 4 feet each; at the rate of 2*s*. „ 6*d*. a square yard?

30. Find the cost of lining a cistern with lead, whose depth, length, and breadth are 3 feet „ 6 inches, 7 feet „ 10 inches, and 5 feet „ 4 inches respectively, at 10*s*. „ $6\frac{1}{2}d.$ per square yard.

31. What should be charged for painting the inside and outside of an iron chest 7 feet „ 4 inches long, 4 feet „ 8 inches wide, and 3 feet „ 10 inches deep, at 9*d*. the square yard?

32. How many tons of water are there in a cistern 18 feet „ 8 inches long, 18 feet „ 4 inches broad, and 6 feet „ 9 inches deep, supposing a cubic foot of water to weigh 1000 ounces?

33 A cistern measured 6 feet „ 3 inches in length, 4 feet „ 2 inches in width, and was 5 feet deep. After being filled with water, it leaked till the surface of the water sunk

7 inches; how many cubic feet and inches of water then remained in the cistern?

34. Find the cost of papering a room 21 feet long, 15 wide, and 12 high with paper $2\frac{1}{2}$ feet wide at 9*d.* a yard, allowing for a door 7 feet high and 3 wide, and 2 windows each 5 feet high and 3 wide.

35. What will be the expense of papering a room that measures 19 feet „ 8 inches in width, 24 feet „ 4 inches in length, and is $13\frac{1}{2}$ feet high, with a paper which is $2\frac{3}{4}$ feet wide, and costs 11*s.* per piece of 12 yards in the piece; the windows and parts not requiring to be papered, making up a sixth part of the whole surface?

36. The floor of a room contains 40 square yards; its height is 5 yards, and the length is 3 yards more than the breadth; find the number of square yards in the 4 walls.

37. If a field of 10 acres be divided into allotments measuring 110 feet by 20 feet, at what sum ought each of the allotments to be let, if the rent be £10 per acre?

38. The expense of carpeting a room 20 feet long was £7 „ 10*s.*; but if the breadth had been 3 feet less than it was, the expense would have been £6; what was the breadth of the room?

39. Transform the number 981 into the binary and ternary scales: and transform 4321 from the quinary to the septenary scale.

40. Transform the numbers 1785 and 281 from the denary to the duodenary scale; and find their product in the duodenary scale.

41. Multiply together the numbers $1e{\cdot}t$ and $15{\cdot}8$ expressed in the duodecimal scale.

42. Transform 2304 from the quinary to the undenary scale.

43. Divide $29t96580$ by $2tt9$ in the duodecimal scale.

CHAPTER XX.

INVOLUTION AND EVOLUTION; OR THE EXTRACTION OF THE SQUARE AND CUBE ROOT.

§ 121. We have already seen (§ 40. *Obs.* p. 48) that the *Powers* of numbers are formed by multiplying the numbers by *themselves* a certain number of times; that 2×2 is called the *second power* of 2, or the *square* of 2, and is written 2^2: that $2 \times 2 \times 2$ is called the *third power* of 2, or the *cube* of 2; and is written 2^3; and so on.

Similarly the powers of fractions are found by raising both numerator and denominator to the power required; thus

$$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}; \quad \left(1\frac{1}{5}\right)^5 = \frac{6^5}{5^5} = \frac{7776}{3125} = 2\frac{1526}{3125};$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2^2} = \frac{1}{4}.$$

So again the power of a decimal may be found by multiplying it by itself the number of times indicated by the power; thus

$$\cdot03^2 = \cdot03 \times \cdot03 = \cdot0009.$$

This raising of numbers, by successive multiplications with the same multiplier, to their powers is called *Involution;* while the converse operation of finding out from the result of an involution what was the original multiplier employed, is called *Evolution.*

Def. The square root of any proposed number is that number which, when multiplied by itself, will produce the proposed number.

Def. The cube root is that number which, when multiplied by itself twice over, will produce the proposed number.

The symbol commonly used to signify the extraction of the square root is $\sqrt{}$; and for the extraction of the cube root is $\sqrt[3]{}$.

In arithmetic we do not attempt more than the extraction of the square and the cube root; the fourth root, which is sometimes required, being only the square root of the square root; and the sixth root only the cube root of the cube root.

§ 122. The reverse process for extracting either a square or a cube root by evolution must be based upon the direct method of obtaining the square by involution. But inasmuch as in the arithmetical process of multiplication the several parts of the number are so mixed up as not to be easily distinguishable, it becomes necessary to exhibit a number separated into its component parts, and to show how we may find the square of it in this altered shape: so that, by reversing the operation, we may be able to exhibit the process for finding the square root[1].

Let it be required to find the square of 83. Write 83 as $80+3$; now to obtain the product of $(80+3)$ multiplied by $(80+3)$ we observe that there is first 80×80; then 80×3;

[1] To those who understand Algebra it will be a help to observe how the Arithmetical process of evolution is based upon the Algebraical. To investigate the algebraical method, first, by involution, find the square of the quantity $a+b$. By multiplication we obtain, as its square, the quantity $a^2+2ab+b^2$. Now for the extraction of the square root, we must devise a method by which from $a^2+2ab+b^2$ we can evolve $a+b$; for this latter we know is the quantity which, when multiplied by itself, will give the proposed quantity. The method is as follows:

$$
\begin{array}{rl}
 & a^2+2ab+b^2(a+b \\
 & \underline{a^2} \\
2a+b) & 2ab+b^2 \\
 & \underline{2ab+b^2} \\
 & \cdot\quad\cdot
\end{array}
$$

Having arranged the terms according to the dimensions of some one letter, we see that the square root of the first term of the proposed quantity is a; write down a on the right hand as the first term in the root, square a, and subtract its square from the proposed quantity; then bring down the remainder $2ab+b^2$. To find b, the second term in the root, divide $2ab$, the first term in this remainder, by $2a$; write b, the result of this division, as the second term in the root; and placing $2a+b$ on the left of the remainder, as if it were a divisor, multiply it by b, and subtract the product, viz. $2ab+b^2$, from the remainder. If there be more terms, consider $a+b$ as a new value of a, and proceed as before.

next 3×80; lastly 3×3: that is, there is $6400 +$ twice $240 + 9$; *i.e.* the square of 83 is $6400 + 480 + 9$, or is 6889.

Now let us, in order to find the *square root* of 6889, reverse the process just used; and writing the number as $6400 + 480 + 9$, let us first find the square root of 6400, which is 80; and subtracting this square of 80 from the given number, we have as a remainder $480 + 9$. In order to find the second figure in the root we double 80, which thus becomes 160, and by this result we divide 480, and take the quotient 3 as the second part of the root. Next writing $160 + 3$ by the side of the remaining portion of the square, and multiplying it by the figure 3 which is written as the second part of the root, we obtain $480 + 9$. The process would be exhibited as follows:

$$\begin{array}{rl} & 6400 + 480 + 9\ (80 + 3 \\ & 6400 \\ \hline 160 + 3) & \quad 480 + 9 \\ & \quad 480 + 9 \end{array}$$

Next, if we wrote the numbers nearly in the ordinary manner, it would not be difficult to trace the various steps of the process: then we should have

$$\begin{array}{rl} & 6889\ (80 + 3 \\ & 6400 \\ \hline 160 + 3, \text{ or } 163) & \ \ 489 \\ & \ \ 489 \end{array}$$

Lastly, if we omitted all ciphers which are superfluous, we might adopt the following shortened form:

$$\begin{array}{rl} & 6889\ (83 \\ & 64 \\ \hline 163) & \ \ 489 \\ & \ \ 489 \end{array}$$

§ 123. Before deducing from this process the practical rule for the extraction of the square root, we must first find out how many figures the root will contain: and both the number and the local value of the figures in the square root may be determined from the following considerations:

since $\quad 1^2 = 1$, $10^2 = 100$, $100^2 = 10000$, and so on;

it appears that

the square of a number consisting of one digit may contain either one or two digits;

the square of a number consisting of two digits may contain either three or four digits;

the square of a number consisting of three digits may contain either five or six digits;

and so on. Whence we observe that in squaring any whole number, every digit of which it consists, with the exception of the extreme left-hand digit, must introduce two digits in the square; while the extreme left-hand digit may introduce *either* one *or* two digits, as the case may be, (for the squares of the numbers 1, 2, and 3 consist each of one figure, and the squares of the other numbers up to 9 of two figures.) Hence it follows that, in any whole number whose square root is to be extracted, if a point be placed over every second figure, beginning with the figure in the place of units, the number will be thus divided into *periods*, of which the one on the extreme left may consist (according as the given number contains an odd or an even number of digits) of either one or two figures: the others will all consist of two figures; and the number of these periods will show the number of figures in the square root, and will thus denote the local value of the first figure of the root.

Having prepared the number, by this method of *pointing*, for the extraction of its square root, we may give the rule for the process as follows:

Find the largest number whose square can be subtracted from the left-hand period; write this as the first figure in the root, subtract its square from the first period, and to the remainder bring down the next period. Double the first figure of the root, place it on the left of the remainder, and using it as a divisor, divide the remainder, omitting the last figure, by it; the quotient is the next figure in the root, which must be annexed to the divisor as well as to the root; by this last figure in the root multiply the divisor as it now stands, and the required subtrahend will be obtained.

If there should be more periods to be brought down, the operation must be repeated; remembering to double only the last figure in the divisor at each successive step.

It will occasionally happen, especially in the early stages of the process, that the figure obtained as a quotient by the trial divisor is too large, and will give a subtrahend larger than the minuend from which it is to be taken: in such a case we must try a smaller number.

Ex. 2. *Extract the square root of* 175233494881.

```
         175233494881 (418609
         16
    81)  152
          81
   828)  7133
         6624
  8366)  50949
         50196
837209)  7534881
         7534881
```

In extracting the square root of a decimal, we observe that, (since in squaring a decimal *each* figure in the decimal multiplier must introduce *two* figures in the result,) the figures must, from the commencement, be taken in periods of *two* figures. Hence, in *pointing* for the extraction of the square root of a decimal, care must be taken always to place the first point over the place of *hundredths*, never over the place of *tenths:* to make up a full period at the end, a cipher, if necessary, can always be added to the right hand.

If the number whose root is required should consist of integers and decimals, the integers are first pointed by placing the point over the place of units, and pointing every alternate figure: then, commencing again from the place of units, and proceeding to place a point over every second figure of the decimals, the first point, (always missing the place of *tenths*,) will fall over the place of *hundredths*, and so on.

Ex. 3. *Extract the square roots of* ·167281; *and of* 332·150625.

```
     ·167281 (·409               332·150625 (18·225
      16                         1
809)  7281                  28)  232
      7281                       224
                           362)  815
                                 724
                          3642)  9106
                                 7284
                         36445)  182225
                                 182225
```

Since the number of *periods* shows the number of figures in the root, it follows that as many *decimal periods* as there may be in the proposed quantity, so many *decimal places* will there be in the root.

When the given number has no exact square root, if it be a decimal, ciphers can be added to it; or if it be an integer, a decimal point can be placed after it, and ciphers attached; and then the process can be carried as far as may be required.

Ex. 4. *Extract the square root of* 3 *to three places of decimals.*

In order to show the exact value of the remainder in this case, we will exhibit the process with the decimal point retained throughout the whole operation.

$$
\begin{array}{r|l}
 & \dot{3}\cdot 0\dot{0}0\dot{0}0\dot{0}\ (1\cdot 732,\ \&c. \\
 & 1 \\
\hline
2\cdot 7) & 2\cdot 00 \\
 & 1\cdot 89 \\
\hline
3\cdot 43) & \cdot 1100 \\
 & \cdot 1029 \\
\hline
3\cdot 462) & \cdot 007100 \\
 & \cdot 006924 \\
\hline
 & \cdot 000176
\end{array}
$$

therefore 1·732, &c. is the approximate root of 3.

We see from this, that when an integer has no integral square root, it has no square root at all in finite terms. Thus 3 has no exact square root; but since the number 1·732 multiplied by itself gives very nearly 3, it is commonly said that 1·732 is very nearly the square root of 3; more properly perhaps, it is the square root of something very near 3.

§ 124. When the root is required to be extracted to many places of figures, it is found that the work may be considerably shortened, if, *when one figure more than half the number of required digits in the root has been obtained by the ordinary process,* we divide the remainder that then arises, by the divisor, after the manner of contracted division, (page 122,) and so obtain the remainder of the digits by simple division.

Ex. 5. *Extract the square root of* 2 *to* 8 *places of decimals.*

$$
\begin{array}{lll}
 & \dot{2}\cdot0\dot{0}0\dot{0}0\dot{0}0\dot{0} & (1\cdot4142 \\
 & 1 & \\
\hline
24) & 100 & \\
 & 96 & \\
\hline
281) & 400 & \\
 & 281 & \\
\hline
2824) & 11900 & \\
 & 11296 & \\
\hline
28282) & 60400 & \\
 & 56564 & \\
\hline
\text{(by division)}\quad 2828{,}2) & 3836 & (1356 \\
 & 1008 & \\
 & 152 & \\
 & 12 & \\
 & 00 &
\end{array}
$$

Hence 1·41421356 &c. is the approximate root of 2.

This plan does not indeed show the correct value of the remainder, and might perhaps make the process appear finite when it is not really so. However, when the object is to find the value of a great many figures in the approximate root, the saving of labour that results from the adoption of the method renders it very advantageous.

Ex. 6. *Extract the square root of* $\frac{169}{529}$, *and of* $\frac{21699}{25009}$, *and of* $\frac{5}{18}$.

The square root of a vulgar fraction may be obtained by extracting the square root of the numerator and denominator; thus,

$$\sqrt{\left(\frac{169}{529}\right)} = \frac{\sqrt{(169)}}{\sqrt{(529)}} = \frac{13}{23}.$$

But if neither the numerator nor the denominator have an exact square root, the vulgar fraction may first be converted into a decimal, and its approximate root may then be extracted.

Thus $\frac{21699}{25009} = \cdot867647$, &c., and by extracting the approxi-

mate square root of ·867647, &c. we shall obtain the approximate square root of the equivalent vulgar fraction:

$$\begin{array}{r|l}
 & \cdot 867647,\ \&c.\ (\cdot 931,\ \&c. \\
 & 81 \\ \hline
183) & 576 \\
 & 549 \\ \hline
1861) & 2547 \\
 & 1861 \\ \hline
 & 686
\end{array}$$

Or sometimes, when the denominator is not an exact square, we may reduce the fraction to an equivalent one whose denominator is a square, (by multiplying both numerator and denominator by the least number that will render the denominator a square,) and then extracting the square root of both numerator and denominator. The least number which will make the denominator a square is found by taking out any factor in the denominator which is a square, and multiplying both numerator and denominator by the remaining factor: thus,

$$\sqrt{\frac{5}{18}} = \sqrt{\frac{5}{9 \times 2}}$$

$$= \sqrt{\frac{5 \times 2}{9 \times 4}} = \sqrt{\frac{10}{36}}$$

$$= \frac{\sqrt{10}}{6} = \frac{3 \cdot 16227,\ \&c.}{6}$$

$$= \cdot 52704,\ \&c.$$

Ex. 7. *Extract the fourth root of* ·0000065536.

The fourth root is the square root of the square root; hence

$$\begin{array}{r|l}
 & \cdot 0000065536\ (\cdot 00256 \\
 & 4 \\ \hline
45) & 255 \\
 & 225 \\ \hline
506) & 3036 \\
 & 3036 \\ \hline
 & \cdots\cdot
\end{array}$$

```
     ·00256000 (·05059, &c.
       25
1005)  6000
       5025
10109) 97500
       90981
        6519
```

therefore ·05059, &c. is the approximate fourth root.

§ 125. From these examples it will be observed that the operation of extracting the square root consists in successive repetitions of two processes; by one of which the given square number is diminished by numbers which, taken together with those previously subtracted, make up the square of the part of the root already found; while by the other process, a divisor is formed for obtaining the next root-figure. Thus in Example 8, given below, supposing the first part of the root to stand as $30+8$ integers, and ciphers to be used, we should first subtract not 9 but 900, and secondly subtract 544; but $900+544=1444$, which is the square of 38.

The operation might therefore be carried on in two parallel columns as follows: find the first figure in the root, set it in the column to the left, which call the first column; set the same figure beneath it, multiply them together, place the product under the second column, subtract it, and to the remainder annex the next period. Then add together the two figures in the first column, and use their sum as a trial divisor to obtain the next figure in the root: look upon the number which is their sum as multiplied by 10, (for each time that a new root-figure is obtained, the local value of those previously established is increased tenfold,) and add to it the new root-figure: (thus in Ex. 8, 6 becomes 60, and with 8 added to it, 68; 76 becomes 760, and so on;) set the new root-figure beneath the new sum in the first column, multiply by it, and place the product in the second column; subtract, and bring down the next remainder. Add the figures in the first column, use the sum as a new trial divisor, look on that sum as multiplied by 10, add the new root-figure to it; and so continue the process.

This process of arranging in parallel columns is not indeed essential; but as it serves to illustrate the process of extracting the cube root which is afterwards given, an example is added, in which the method is exhibited at length.

Ex. 8. *Extract the square root of* 15 *to ten places of decimals.*

I.	II.	
3	15·	(3·87298
3	9	
68	600	
8	544	
767	5600	
7	5369	
7742	23100	
2	15484	
77449	761600	
9	697041	
774588	6455900	
8	6196704	
774596	259196	

From this point proceeding by contracted division we obtain five more figures in the root:

```
77459,6) 259196 (33462
         26817
          3579
           481
            16
             1
```

Hence the square root correct to the tenth place of decimals is 3·8729833462.

§ 126. In investigating a method for the extraction of the cube root, we must first show, (by separating a number into its component parts, as we did when forming its square,) how we may represent the cube of any number; and then by a converse operation, how the cube root may be obtained.

We saw that the *square* of $80+3$ might be written as $80\times 80+$ twice $80\times 3+3\times 3$; *i.e.* as 80^2+ twice $80\times 3+3^2$. Now to obtain the *cube* of $80+3$, we again multiply the *square*, or (80^2+ twice $80\times 3+3^2$) by $(80+3)$. The results obtained are

$$80^2\times 80+\text{twice }80^2\times 3+3^2\times 80$$
$$+\quad 80^2\times 3+\text{twice }80\times 3^2+3^3;$$

and these results combined are

$$80^3 + \text{thrice } 80^2 \times 3 + \text{thrice } 80 \times 3^2 + 3^3.$$

§ 127. Before commencing the process of evolution[1] by which we may find the cube root, let us remark that

Since $1^3 = 1$, $10^3 = 1000$, $100^3 = 1000000$, and so on; it appears

that the cube of a number consisting of one digit may contain either one, two, or three digits;

that the cube of a number consisting of two digits may contain four, five, or six digits;

that the cube of a number consisting of three digits may contain seven, eight, or nine digits;

and so on. Whence we observe that in cubing any whole number, every digit of which it consists, with the exception of the extreme left-hand digit, will introduce three digits in the cube; while the extreme left-hand digit may introduce either one, two, or three digits, as the case may be, (for the cubes of the numbers 1 and 2 consist of one figure each; of 3 and 4 of two figures each; and of the other numbers up to 9 of three figures each). Hence it follows that, if, when about to extract the cube root, we place a point over every third figure,

[1] Here also the algebraical form may be of some assistance in understanding the arithmetical process: By involution we find that the cube of $a+b$ is $a^3+3a^2b+3ab^2+b^3$: from this quantity, then, we are to devise a method of evolving $a+b$, which we know to be its cube root. The method is as follows: arrange the terms according to the dimensions of some one letter, a, and take the cube root of a^3, the first term; this gives a as the first term in the root. Subtract a^3 from the given cube quantity, bring down the three terms $3a^2b+3ab^2+b^3$, and divide the first term of this remainder by $3a^2$, by which we obtain b, the second term of the root, which is to be added to a already standing to the right of the cube quantity. Arrange $3a^2+3ab+b^2$ in a loop to the left of the remainder, multiply by b, and subtract the result, viz: $3a^2b+3ab^2+b^3$ from the remainder: the whole cube of $a+b$ has now been subtracted; and if no remainder be left, as in the present case, the operation would be finished. If however an expression containing more terms had been given, we should proceed by considering $a+b$ to be the new first term in the root, and should treat it as before we treated a. The form would stand as follows:

$$\begin{array}{r|l}
 & a^3+3a^2b+3ab^2+b^3\ (a+b \\
 & \underline{a^3} \\
3a^2+3ab+b^2) & 3a^2b+3ab^2+b^3 \\
 & \underline{3a^2b+3ab^2+b^3} \\
 & \cdot \quad \cdot \quad \cdot
\end{array}$$

beginning with the figure in the place of units, the number will be thus divided into *periods*, of which the period on the extreme left may consist of one, two, or three figures; the others must all consist of three figures; and these periods will show the number of figures in the cube root, and will thus denote the local value of the first figure in the root.

§ 128. Having explained how to prepare the number, by thus *pointing* it, we may now take an example, and exhibit the process of extracting the cube root, as follows:

Ex. 1. *Let it be required to extract the cube root of* 571787.

By pointing, we divide the given number into two periods, and thus learn that the required root consists of two digits, of which the left-hand digit will be in the place of tens. Consequently we must seek for the largest multiple of ten whose cube is less than 571000; this being found to be 80, write the number 571787, duly pointed, in one column, which call the third column, with the number 80 in a loop to the right of it: cube 80, write the result, viz. 512000, beneath 571787, and subtract it: (we have now removed the cube of 80, and have left as a remainder a number which must be equal to $80^2 \times$ the second root-figure + thrice $80 \times$ the square of the second root-figure + the cube of the second root-figure). Now arrange on the left two other columns: in the one to the extreme left, which call the first column, place *thrice* 80, that is, 240; in the next, or second column, place *thrice the square of* 80, that is, 19200. Then divide the remainder in the third column by the number in the second column; that is, divide 59787 by 19200: the quotient is 3; and this, presumably, is the second figure in the root. Add the 3 thus obtained to the first column, and multiply the sum by 3; that is, multiply (240 + 3) by 3; add the result, viz. 729, to the number in the second column, and multiply the sum, which is 19929, by 3: place the product, viz. 59787, beneath the number left in the third column and subtract it from it; the remainder is 0; whence we conclude that the required root is 83. The entire process will stand thus:

I.	II.	III.
240		$57\dot{1}78\dot{7}$ (80 + 3
3	19200	512000
$\overline{243} \times 3$	729	$\overline{59787}$
	$\overline{19929} \times 3$	59787

The ciphers may be omitted, as superfluous, in the third column, and in the root: they *may* likewise be omitted in the first and second columns; but for the sake of clearness, it would seem better to retain them in those two columns, and only in practice omit them in the third column, and in the root: the form, with these ciphers omitted, will be

I.	II.	III.
240		$57\dot{1}78\dot{7}$ (83
3	19200	512
243 × 3	729	59787
	19929 × 3	59787

It sometimes happens that the number resulting from the division of the first remainder in the third column by the trial divisor in the second column, is too large, and then a lower number must be tried: this will not unfrequently occur with the second, and perhaps even with the third figure of the root; but is not likely to occur in the later stages of the operation. A little practice will soon enable the learner to find the correct number.

§ 129. To enable us to *point* correctly when the given cube number consists in part or entirely of decimals, we observe that, in cubing a decimal, *each* figure in the decimal multiplier must introduce *three* figures in the result; and that consequently, in extracting a cube root, the decimal figures must, from the commencement, be taken in periods of *three* figures. Hence if the given cube be a mixed number, consisting of integers and decimals, place a point over the units figure, and over every third figure to the left, and to the right of it; if it consist entirely of decimals, passing over the figures in the place of tenths and hundredths, set the first point over the figure in the place of thousandths, and point over every third figure; making up a full period at the extreme right by the addition of one or two ciphers, should it be necessary to do so. The number of decimal *periods* in the cube, will indicate the number of decimal *places* in the root.

Ex. 2. *Extract the cube root of* ·493039; *and of* ·000185193.

I.	II.	III.
210		·$49\dot{3}03\dot{9}$ (·79
9	14700	343
219 × 9	1971	150039
	16671 × 9	150039

I.	II.	III.
150		$\cdot 00\dot{0}18\dot{5}19\dot{3}$ ($\cdot 057$
7	7500	125
157 × 7	1099	60193
	8599 × 7	60193

Observe in the second case that the trial divisor 7500 gave as a quotient figure 8; but as this was found to be too large, the next lower number 7 was taken, and found to succeed.

§ 130. When there are more than two figures in the root, the process has to be continued; and in order to do this correctly, we must make the number in the first column equal to *three times the part of the root already found;* and the number in the second column equal to *three times the square of the part of the root already found.*

Now the number in the first column is already thrice the first figure in the root, with the second figure added to it; so that, if we increase this number further by twice the second figure, we shall have thrice the first and thrice the second figures of the root: and as by the addition of a third figure to the root the local value of the first and second figures will be increased tenfold, we must multiply the above-named result by 10, by annexing a cipher to it; and we shall have *three times the part of the root already found.*

Moreover in the second column there is already thrice the square of the first figure, increased by thrice the sum of the first and second figures multiplied by the second figure: so that this result needs only to be increased again by thrice the sum of the first and second figures multiplied by the second figure, (*i.e.* by the number standing second in the column,) and by the square of the second figure, in order to become thrice the square of the first and second root-figures. If therefore at the bottom of the second column we write the square of the second figure, and then add together that square, the number next above, and the number next above that, we shall have thrice the square of the first and second figures of the root. To this, as the local value of these figures will be increased tenfold, we must annex two ciphers, in order to increase their square an hundredfold. Thus finally we shall obtain *three times the square of the part of the root already found.* With these fresh numbers we continue the operation as before.

Ex. 3. *Extract the cube root of* 189·119224.

I.	II.	III.
150		189·119224 (5·74
7	7500	125
157 × 7	1099	64119
14	8599 × 7	60193
1710	49	3926224
4	974700	
1714 × 4	6856	
	981556 × 4	3926224

Just as in the extraction of the square root, we can find the *approximate cube root* of a number which is not a perfect cube, by adding ciphers after the decimal point, and bringing down as many periods of these as may be required. And it has been found that when *two figures more than half of the required digits in the root have been obtained by the ordinary process, the remainder of the digits required may be obtained by division.*

Ex. 4. *Extract the cube root of* 1·808 *to seven places of decimals.*

I.	II.	III.
30		1·808 (1·2182
2	300	1
32 × 2	64	808
4	364 × 2	728
360	4	80000
1	43200	
361 × 1	361	
2	43561 × 1	43561
3630	1	36439000
8	4392300	
3638 × 8	29104	
16	4421404 × 8	35371232
36540	64	1067768000
2	445057200	
36542 × 2	73084	
	445130284 × 2	890260568
		177507432

From this point we may proceed by division to find 3 figures more in the root; and only taking sufficient figures to ensure, by the process of contracted division, the correctness of 3 figures in the quotient, we have

```
4452) 17750 (398
      4394
       387
        31
```

Hence the approximate cube root required is 1·2182398, &c.

Exercise XXIV.

1. Find the square of $4\frac{1}{4}$; the cube of 37·6; the 4th power of $\frac{3}{4}$; the 5th power of $3\frac{1}{2}$; the 7th power of ·4.

2. Express as a decimal the value of $\frac{1}{3^5}$, and of $\frac{1}{9^5}$; and find the fraction which is the quotient of the 5th power of $\frac{2}{3}$ divided by the square of $\frac{8}{9}$.

3. Extract the square root of the following:

(1) 841.	(2) 1521.	(3) 8649.
(4) 11664.	(5) 60516.	(6) 142884.
(7) 540225.	(8) 667489.	(9) 175233494881.

4. Extract the square root of

(1) 32·49.	(2) 1·4161.	(3) 12·8164.
(4) 3782·25.	(5) ·00749956.	(6) ·0000974169.

5. Extract the square root of (giving in each case the *true* value of the remainder)

(1) 2 to six places of decimals.
(2) 7 to seven places of decimals.
(3) 10 to five places of decimals.
(4) 77 to six places of decimals.
(5) 126 to four places of decimals.
(6) 881 to five places of decimals.

6. Extract the square root of

(1) 10·3041.	(2) 29·506624.	(3) 1730·56.
(4) 290·225296.	(5) ·0004.	(6) ·00001024.

7. Extract the square root of 5, ·5, ·05, and ·005 each to 5 places of decimals.

8. Extract the square root of $\frac{14161}{14641}$; and of $\frac{2}{3}$ to seven places of decimals.

9. Extract the square root of ·16; also of ·016, and $\cdot\dot{4}$, each to three places of decimals.

10. Extract the square root of $\frac{167281}{169}$; and of $6\frac{7}{8}$ to 8 places of decimals; and of $\frac{3}{8}$ to 9 places of decimals.

11. What are the quantities of which ·001 and ·073 are the square roots?

12. Extract to three places of decimals the square roots of $\cdot\dot{3}\dot{4}$, and of $\cdot 1\dot{2}\dot{3}$; also the square root of $\cdot\dot{9}$.

13. Extract the cube root of

(1) 21952. (2) 185193. (3) 970299.
(4) 1367631. (5) 12812904. (6) 28934443.

14. Extract the cube root of

(1) ·193823. (2) 857·375. (3) 245·314376.
(4) ·065939264. (5) ·023639903. (6) ·000030664297.
(7) $\frac{343}{729}$. (8) $52034\frac{10}{27}$.
(9) ·1 to 4 places of decimals.

15. Extract the cube root of

(1) 2 to three places of decimals.
(2) 7 to eight places of decimals.
(3) 28 to three places of decimals.
(4) $\cdot\dot{8}$ to two places of decimals.
(5) ·065 to five places of decimals.
(6) ·05 to three places of decimals.
(7) ·7 to seven places of decimals.

16. Extract the fourth root of

(1) 923521. (2) 614656. (3) 1679616.

17. Determine the side of a square whose area is equal to a rectangle, which is 81 feet long and 5 feet $0\frac{3}{4}$ inches wide.

18. The area of a square field is 10 acres; what will it cost to build a wall round it at 4*s*. „ 9*d*. per square yard of walling, if the wall be 2 yards high?

19. The length of a metre is 39·37 inches; find the number of solid inches in a cube whose side is a metre; and find the length of the side of a cube which contains 8060150125 solid inches.

20. A body of men in column form 324 ranks 9 abreast; if they were drawn up in a solid square, how many would there be in each face?

21. If a cubic foot of water weigh 1000 ounces, find the length of the side of a cube of water, which weighs 1 ton „ 6 lbs. „ 1 oz.

22. How long will it take to walk along the four sides of a square field which contains 16 acres „ 401 square yards, at 3 miles per hour?

23. A certain number of persons agree to subscribe as many guineas each as there are subscribers in all; the whole subscription being £1047901 „ 1*s.*, how many subscribers were there?

24. Find the number of which ·0101 is the square root; and find the number which when squared gives ·010164.

25. The length of a room is twice its breadth, and its area contains 1152 square feet; what is the length of the room?

26. Find the hypotenuse of a right-angled triangle, whose sides are 35 inches and $42\frac{2}{3}$ inches. [Euclid, Bk. I. 47.]

27. Reduce to decimals $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, and find which is the greater $\sqrt[3]{2}$ or $\sqrt[3]{3}$.

28. Find the edge of a cube which contains 37 cubic feet „ 64 cubic inches.

29. There are 640 acres in a square mile; how many feet must there be in the side of a square field which contains $3\frac{1}{2}$ acres?

30. The edges of a rectangular chest which contains 64 cubic feet are in the proportion of 1, 2, 4; what are the actual lengths of the edges?

31. A certain cubical tank is found to hold the same quantity of water as a tank, the length, breadth, and depth of which are respectively 11 feet, 12 feet, and 10 feet „ 1 inch. What is the length of an edge of the cubical tank?

32. A piece of cloth is 5 times as long as broad, and costs £19; supposing the price to be 4*s.* „ 9*d.* a square yard, find the dimensions of the piece.

ANSWERS TO THE EXERCISES.

Exercise I. (Page 9.)

1. (1) 19006. (2) 1600402. (3) 8308791.
(4) 166402009. (5) 1000000000. (6) 2300000405607.

2. (1) One hundred and twenty-three million, four hundred and fifty-six thousand, seven hundred and eighty-nine.

(2) Nine thousand and nine million, nine thousand and nine.

(3) Seven hundred and seventy-seven million, seven hundred and seventy-seven.

(4) Eight hundred and ninety-six thousand seven hundred and eighty-seven million, five hundred and forty-two thousand, one hundred and thirty-four.

(5) Four billion, five hundred and sixty-three thousand two hundred and eighteen million, seven hundred and sixty-four thousand five hundred and twenty-nine.

(6) Three hundred and seventy-eight billion, six hundred and fifty-eight thousand four hundred and fifty-nine million, three hundred and seventy-two thousand one hundred and fifty six.

3. Cf. § 5, 6.

4. Cf. § 10. In the quinary scale by 12. If only 7 digits besides the cipher were used the scale would be the octenary, *i.e.* 8 would be written 10, and 13 would be written 15.

5. Cf. § 11, 12.

6. Cf. § 13, 14, and note on p. 4.

Exercise II. (Page 32.)

1. (1) 10964. (2) 766337. (3) 1727271.
(4) 12659262. (5) 9999999. (6) 338901713.
2. (1) 3175. (2) 4214. (3) 268586.
(4) 2. (5) 88408512. (6) 370651673.
3. First remainder 1767122. Second remainder 11000.
4. First remainder 2715952. Second remainder 15052.
5. (1) 3634496. (2) 839068254.
(3) 14984242647. (4) 183478853167.
(5) 6467176188. (6) 15160301184204.
6. (1) 35. (2) 405. (3) 462 with remainder 2140103.
(4) 20303 with remainder 8534579. (5) 9009.
(6) 8887 with remainder 136725.

Exercise III. (Page 33.)

(1) 12511. (2) £6 „ 6*s*. „ 7*d*.
(3) 339. (4) £1008 „ 13*s*. „ 7*d*.
(5) 2008360. (6) £5007 „ 4*s*.
(7) 370 quotient, 1568 remainder.
(8) 550974. (9) 7 times. (10) 1.
(11) 4305. (12) Cf. § 23; 3. (13) 176517.

(14) The general proposition is—"If to the sum of any two numbers there be added their difference, the result is equal to twice the greater of the numbers; but if from the sum there be subtracted the difference, the result is equal to twice the smaller number."

(15) £149. (16) £536 „ 10*s*. „ 8*d*.
(17) 45678, remainder 102. (18) £9410 „ 1*s*.
(19) £2650; and 1769 quarters. (20) 1*s*. „ 4*d*.
(21) £71 „ 10*s*.; and 14*s*. „ $3\frac{3}{5}$*d*. (22) £24 „ 3*s*. „ 4*d*.

(23) (1) 34, and remainder 4199. (2) 1032. (3) 443, and remainder 57.

(24) By dividing £6 by £3 we obtain 2, by dividing £3 by £6 we obtain $\frac{1}{2}$, and we may multiply 10 yards by either 2 or $\frac{1}{2}$.

EXERCISE IV. (Page 50.)

I. (1) 13. (2) 493. (3) 1235. (4) 4199. (5) 221. (6) 13.

II. (1) 10296. (2) 28152. (3) 722484. (4) 1779050.

III. (1) 37. (2) 40278. (3) 1912.
(4) 571. (5) 47. (6) 53.

IV. (1) 18522000. (2) 30968. (3) 119025.
(4) 7847. (5) 17748. (6) 61688187.

V. (1) $2^3 \times 3^2 \times 5^2$. (2) $2^3 \times 3^2 \times 7^2$. (3) $3 \times 5^2 \times 7^2 \times 11$.
(4) $2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 23$.

VI. (1) $2 \times 3^4 \times 7^3 \times 11 \times 29$, $2^3 \times 3^2 \times 7^2 \times 13 \times 23$, $2^4 \times 3^2 \times 7^2 \times 11 \times 31$. Hence the G.C.M. required is $2 \times 3^2 \times 7^2$, which is 882; and the L.C.M. is $2^4 \times 3^4 \times 7^3 \times 11 \times 13 \times 23 \times 29 \times 31$, which is 1314385280208.

(2) $2 \times 3^4 \times 11 \times 191$; $2 \times 3^2 \times 11 \times 191$; and $2 \times 3^5 \times 7 \times 11 \times 191$. Hence G.C.M. $2 \times 3^2 \times 11 \times 191$; and L.C.M. $2 \times 3^5 \times 7 \times 11 \times 191$; *i.e.* G.C.M. is 37818, and L.C.M. is 7147602.

VII. 720, and 1680. VIII. 23.

IX. 843. X. 362. XI. 57. XII. 4.

XIII. The 1st, 2nd, and 4th are divisible by 9 and 11; the 3rd and 5th by 8.

XIV. The 1st, 2nd, and 3rd are divisible by 7; the 2nd and 4th by 13.

EXERCISE V. (Page 66.)

I. (1) £26416 „ 1*s.* „ 11*d.* (2) £4077 „ 15*s.* „ 1*d.*
(3) £7447 „ 8*s.* „ 4*d.* (4) £5555 „ 3*s.* „ 6½*d.*
(5) £25978 „ 2*s.* „ 9*d.* (6) £29666 „ 7*s.* „ 8*d.*

II. (1) £2773 „ 11*s.* „ 1¾*d.* (2) £2105 „ 9*s.* „ 4¾*d.*
(3) £317 „ 6*s.* „ 9¾*d.* (4) £12116 „ 10*s.* „ 5¾*d.*

III. (1) £106561 „ 14*s.* „ 7½*d.* (2) £103419 „ 13*s.* „ 1½*d.*
(3) £89924 „ 0*s.* „ 8¼*d.* (4) £251782 „ 12*s.* „ 2½*d.*
(5) £2512853 „ 17*s.* „ 4*d.* (6) £978717 „ 18*s.* „ 8½*d.*
(7) £6678 „ 6*s.* „ 3¼*d.* (8) £5829 „ 16*s.* „ 11½*d.*
(9) £23308 „ 9*s.* „ 5¼*d.* (10) £1506 „ 15*s.* „ 10½*d.*
(11) £7911 „ 16*s.* „ 10½*d.* (12) £17726 „ 16*s.* „ 11¾*d.*

IV. (1) £3267 „ 3*s.* „ $7\frac{3}{4}$*d.* (2) £66 „ 18*s.* „ 3*d.* „ $1\frac{9}{13}$ *far.*
(3) £3 „ 3*s.* „ 11*d.* „ $1\frac{81}{82}$ *far.* (4) £7 „ 18*s.* „ 6*d.* „ $1\frac{307}{379}$ *far*
(5) £10 „ 0*s.* „ 8*d.* „ $2\frac{294}{401}$ *far.* (6) £44 „ 9*s.* „ 0*d.* „ $2\frac{5}{1307}$ *far.*

V. (1) £4 „ 2*s.* „ 6*d.* (2) £659 „ 12*s.* „ 6*d.*
(3) £734 „ 14*s.* „ 8*d.* (4) 16*s.* „ 8*d.*
(5) £6 „ 6*s.* „ $10\frac{1}{4}$*d.* (6) £30979 „ 9*s.* „ 3*d.*

VI. £4 „ 11*s.* „ 0*d.* VII. £1 „ 6*s.* „ $4\frac{3}{4}$*d.*

VIII. 2*s.* „ 9*d.* IX. £62403 „ 15*s.*

X. £2 „ 14*s.* „ 9*d.* „ $2\frac{10}{13}$ *far.*

XI. 4*s.* „ 9*d.* „ $1\frac{13}{23}$ *far.*

XII. £111 „ 14*s.* „ 8*d.*; and $8\frac{3}{52}$*d.*

Reduction.

XIII. (1) 22686. (2) 48001. (3) 61546. (4) £10 „ 5*s.* (5) £4 „ 13*s.* „ $6\frac{1}{2}$*d.* (6) 5040. (7) 962. (8) 1085. (9) 5760. (10) 96. (11) 3744. (12) $1248\frac{3}{4}$. (13) 24000. (14) 4800. (15) 84. (16) 84. (17) £5 „ 16*s.* „ 11*d.* (18) 6400. (19) 882. (20) 3923. (21) 38220. (22) 40. (23) 37. (24) 64.

XIV. Avoirdupois:

	tons	cwt.	qrs.	lbs.	oz.	drs.
(1)	40 „	11 „	3 „	26 „	12.	
(2)	12 „	17 „	0 „	6 „	5 „	2.
(3)	3 „	16 „	3 „	25 „	2.	
(4)	8 „	2 „	0 „	21 „	10 „	11.
(5)	227 „	4 „	0 „	22 „	0.	
(6)	4395 „	2 „	3 „	20 „	8 „	12.
(7)				16 „	6 „	7.
(8)		14 „	0 „	10 „	2.	

XV. Troy Weight:

	lbs.	oz.	dwt.	grs.		lbs.	oz.	dwt.	grs.
(1)	11 „	5 „	13 „	15.	(2)	29 „	1 „	18 „	11.
(3)		6 „	10 „	20.	(4)	6 „	5 „	2 „	7.
(5)	807 „	4 „	8 „	18.	(6)	400 „	10 „	1 „	18.
(7)		11 „	9 „	6.	(8)		6 „	15 „	$21\frac{7}{30}$.

XVI. **Apothecaries' Weight:**

	lbs.	oz.	drs.	scru.	grs.
(1)	19 „	6 „	0 „	1 „	3.
(2)	49 „	5 „	7 „	0 „	9.
(3)		11 „	7 „	2 „	19.
(4)	198 „	7 „	5 „	1 „	19.
(5)	219 „	9 „	4 „	0 „	0.
(6)	1355 „	3 „	0 „	1 „	0.
(7)			3 „	1 „	17.
(8)			3 „	2 „	1.

XVII. **Long Measure:**

	m.	fur.	pl.	yd.	ft.	in.	
(1)			19 „	0 „	1 „	11.	
(2)	34 „	7 „	7 „	2 „	1 „	10.	
(3)		4 „	31 „	$2\frac{1}{2}$ „	1 „	6,	*i.e.* 4 „ 31 „ 3 yds.
(4)	161 „	7 „	39 „	$4\frac{1}{2}$ „	2 „	11,	*i.e.* 161 „ 7 „ 39 „ 5 „ 1 „ 5.

	fur.	pl.	yd.	ft.	in.
(5)	9 „	1 „	1 „	1 „	6.
(7)				2 „	5.

	m.	fur.	pl.	yd.	ft.	in.
(6)	96 „	6 „	6 „	1 „	1 „	6.
(8)			30 „	4 „	0 „	6.

Exercise VI. (Page 90.)

Addition of Fractions:

(1) $1\frac{73}{120}$. (2) $2\frac{6}{7}$. (3) $1\frac{53}{168}$. (4) $\frac{298}{315}$. (5) $23\frac{143}{420}$.
(6) $1\frac{277}{528}$.

Subtraction of Fractions:

(1) $\frac{8}{17}$. (2) $2\frac{79}{126}$. (3) $\frac{2}{15}$. (4) $\frac{11}{24}$. (5) $2\frac{1}{70}$.

Multiplication of Fractions:

(1) $1\frac{3}{17}$, $\frac{20}{51}$, $\frac{20}{357}$, $2\frac{38}{51}$. (2) $\frac{33}{128}$. (3) $\frac{5}{12}$.
(4) $\frac{1}{2}$. (5) $2\frac{51}{200}$.

Division of Fractions:

(1) $\frac{1}{15}$, $\frac{27}{35}$, $\frac{27}{385}$, $8\frac{17}{35}$. (2) $2\frac{577}{919}$. (3) $2\frac{8}{304}$.
(4) The latter is the larger by $\frac{639}{1012}$.

Miscellaneous Examples.

(1) $\frac{1}{9}$ and $\frac{64}{4913}$. (2) $1\frac{13}{120}$; also $\frac{3}{8}$ of $\frac{5}{6}$ is greater by $\frac{1}{720}$.

(3) $\frac{2}{5}$. (4) The fractions become $\frac{70}{105}$, $\frac{84}{105}$, $\frac{90}{105}$.

(5) $\frac{7}{9}$. (6) $\frac{54}{43}$.

(7) Product $\frac{5}{6}$, quotient $\frac{169}{180}$; of which the latter is the greater, and the difference in the lowest term is $\frac{5}{36}$. (8) $\frac{8}{9}$.

(9) $\frac{1}{864}$. (10) $\frac{2}{3}$. (11) $\frac{38}{49}$. (12) $\frac{19}{48}$. (13) $\frac{29}{63}$.

(14) $13\frac{27}{32}$. (15) $\frac{42}{315}$, $\frac{225}{315}$, $\frac{81}{315}$; the sum $1\frac{11}{105}$.

(16) $1\frac{11}{105}$. (17) $\frac{1}{24}$ and $\frac{2}{35}$.

(18) Sum $\frac{45}{26}$; difference $\frac{9}{26}$. (19) $\frac{1}{48}$. (20) $\frac{15}{16}$.

(21) $\frac{5}{18}$. (22) 1. (23) $1\frac{884}{997}$. (24) $\frac{2}{7}$.

(25) $2\frac{4}{17}$, and $2\frac{15}{53}$. (26) $\frac{26}{35}$. (27) Sum is 8, quotient $5\frac{5}{7}$.

(28) 4. (29) $\frac{11}{36}$. (30) 1. (31) $\frac{52}{219}$, $\frac{468}{2555}$, and $\frac{27}{35}$.

(32) Sum is 5, difference $\frac{1}{45}$, quotient 225. (33) $10\frac{9}{2000}$.

(34) $\frac{3}{11}$. (35) $\frac{1}{69120}$. (36) $\frac{13}{51}$.

Exercise VII. (Page 100.)

(1) $\frac{479}{480}$. (2) $\frac{10}{13}$. (3) £1 „ 6*s.* „ $0\frac{1}{4}$*d.*

(4) $\frac{4}{3}$. (5) 12*s.* „ $2\frac{1}{2}$*d.* (6) $\frac{2}{245}$. (7) $\frac{7}{40}$.

(8) £6 „ 11*s.* (9) $\frac{3}{5}$. (10) $\frac{542}{675}$. (11) $\frac{1}{7}$.

(12) $\frac{5}{126}$; and 3 hours „ 36 min. (13) 10*s.* „ 11*d.* „ $2\frac{8}{35}$ *far.*

(14) $\frac{1}{256}$. (15) $\frac{21}{800}$, and 3 qrs. „ 21 lbs.

(16) The relative values are as 62, 61, 54 (the absolute values being 15*s.* „ 6*d.*, 15*s.* „ 3*d.*, and 13*s.* „ 6*d.*).

(17) $\frac{1}{1120}$, and $\frac{9}{32}$. (18) 4*s.* „ $9\frac{1}{2}$*d.* (19) £1 „ 7*s.* „ 3*d.*

(20) £1 „ 8*s.* (21) 13*s.* „ 4*d.*

(22) The relative values are as 45, 46, 47, (the absolute values being 1*s.* „ $10\frac{1}{2}$*d.*, 1*s.* „ 11*d.*, and 1*s.* „ $11\frac{1}{2}$*d.*).

(23) $\frac{7}{60}$. (24) 16 cwt. (25) $\frac{1}{64}$ mile. (26) $\frac{1}{14}$, and $\frac{72}{1225}$. (27) $\frac{1}{12}$ of a week; and 8 min. „ 15 sec.

(28) 31 square inches. (29) $\frac{1}{60}$ of a furlong.

(30) $\frac{13}{32}$ of a quarter. (31) $\frac{7}{1860}$.

(32) 199 qrs., and $\frac{100}{17}$ yds. (33) $5\frac{31}{37}$ square inches.

(34) $\frac{1}{84480}$. (35) $\frac{526}{875}$. (36) $\frac{20125}{36864}$.

Exercise VIII. (Page 108.)

(1) Cf. § 64. (2) Cf. § 68.

(3) Two hundred and eighty-three *thousandths;* five thousand three hundred and twenty-one *ten thousandths;* seventy-four thousand eight hundred and ninety-five *hundred thousandths;* eight hundred and twenty-one thousand and fifty-six *millionths;* twenty-seven, together with eight thousand three hundred and fifty-four *ten thousandths;* thirty-four, together with nine *ten thousandths;* forty-three, together with one hundred and one thousand and seven *millionths.*

(4) 53·9; 47·73; 6·0069; 1·000001; 3·7; 35·721341; 9·000400537.

(5) $\frac{7}{10}$; $\frac{7}{100}$; $\frac{7}{1000}$; $\frac{7}{1000000}$; $\frac{327}{1000}$; $\frac{327}{100}$; $\frac{327}{10}$; $\frac{45697}{100000}$; $\frac{45697}{100}$; $\frac{893}{1000}$; $\frac{893}{10000000}$.

(6) ·073; ·0197; ·000001; ·00261; ·0001001.

(7) $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{1}{20}$; $\frac{1}{40}$; $\frac{1}{5}$; $\frac{1}{500}$; $\frac{3}{8}$; $\frac{127}{2000}$; $\frac{1001}{200000}$; $\frac{5907}{125}$.

(8) 3·79, 37·9, 379.

(9) ·00703, ·0000703, ·00000703

(10) Cf. § 66.

(11) $\frac{73}{200}$; $\frac{1}{8}$; $\frac{7}{2000}$; $\frac{3}{250}$; $\frac{7}{4}$; $\frac{669}{80}$.

(12) ·7453, 7·453, 74·53; 48956·21, 489562·1, 4895621; 8764·30071, 87643·0071, 876430·071; also ·000531674, ·0000531674, ·00000531674; ·000000000317, ·0000000000317, ·00000000000317; 902·030401, 90·2030401, 9·02030401.

Exercise IX. (Page 122.)

1. Addition:

(1) ·59327.	(2) 2·919563.	(3) 554·40861.
(4) 3·41203.	(5) 9111·0257962.	(6) 1934·16261.
(7) 3030·30303.	(8) 1809·0998.	(9) 58624·27673.
(10) 73088·4933367.		

2. Subtraction:

(1) 3·431.	(2) ·0011.	(3) 39·8489194.
(4) ·336606.	(5) 8·9524.	(6) 6·999613.
(7) ·00027.	(8) ·0000046.	(9) ·45678.
(10) 91·48873.		

3. Multiplication:

(1) ·0000378.	(2) ·1487992.	(3) 2·71984.
(4) ·0058028.	(5) ·00924397488.	(6) ·00003738028.
(7) 9864·16[illegible]175.	(8) 586·3672853.	(9) ·93586685.
(10) ·0237666.	(11) 220073·2212.	(12) 1870950·4885448.

4. Division:

(1) 711·858.	(2) 2280·28.	(3) 234508.
(4) 12500.	(5) ·0001.	(6) 15000.

(7) ·013. (8) 4·57. (9) ·008.
(10) ·050005. (11) 70·299. (12) 66·39.
(13) ·61164. (14) 830000. (15) ·008765.
(16) 100. (17) ·00293. (18) 23·84.
(19) ·039. (20) 925·4. (21) 3·424, &c.
(22) ·00879, &c.

5. Contracted multiplication:

(1) ·010271. (2) 12·60295. (3) 46·90415.
(4) ·100175. (5) 26·38702.

6. Contracted division:

(1) 3·77192. (2) 13·54909. (3) 3·46410.
(4) 3·19467. (5) 704·66269. (6) ·03749.

Exercise X. (Page 142.)

1. (1) ·17708$\dot{3}$. (2) ·2375. (3) ·88125.
(4) ·96875. (5) ·598958$\dot{3}$.
2. (1) 7*s*. „ 6*d*. (2) 3*s*. „ 4*d*. (3) 18*s*. „ 7*d*.
(4) 13*s*. „ 7½*d*. (5) 15*s*. „ 9*d*. „ 2·16 *far*.
3. ·6; also ·12. 4. ·325. 5. ·$\dot{7}$. 6. 9*s*. „ 10·16*d*.
7. ·248958$\dot{3}$; and 2*d*. „ 1·3744 *far*. and ·286458$\dot{3}$.
8. 7½*d*. 9. ·000372. 10. ·$\dot{5}3846\dot{1}$.
11. 3*s*. „ 4½*d*. 12. 3 flor. „ 2 cent. „ 5 mils.
13. (1) 14*s*. „ 10*d*. (2) 17*s*. „ 7*d*. (3) 6*s*. „ 3¼*d*.
(4) 11*s*. „ 8½*d*. (5) 1*s*. „ 2½*d*.
14. (1) 15*s*. „ 3½*d*. (2) 19*s*. „ 11¾*d*. (3) 7*s*. „ 4¼*d*.
(4) £1 „ 9*s*. „ 5*d*. (5) 12*s*. „ 5½*d*. (6) 11*s*. „ 8¾*d*.
15. (1) 4fl. „ 0c. „ 9m. (2) 8fl. „ 6c. „ 1m. (3) 1fl. „ 9c. „ 7m.
(4) 9fl. „ 9c. „ 5m. (5) 6fl. „ 3c. „ 4m. (6) 7fl. „ 9c. „ 1m.

Exercise XI. (Page 143.)

1. ·1875, ·225, ·208, ·136, ·848, ·0176, 4·9. 2. 1·4390625.
3. ·0$\dot{1}\dot{8}$, ·06$\dot{1}\dot{8}$, ·0$\dot{0}6\dot{6}$, ·23$\dot{4}\dot{5}$, 1·29$\dot{3}\dot{1}$, 46·$\dot{3}1\dot{2}$, ·61$\dot{7}2\dot{6}$.
4. 1·69$\dot{1}7027\dot{4}$.
5. $\frac{1}{2}$, $\frac{1}{200}$, $\frac{1}{40}$, $\frac{1}{400}$, 7½, $\frac{3}{40}$, $\frac{1}{250}$, $\frac{2}{5}$, $\frac{13}{1280}$, $\frac{2863}{4000}$, $\frac{2863}{400000}$.
6. ·00000208, 1·487992, ·271984, ·005334, ·61915, ·003738028.

7. 500, 1200, 150000, 234·508, 2280·28, 7118·58.

8. $7{\cdot}\dot{3}$, $\cdot 1\dot{0}\dot{5}$, 4·9, $30{\cdot}1\dot{7}1428\dot{5}$. 9. 2375, ·1, $\cdot 0\dot{6}$, ·1875.

10. ·2666204, ·666551, ·00021329632. 11. ·3.

12. £1 „ 13*s*., and 2·0625. 13. $16{\cdot}2\dot{0}5916\dot{3}$. 14. $\cdot\dot{1}9047\dot{6}$.

15. 13 hours „ 10 min. „ $5\frac{1}{4}$ sec. 16. 29 yds. „ 2 ft. „ $0\frac{3}{5}$ in.

17. $5\frac{19}{70}$, and $5{\cdot}2\dot{7}1428\dot{5}$. 18. ·51454246976, &c.

19. ·4375, and $1{\cdot}0049\dot{2}\dot{4}$.

20. 7 miles „ 7 furlongs „ 152·052 yards.

21. 2 acres „ 1 ro. „ 39 po. „ 20 sq. yds. „ 2 sq. ft. „ $36\frac{1}{2}$ (nearly) sq. inches.

22. ·4921875. 23. 2*s*. „ $1\frac{1}{2}$*d*. 24. 36.

25. 8 flo. „ 7 cent. „ $6{\cdot}041\dot{6}$ mils ; and 16*s*. „ 8*d*. „ 3·52 *far*.

26. Sum is £1·865, product is £55·95, and this is £55 „ 19*s*.

27. Each share is £7 „ 10*s*. „ 2*d*. „ 3·52 *far*.

28. Each is 19*s*. „ 6*d*.

Exercise XII. (Page 148.)

1. ·914382 metre ; and 1·609314 kilom.

2. 10·9363305 yds. ; and 6·213824 miles.

3. 80·46575 kilom. ; and 49·71059 miles.

4. 201·164314 metres ; and 10000

5. (1) 17*s*. „ $7\frac{1}{4}$*d*. (2) £3 „ 5*s*. „ $2\frac{1}{2}$*d*. (3) £3 „ 13 „ $2\frac{1}{2}$*d*. (4) £10 „ 15*s*. „ $9\frac{1}{2}$*d*. (5) £2 „ 15*s*. „ $7\frac{1}{4}$*d*. (6) £8 „ 13*s*. „ 9*d*.

6. (1) 21·875. (2) 35·312. (3) 42·187. (4) 120. (5) 170. (6) 125. (7) 1085.

7. £991 „ 4*s*. „ $4\frac{1}{2}$*d*. ; and 1261·075 francs.

8. He gained £3 „ 10*s*. „ $3\frac{1}{4}$*d*. 9. ·404671 hectare.

10. 74·13429 acres. 11. 5·6072438. 12. 2·85119.

13. 22·009625. 14. 2·60965. 15. £127 „ 11*s*. „ $7\frac{1}{2}$*d*.

16. $\frac{1}{2}$ pint short. 17. 2751·203125.

18. 3·732419 ; and ·453592. 19. 3 qrs. „ 10·2475 lbs.

20. 506·34547. 21. ·998339. 22. 91·9075.

23. 69 lbs. „ $13\frac{3}{4}$ oz. (nearly) ; and 3 miles „ $164\frac{1}{4}$ yds.

24. 60·79559 kilogs. ; and 5431·43649 metres.

25. ·99412. 26. 24855·295.

27. 7898·13 miles; and 11738343·61 metres.

28. ·764513; and 142·382044.

29. 321 cub. yds. „ 19·11 &c. cub. feet. 30. £547 „ 13*s.*

31. 1296$\frac{3}{4}$ miles. 32. 357 yds. 2 feet; and 834 sq. yds.

EXERCISE XIII. (Page 158.)

1. £17. 2. £378. 3. £1605. 4. £4628.

5. £965. 6. £7281 „ 1*s.* „ 3*d.* 7. 9*s.* „ 10$\frac{3}{4}$*d.*

8. £836 „ 16*s.* „ 4$\frac{7}{8}$*d.* 9. £18 „ 17*s.* „ 2$\frac{1}{4}$*d.*

10. £117 „ 0*s.* „ 3*d.*

11. £1657 „ 5*s.* „ 6$\frac{1}{2}$*d.* (nearly).

12. £4612 „ 14*s.* „ 0$\frac{3}{4}$*d.*

13. £21 „ 18*s.* „ 4$\frac{41}{56}$*d.* 14. £49 „ 5*s.* „ 9$\frac{15}{28}$*d.*

15. £140 „ 18*s.* „ 4$\frac{13}{112}$ *d.* 16. £89 „ 6*s.* „ 1$\frac{1}{8}$*d.*

17. £86. 18. £19 „ 1*s.* „ 10$\frac{1}{2}$*d.*

19. £696 „ 6*s.* „ 4$\frac{1}{20}$*d.* 20. £187 „ 18*s.* „ 10*d.* „ 3$\frac{1}{2}$ *far.*

21. £30 „ 14*s.* „ 11$\frac{1}{8}$*d.* 22. £330 „ 14*s.* „ 6$\frac{1}{4}$*d.* (nearly).

23. £1124 „ 9*s.* „ 2$\frac{1}{2}$*d.* (nearly). 24. £23 „ 18*s.* „ 4$\frac{7}{8}$*d.*

25. £4412 „ 12*s.* „ 3$\frac{3}{4}$*d.* 26. £98 „ 5*s.* „ 4$\frac{1}{8}$*d.*

27. £23 „ 14*s.* „ 0$\frac{3}{4}$*d.* 28. £225 „ 13*s.* „ 3$\frac{1}{4}$*d.*

29. £513 „ 6*s.* „ 6$\frac{1}{4}$*d.*

30. (1) £2717 „ 2*s.* (2) £982 „ 6*s.* (3) £164 „ 8*s.*
(4) £818 „ 8*s.*

31. £736 „ 13*s.* „ 3$\frac{1}{4}$*d.* 32. £61412 „ 15*s.* „ 3$\frac{1}{2}$*d.*

33. £26 „ 7*s.* „ 8$\frac{3}{4}$*d.* 34. 8*s.* „ 5*d.*

35. £5 „ 7*s.* „ 9$\frac{1}{4}$*d.* 36. £91 „ 14*s.* „ 10$\frac{1}{2}$*d.*

EXERCISE XIV. (Page 183.)

1. 7697. 2. 16*s.* „ 6$\frac{3}{4}$*d.* 3. £39 „ 6*s.* „ 11*d.*

4. £59 „ 14*s.* „ 8*d.* 5. £499 „ 0*s.* „ 5$\frac{19}{121}$*d.*

6. £2756 „ 5*s.*, or 2625 guineas. 7. 17*s.* „ 4*d.*

8. £179 „ 4*s.* 9. £840. 10. £1000.

11. £178 „ 1*s.* „ 6*d.* 12. 13 feet „ 2·382 inches.

13. 420 revolutions. 14. £21 „ 15*s.* „ 9$\frac{3}{4}$*d.* 15. £172.

16. $8d$. 17. £4 „ $15s$. „ $10\frac{1}{2}d$. 18. £7 „ $19s$. „ $10\frac{50}{149}d$.
19. £49 „ 9 flo. „ 5 cents „ 5 mils. 20. £16 „ $16s$. „ $10\frac{14}{189}d$.
21. £3 „ $13s$. „ $2\frac{6}{35}d$. 22. 21 lbs. „ 5 oz. „ 16 dwts. „ 6 grs.
23. £5 „ $16s$. „ $0\frac{48}{55}d$. 24. 45 feet. 25. $79\frac{7}{11}$ feet.
26. 14. 27. 361. 28. 22·543, &c. 29. $\frac{1}{2}$.
30. 13 „ $13s$. „ $0\frac{4}{5}d$. 31. $4\frac{477}{875}d$., or $4\frac{1}{2}d$. nearly.
32. £2499. 33. 2 tons „ 9 cwt. „ 1 qr. „ 11·2 lbs.
34. 1 min. „ $40\frac{4}{5}$ sec. 35. £22. 36. £562 „ $19s$. „ $1\frac{37}{47}d$.
37. $7d$. 38. £160 „ $17s$. „ $2\frac{3}{4}d$. 39. 21 men.
40. 4 weeks „ 2 days. 41. 15 men. 42. 14 men.
43. 12 hours. 44. £385. 45. 4400 men.
46. 9 months. 47. 75 lbs. 48. £114 „ $6s$.
49. $18\frac{156}{511}$ miles. 50. $6\frac{2}{3}$ hours. 51. 9 days.
52. 15 days. 53. 7 months. 54. $1968\frac{3}{4}$ lbs.
55. Breadth 4 yds., length 20 yds. 56. $802{\cdot}15\dot{6}$ yds.
57. 5 days. 58. 3000. 59. 15 hours. 60. 126 days.
61. $2531\frac{1}{4}$ dinaras. 62. $12\frac{43}{256}$ dronas. 63. 108 days.
64. $1\frac{1}{2}$ days. 65. 4 cwt. „ 2 qrs. „ $17\frac{97}{103}$ lbs.
66. $4677\frac{1}{2}$ yds. 67. 6·3 days. 68. 2 months.
69. £6 „ $13s$. „ $4d$. 70. 19·36 days. 71. 4·02 inches.
72. 268800. 73. 105 days. 74. $32s$.
75. £7 „ $0s$. „ $7\frac{1}{2}d$. 76. They did $\frac{6}{7}$ and $\frac{1}{7}$ respectively.
77. 24. 78. £1458 „ $12s$. 79. $74\frac{2}{27}$ hours.
80. £11 and £70. 81. £132. 82. $2s$. „ $4d$.

EXERCISE XV. (Page 196.)

1. 40 and 60. 2. 21, 15, 9. 3. 1140, 855, 684.
4. £8048 „ $15s$. „ $6d$. and £13414 „ $12s$. „ $6d$.
5. 1122, 1726. 6. 18 brandy, 45 wine, 60 water.
7. £$255\frac{27}{203}$, £$223\frac{40}{203}$, £$201\frac{33}{203}$, £$150\frac{94}{203}$.
8. £343 „ $9s$. „ $6\frac{102}{237}d$., £628 „ $9s$. „ $9\frac{111}{237}d$., £760 „ $0s$. „ $8\frac{24}{237}d$.
9. £23 „ $2s$., £34 „ $13s$., £69 „ $6s$.
10. The share of each man is $16s$. „ $8d$., of each woman $10s$., of each child $6s$. „ $8d$.
11. £1 „ $1s$., £1 „ $11s$. „ $6d$., £2 „ $12s$. „ $6d$.

12. The canna of cloth was worth 3 florins, the lira of saffron 2 florins.

13. 1st butt, $12\frac{24}{25}$ gals. Mal., $8\frac{16}{25}$ Gk. wine, $14\frac{3}{5}$ Rom.
2nd butt, $8\frac{16}{25}$ ...., $5\frac{19}{25}$ ...., $9\frac{3}{5}$ ......
3rd butt, $14\frac{2}{5}$ ...., $9\frac{3}{5}$ ...., 16 ......

14. By friar $26\frac{36}{41}$ soldi, by barber $111\frac{21}{41}$ soldi, by artisan $350\frac{22}{41}$ soldi, by gentleman $711\frac{3}{41}$ soldi.

15. *A*'s share was £1548, *B*'s share was £1989.

16. Each man has £$29\frac{7}{17}$, each woman £$14\frac{12}{17}$, each child £$4\frac{36}{51}$.

17. Each child £1 „ 16*s*., each woman £3, each man £4 „ 4*s*.

18. 1 man. 19. 51 oz.; and the value, at 5*s*. „ 6*d*. per oz., is £14 „ 0*s*. „ 6*d*.

20. 184 oz. silver, 32 oz. gold; and 5*s*. „ $1\frac{1}{2}$*d*. 21. 188 oz.

22. 80 gallons brandy, 40 gallons water.

23. 60 gallons wine, 20 gallons water. 24. 70 gallons.

25. $\frac{5}{12}$. 26. *A*'s share $\frac{9}{23}$, *B*'s share $\frac{6}{23}$, *C*'s share $\frac{8}{23}$.

27. $\frac{1}{3}$.

Exercise XVI. (Page 211.)

1. £2 „ 16*s*. „ 11*d*. „ $2\frac{1}{5}$ *far*.
2. £446 „ 8*s*. „ $9\frac{15}{73}$*d*.
3. £436 „ 7*s*. „ $6\frac{9}{125}$*d*.
4. £1 „ 13*s*. „ 7·1424*d*.
5. £8 „ 8*s*. „ $4\frac{1}{5}$*d*.
6. £20 „ 15*s*. „ $0\frac{3}{5}$*d*.
7. £1213 „ 10*s*. „ 9*d*.
8. £69 „ 10*s*.; and £347 „ 10*s*.
9. £212 „ 10*s*.
10. £1516 „ 10*s*. „ $3\frac{3}{8}$*d*.
11. £394 „ 8*s*. „ $0\frac{1}{2}$*d*. nearly.
12. £10 „ 1*s*. „ $8\frac{1}{4}$*d*. nearly.
13. £44 „ 6*s*. „ $2\frac{2}{5}$*d*.
14. £3 „ 11*s*. „ 9*d*.
15. £402 „ 10*s*.
16. £1 „ 0*s*. „ $8\frac{1}{4}$*d*.
17. £1118 „ 15*s*.
18. £160.
19. £3 „ 12*s*. „ $2\frac{23}{73}$*d*.
20. £13692 „ 6*s*. „ $1\frac{11}{13}$*d*.
21. £6 „ 10*s*. „ $10\frac{1}{4}$*d*.
22. £7 „ 11*s*. „ $0\frac{3}{5}$*d*., and £18 „ 8*s*. „ 9*d*.
23. $3\frac{13}{20}$ per cent.
24. £120 „ 10*s*.
25. $\frac{3}{4}$ of a year.
26. $3\frac{1}{4}$ per cent.
27. $4\frac{1}{4}$ years.
28. £8500.
29. $3\frac{1}{2}$ per cent.
30. $5\frac{1}{2}$ years.
31. £776.
32. $3\frac{1}{3}$ per cent.
33. 3 years, 7 months.
34. £142 „ 10*s*.
35. £573 „ 6*s*. „ 8*d*.
36. $4\frac{1}{2}$ per cent.

Exercise XVII. (Page 213.)

N.B. The fractional parts of the farthing are mostly omitted.

1. £267 „ 16*s.* „ $1\frac{1}{2}d.$
2. £787 „ 8*s.* „ 1·11*d.*
3. £1215 „ 10*s.* „ $1\frac{4}{5}d.$
4. £411 „ 16*s.* „ $8\frac{3}{4}d.$
5. £24 „ 14*s.* „ 7·115*d.*
6. £2510 „ 3*s.* „ 6·288*d.*
7. £61 „ 4*s.*; also £811 „ 16*s.* „ $5\frac{3}{4}d.$
8. £1389 „ 3*s.*
9. £6 „ 2*s.* „ $4\frac{1}{4}d.$
10. £607 „ 8*s.* „ $6\frac{1}{4}d.$
11. £150 „ 9*s.* „ $9\frac{3}{4}d.$
12. £899 „ 12*s.* „ 5*d.*
13. £1 „ 2*s.* „ $2\frac{1}{4}d.$
14. £160.
15. £6 „ 16*s.* „ $4\frac{1}{4}d.$
16. £155 „ 7*s.* „ $5\frac{1}{2}d.$
17. £1 „ 1*s.* „ $11\frac{1}{4}d.$
18. £270.
19. £458 „ 7*s.* „ $10\frac{1}{4}d.$
20. £255 „ 11*s.* „ $2\frac{1}{4}d.$
21. £29 „ 4*s.* „ $3\frac{1}{4}d.$
22. £1214 „ 17*s.* „ $0\frac{3}{4}d.$
23. £276 „ 2*s.* „ $11\frac{1}{2}d.$
24. £1200.
25. £725.

Exercise XVIII. (Page 223.)

1. £15 „ 3*s.* „ $7\frac{31}{33}d.$
2. £3 „ 10*s.*
3. £70.
4. £1 „ 7*s.* , $10\frac{3}{4}d.$
5. £16 „ 1*s.* „ $2\frac{2}{137}d.$
6. £4 „ 16*s.* „ $9\frac{8}{31}d.$
7. $3\frac{149}{505}d.$
8. £734 „ 3*s.* „ $0\frac{1}{4}d.$
9. £3 for both.
10. £11 „ 3*s.* „ $4\frac{48}{77}d.$
11. £1 „ 4*s.* „ $10\frac{1}{5}d.$
12. £49 „ 10*s.* „ $4\frac{3}{4}d.$
13. £558 „ 0*s.* „ $11\frac{1}{2}d.$
14. £116 „ 2*s.* „ $6\frac{30}{31}d.$
15. £1 „ 18*s.* „ $4\frac{4}{23}d.$
16. £45 „ 2*s.*
17. 7·821 pence, or $7\frac{3}{4}d.$ nearly.
18. £26 „ 17*s.* „ 6*d.*, and £125.
19. £1216 „ 13*s.* „ 4*d.*
20. £70 „ 19*s.* „ $6\frac{1}{2}d.$
21. £164 „ 10*s.* „ $3\frac{27}{31}d.$
22. £1 „ 1*s.* „ $3\frac{3}{4}d.$
23. £676 „ 19*s.* „ $7\frac{125}{821}d.$
24. $3\frac{3}{4}.$
25. £239 „ 19*s.* „ 6*d.*
26. £82 „ 5*s.* „ $1\frac{29}{31}.$
27. £85000.
28. £117 „ 14*s.* „ 4*d.* nearly.
29. £750.
30. £43 „ 9*s.* „ $4\frac{3}{4}d.$
31. £500
32. $3\frac{1}{4}$ per cent.
33. $3\frac{1}{2}$ years.

34. 4 per cent. 35. £676 „ 13*s*. „ 4*d*. 36. 15 months.

37. 4 per cent. 38. £520 ; and 6 per cent.

39. Cf. § 99 and § 102. 40. 16 months.

41. £9 „ 12*s*. „ $3\frac{8}{13}d$.; and £9 „ 16*s*. 42. £$16\frac{11}{13}$, and £16.

Exercise XIX. (Page 228.)

1. 1 year „ 4 months. 2. $11\frac{1}{3}$ months.

3. 1 year „ 9 months. 4. £666 „ 6*s*. „ 8*d*.

Exercise XX. (Page 241.)

1. 1700 stock. 2. £1410. 3. £32 „ 9*s*. „ $6\frac{234}{735}d$.

4. £25 „ 9*s*. „ $2\frac{230}{707}d$. 5. £3000. 6. $3\frac{27}{91}$.

7. 3750 stock, and the diminution of income £7 „ 10*s*.

8. £18. 9. 12*s*. „ $11\frac{65}{77}d$. difference. 10. £4200.

11. £3025. 12. $3\frac{7}{31}$. 13. 1200 stock.

14. £1658 „ 5*s*. 15. The latter. 16. $114\frac{27}{43}$.

17. £33 „ 17*s*. „ $11\frac{85}{89}d$.

18. £116 „ 14*s*. In the latter the income would be £124 „ 3*s*.

19. £34 „ 7*s*. „ $9\frac{61}{63}d$. 20. £2 „ 10*s*. 21. $5\frac{95}{749}$.

22. £818 „ 8*s*. 23. An increase of £15 „ 3*s*. „ $0\frac{4}{11}d$.

24. 30000. 25. £58 „ 15*s*. 26. £1 „ 18*s*. „ $6\frac{3}{4}d$. nearly.

27. $1333\frac{1}{3}$. 28. £53 „ 13*s*. „ $1\frac{103}{221}d$. 29. $81\frac{1}{3}$.

30. £2612. 31. 4800, and £134 „ 8*s*.

32. Diminish it by 10·392*d*.

33. Income £46 „ 7*s*. „ $10\frac{2}{97}d$., loss £54 „ 2*s*. „ $5\frac{87}{97}d$.

34. £1350 invested. Increase in income £9.

35. 13*s*. „ $2\frac{2}{81}d$. 36. $3\frac{68}{233}$. 37. $96\frac{1}{4}$.

38. The rates are $\frac{8}{3}$ and $\frac{100}{27}$, which are as 18 : 25.

39. 1000 stock.

40. The rates are $4\frac{4}{249}$ and $4\frac{9}{31}$, and these are as 6800 : 7221.

41. As 3495 : 2778, or as 39 : 31 nearly. 42. 12600 stock.

EXERCISE XXI. (Page 249.)

1. 19758 florins „ 3 stivers. 2. £734 „ 4*s*. 3. 4*s*. „ 2*d*.
4. $6\frac{160}{161}$ per cent. 5. $173\frac{1}{13}$. 6. 648.
7. £5 „ 13*s*. „ 4*d*. loss.

EXERCISE XXII. (Page 256.)

1. 24*s*. 2. 11*s*. 3. $16\frac{2}{3}$ per cent.
4. 60 per cent. I gained, and he lost 25 per cent.
5. $16\frac{2}{3}$. 6. $14\frac{7}{12}$. 7. 4*s*. „ $7\frac{1}{5}$*d*.
8. The former is better than the latter in the ratio of 9 : 8.
9. $12\frac{1}{2}$. 10. 18*s*. „ 4*d*. 11. $7\frac{2}{7}$*d*. 12. 70 per cent.
13. 1 lb. of cheaper to 2 lbs. of dearer tea. 14. 6*s*. „ $8\frac{61}{115}$*d*.
15. 3 : 13. 16. $41\frac{6}{431}$*s*. 17. $15\frac{15}{29}$ per cent.
18. $33\frac{1}{3}$ per cent. 19. $4\frac{263}{1120}$*d*. 20. £1 „ 11*s*. „ $11\frac{1}{4}$*d*.
21. $3\frac{3}{4}$ per cent. loss. 22. 4*s*. 23. 50 per cent.
24. $42\frac{6}{7}$ per cent. 25. £11·423, &c. 26. 2·94 pence.
27. 4 per cent. 28. $15\frac{5}{8}$ per cent. 29. £50.
30. 14*s*. 31. 72*s*. „ 6*d*. 32. 16 per cent.
33. 1*s*. „ 8*d*. 34. 25 per cent. 35. 6 lbs. 36. £540.
37. 50 per cent. 38. 20 per cent.

EXERCISE XXIII. (Page 275.)

1. 311 square feet „ 66 square inches.
2. 235 square feet „ $131\frac{8}{9}$ square inches.
3. 4915 cubic feet „ 870 cubic inches.
4. 128 square feet „ $31\frac{17}{18}$ square inches.
5. 8 square feet „ $115\frac{1}{2}$ square inches; and £6 „ 12*s*. „ $0\frac{3}{8}$*d*.
6. £6 „ 3*s*. „ $11\frac{1}{2}$*d*. 7. 49 square feet „ 62 square inches.
8. 26 yards „ 0 feet „ 4 inches.
9. 329 cubic feet „ 918 cubic inches.
10. 1 foot „ 3 inches. 11. 14 feet „ 4 inches.
12. £4 „ 3*s*. „ 4*d*. 13. £7 „ 7*s*. „ $5\frac{5}{8}$*d*.
14. 76 square inches. 15. 512 square feet; £5 „ 13*s*. „ $9\frac{1}{3}$*d*.

16. £12 „ 11*s.* „ $0\frac{1}{2}d.$ 17. £51 „ 7*s.* „ $6\frac{7}{12}d.$

18. 4 yds. „ 2 ft. „ $9\frac{2}{7}d.$ 19. $45\frac{1}{2}$ yards.

20. 10 pieces. 21. £1 „ 0*s.* „ $2\frac{7}{8}d.$

22. £22 „ 17*s.* „ $2\frac{1}{4}d.$ 23. $3429\frac{9}{16}$, and £1 „ 8*s.* „ $2\frac{1}{3}d.$

24. 210 square yards. 25. 5 too many.

26. 3 yds. „ $7\frac{8}{8}$ ft. and £1 „ 1*s.* „ $0\frac{1}{12}d.$

27. $\frac{1039696}{4147515}$ of an inch. 28. $785\frac{7}{9}$ square feet.

29. £9 „ 4*s.* „ 2*d.* 30. £7 „ 16*s.* „ $10\frac{215}{324}d.$

31. £1 „ 6*s.* „ $8\frac{8}{9}d.$ 32. 89 tons „ 6 cwt. „ 1 qr. „ 2 lbs. „ 8 oz.

33. 115 cubic feet „ 30 cubic inches. 34. £4 „ 1*s.* „ $3\frac{3}{8}d.$

35. £5 „ 10*s.* 36. 130 square yards.

37. 10*s.* „ $1\frac{7}{33}d.$ 38. 15 feet.

39. 1111010101; and 11001100; and 1465.

40. 202329. 41. 2*e*1·08. 42. 27*t*. 43. *e*7*t*8.

Exercise XXIV. (Page 294.)

1. $18\frac{1}{16}$; 53157·376; $\frac{81}{256}$; $525\frac{7}{32}$; ·0016384.
2. ·004115226; ·012345679; $\frac{1}{6}$.
3. (1) 29. (2) 39. (3) 93. (4) 108. (5) 246.
   (6) 378. (7) 735. (8) 817. (9) 418609.
4. (1) 5·7. (2) 1·19. (3) 3·58. (4) 61·5.
   (5) ·0866. (6) ·00987.
5. (1) 1·414213 with remainder ·000001590631.
   (2) 2·6457513 with remainder ·00000003834831.
   (3) 3·16227 with remainder ·0000484471.
   (4) 8·774964 with remainder ·000006798704.
   (5) 11·2249 with remainder ·00161999.
   (6) 29·68164 with remainder ·0002469104.
6. (1) 3·21. (2) 5·432. (3) 4·16. (4) 17·036.
   (5) ·02. (6) ·0032.
7. 2·23606, ·70710, ·22360, ·07071.
8. $\frac{119}{121}$ and ·8164966. 9. ·4, ·126, ·666, &c.
10. $\frac{409}{13}$; and 2·52982015; and ·612372435.
11. ·000001, and ·005329. 12. $\cdot\dot{9}$, ·586, ·350.

13. (1) 28. (2) 57. (3) 99. (4) 111. (5) 234. (6) 307.

14. (1) ·47. (2) 9·5. (3) 6·26. (4) ·404. (5) ·287. (6) ·0313. (7) $\frac{7}{8}$. (8) $37\frac{1}{2}$. (9) ·4601.

15. (1) 1·259. (2) 1·9293118. (3) 3·036. (4) ·96. (5) ·40207. (6) ·368. (7) ·8879040.

16. (1) 31. (2) 28. (3) 36.

17. 20·263, &c. feet. 18. £418.

19. 61023·377953; and 2005. 20. 54. 21. 3·3 feet.

22. $12\frac{1}{2}\frac{4}{2}$ minutes. 23. 999.

24. ·00010201, and ·1008, &c. 25. 48 feet.

26. $55\frac{1}{4}$ inches. 27. 9·898979, &c.; the latter.

28. 3·33, &c. feet. 29. 28·875 feet. 30. 2, 4, 8 feet.

31. 11 feet. 32. 20 yards long, 4 broad.

APPENDIX,

CONTAINING EXAMINATION PAPERS.

I. *Winchester College. Election*, 1868. *Arithmetic for Boys under Thirteen years.*

1. Write down the tables of *linear* and of *superficial measure*, and shew how the number of *yards* in a *pole* of the latter may be deduced from that of the former.

2. Multiply £3764 „ 18*s.* „ $6\frac{3}{4}$*d.* by 95; and divide 55 tons „ 19 cwt. „ 3 qrs. „ 27 lbs. by 96.

3. Four new houses are to be built: the estimated cost of (1) is £4949 „ 15*s.* „ 8*d.*, of (2) £4798 „ 8*s.* „ 1*d.*, of (3) £4501 „ 19*s.* „ 9*d.*, and of (4) £5129 „ 11*s.* „ 11*d.*; what will be their total amount? and what, if the estimates be exceeded 10 per cent.?

4. How many shillings in a continuous straight line would reach from Winchester to London, 64 miles, the diameter of a shilling being $\frac{3}{4}$ of an inch? and what would be their total value in pounds?

5. A yard, 63 feet long and 18 feet broad, is paved with bricks, each measuring 9 in. by $4\frac{1}{2}$ in.; required the number of bricks.

6. If 15 men, working 13 hours a day, earn £95 „ 1*s.* „ 3*d.* in 26 days; how many hours a day must 17 men work, that their wages may amount to £84 „ 3*s.* in 24 days?

7. Find the sum, difference, product, and quotient of $3\frac{7}{8}$ and $2\frac{1}{3}$.

8. How long will an 18-gallon cask of beer last if $4\frac{1}{2}$ pints be drawn from it daily?

9. Brown can run 10 yards whilst Green runs 11; what start ought Green to give Brown in a half-mile course in order to make an even race?

10. How far may a boy ride with his father, who leaves Winchester at 12·30, and will drive to Romsey (12 miles) in 2 hours, so that he may, by walking at the rate of 3 miles an hour, be back by a quarter before 3,—in good time for school?

II. *Winchester College. Election*, 1868. *Arithmetic for Boys of Thirteen years and over.*

1. Write down the tables of *linear* and of *superficial measure*, and shew how the number of *yards* in a *pole* of the latter may be deduced from that of the former.

2. Multiply £3764 „ 18*s*. „ $6\frac{3}{4}$*d*. by $95\frac{1}{2}$; and divide 55 tons „ 19 cwt. „ 3 qrs. „ 27 lbs. by 96.

3. A cistern is to be made 4 ft. 9 in. long, 3 ft. „ 6 in. wide, 2 ft. „ 4 in. deep; how many square feet of lead will line it, and how many cubic feet of water will fill it?

4. What is a gentleman's income who, after paying sixpence out of every pound of it, has £962 „ 16*s*. „ 3*d*. left?

5. If a boy can paddle his own canoe three quarters of a mile *down* the river in ten minutes, but without the aid of the stream would take a quarter of an hour; what is the rate of the stream per hour? and how long will it take him to return against it?

6. Find the sum and the difference of the fractions $21\frac{3}{8}$ and $16\frac{4}{9}$: also divide their sum by their difference, and their difference by their sum.

7. Reduce $\frac{5}{9}$ of a guinea to the fraction of 15 shillings: and $\frac{7}{15}$ of a crown to the decimal of 12*s*. 6*d*.

8. In latitude 51° the length of a degree of longitude is about 37·76 geographical miles: how many miles is Winchester from Dresden—both nearly in that latitude—the longitude of the former being 1° 20′ west, and of the latter 13° 40′ east?

9. How many small cubes, whose edges are 2 inches, may be cut out of a large cube whose edge is 12 inches?

10. At what rate simple interest would a sum of money amount in 2 years to the same, as at 4 per cent. compound interest?

11. A poor fellow, ignorant of arithmetic, had 6*s.* „ 6*d.* in his purse; he talked of spending $\frac{1}{2}$ of it on cakes, $\frac{1}{3}$ of it on fruit, and $\frac{1}{4}$ of it on a knife: shew (1) his mistake, and (2) how he might have spent his money in *proportion* to the fractions he intended.

III. *Winchester College. Duncan Prize. Easter*, 1869.

1. Write down in words 7392586044001; and in figures five billions, twenty-one millions and thirty.

2. Multiply 174550613 by 600417; and divide 2358293184 by 3904.

3. Add together

$$\frac{13}{24} \text{ of } \frac{125}{156} \text{ of } \frac{7}{9\frac{3}{8}} \text{ and } \frac{14}{15} \text{ of } \frac{25}{42} \text{ of } \frac{4\frac{11}{16}}{6\frac{12}{16}},$$

and simplify

$$\frac{9\frac{5}{8} \text{ of } \frac{2}{11}}{6\frac{3}{10} \text{ of } \frac{5}{9}} + \frac{6\frac{2}{3} \text{ of } \frac{9}{10}}{4\frac{1}{8} \text{ of } 2\frac{10}{11}}.$$

4. Multiply ·5678 by ·08765; and divide ·75445 by ·00625.

5. Reduce $\frac{747}{960}$, and $\frac{2}{5}$ of $\frac{7}{12}$ of $\frac{11}{21}$ of $2\frac{1}{2}$ to decimal fractions.

6. From 261 times £35 „ 4*s.* „ 2*d.* take £9089 „ 5*s.* and divide the remainder by 89.

7. Extract the square root of 30712·5625 of $\frac{625}{2401}$; and of ·000000133225.

8. Find the interest of 125 guineas for $2\frac{1}{2}$ years at 4 per cent.: and the discount of £63 „ 15*s.* for 15 months at 5 per cent.

9. Find the value of $\frac{14 \text{ lbs.} „ 8 \text{ oz.} „ 18 \text{ dwt.}}{1 \text{ lb.} „ 0 \text{ oz.} „ 10 \text{ dwt.}}$ of 5*s.* „ $2\frac{1}{2}$*d.*

10. Find the greatest common measure, and the least common multiple of 128, 384, 768, and 2304.

11. If by selling wine at 12*s.* per gallon I lose 25 per cent., at what must I sell it per gallon to gain 25 per cent.?

12. What will be the price of carpeting a room, 24 ft. „ 9 in. long, 19 ft. „ 6 in. wide, at 13*s.* „ 4*d.* per square yard?

13. If a crew, which can row from *A* to *B* in 60 minutes, can row from *B* to *A* in 55 minutes, compare the rates of the stream and boat.

IV. *Mathematical Collections. Eton. Lent*, 1868.

1. A man commencing on January 1, 1867, spends 5*s.* „ $6\frac{1}{2}$*d.* a day, how much will he save in 5 years if his yearly salary be £120?

2. Simplify

$$(4\tfrac{4}{7} \text{ of } 3\tfrac{1}{16}) - (2\tfrac{2}{3} \text{ of } 1\tfrac{5}{16}) + \tfrac{8}{9} - (1\tfrac{3}{4} \text{ of } 4\tfrac{4}{7} \text{ of } \tfrac{3}{14}).$$

3. $\frac{13}{18}$ of a number exceeds $\frac{5}{6}$ of half the number by $40\frac{1}{3}$, find the number.

4. The number of boys in Upper School at Election, 1866, was 760; at Election, 1867, 798. What is the increase on every hundred boys?

5. Simplify (and give the answer in decimals) $\dfrac{\frac{5}{8} \text{ of } 16{\cdot}125}{4\frac{7}{8}}$.

6. If 40 men can reap $400\frac{3}{5}$ acres in $12\frac{3}{4}$ days, how many acres should 30 men reap in $3\frac{2}{5}$ days?

7. Find the value in days of $\cdot\dot{2}8571\dot{4}$ of a lunar month.

8. If money be invested in the 3 per cents at 85, what is the real interest per cent. obtained on the money invested?

9. Find the discount on paying at once £1111 „ 11*s.* due 3 months hence, interest being reckoned at 4 per cent.

10. A gardener plants an orchard with 7225 trees, and arranges them so that the number of rows of trees equals the number of trees in each row. How many rows were there?

V. *Mathematical Collections. Eton. Election*, 1868.

1. Find the present worth of £682 „ 18*s*. „ $9\frac{1}{2}d$. due $3\frac{3}{4}$ years hence at $4\frac{1}{2}$ per cent.

2. A person sells £5000 consols at $94\frac{7}{8}$, and on their rising, he sells £5000 more at $95\frac{5}{8}$; he afterwards buys back the whole £10,000 at 96, how much does he lose?

3. What is the price of wheat when an additional profit of $3\frac{3}{4}$ per cent. would raise the price 1*s*. „ 9*d*. per quarter?

4. The wheels of a carriage are 11 ft. and 7 ft. „ 6 in. in circumference. How many times will each wheel have performed a complete number of revolutions simultaneously in a mile?

5. If the price of gold in London is £3 „ 10*s*. per oz., and the rate of exchange with Paris at par is 25·20 for 1 sov., what is the price of gold at Paris in English money when the exchange is 25·44?

6. State and prove the rule for reducing a recurring decimal to a vulgar fraction.

7. In a cricket match the scores in each successive innings are $\frac{1}{4}$ less than in the preceding innings, and the side, which has had the first innings, wins by 50. What are the scores in each innings?

8. A train having to perform a journey of 250 miles is obliged after 103 miles to reduce its speed by $\frac{1}{8}$. The result is that the train arrives at its destination 1 hr. „ 10 min. behind time, what is its ordinary rate?

9. A cistern is kept constantly supplied with water. Supposing that it is filled to begin, it is found that 24 equal taps opened together will empty it in $5\frac{1}{2}$ minutes, and 15 of them will empty it in 13, how many of them will empty it in 33 minutes?

VI. *Arithmetic. Eton. Easter*, 1869.

1. How much is $\frac{59}{160}$ of an acre?

2. How much paper 2 feet „ 6 in. wide is required for a room 27 feet „ 10 in. × 19 feet „ 8 in., and 15 feet high?

3. Find the value of $\cdot 45$ of £1 „ 3*s.* „ 9*d.* + $\cdot 2\dot{5}\dot{7}$ of £11 „ 5*s.* „ 6*d.* + $\cdot 3125$ of £5.

4. Find the interest on £1925 „ 15*s.* for 2 years „ 8 months at $3\frac{3}{8}$ per cent.

5. What is the present value of £2857 „ 10*s.* due $12\frac{1}{2}$ years hence at $3\frac{1}{2}$ per cent.?

6. Messrs. Brown, Jones, and Robinson engage in business. Brown puts in £500 for the first 4 months, and afterwards doubles it, Jones puts in £3000 at first and trebles it at the end of 6 months. Robinson has £3500 in for 8 months, and then leaves the concern. How ought the first year's profit amounting to £10,000 to be divided among them?

7. A train which goes at the rate of $21\frac{1}{2}$ miles per hour, has got $57\frac{1}{2}$ miles on its journey at 6 o'clock. If it is timed to arrive at its destination at 10·18, what is the least pace at which a special can be sent after it to overtake it?

VII. *Harrow School. Fifth Form. December*, 1867.

1. Find the least number which can be divided by 6, 9, 12, 15, 21 respectively with remainder equal to their greatest common measure.

2. Simplify:

(1) $3\frac{1}{2}+4\frac{1}{3}+5\frac{1}{4}+\frac{3}{4}$ of $\frac{7}{9}+\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{8}$.

(2) $$\left(\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}\right)\div\left(\frac{\frac{1}{4}-\frac{1}{6}}{\frac{1}{4}+\frac{1}{6}}-\frac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}\right).$$

3. Add together $\frac{3}{16}$, $\frac{13}{40}$, $\frac{21}{250}$, and $\frac{9}{625}$; (1) as vulgar fractions, (2) as decimals, and prove that the two sums agree.

4. Reduce $\frac{11}{847}$ to a circulating decimal, and $3\cdot 04\dot{0}\dot{9}$ to its equivalent fraction.

5. If 25 francs „ 25 centimes are equivalent to one pound, what fraction of a shilling is a franc?

6. Find the value of $\frac{9}{8}$ of a guinea + $\frac{9}{16}$ of a crown + $\frac{9}{10}$ of 7*s.* „ 6*d.* − $\frac{9}{8}$ of 2*d.*, and express it as the decimal of 16*s.*

7. If 5 men can reap a field 1400 feet long and 400 broad in 3 days of 14 hours each, in how many days of 12 hours each can 7 men reap a field 1600 feet long and 700 feet broad?

8. Find the difference between the interest and discount of £125 „ 8*s.* „ 6*d.* for half a year at $3\frac{1}{4}$ per cent.

9. A person sells out of the Three per Cent. Consols at $91\frac{1}{8}$, and buys in again when they have fallen $2\frac{1}{2}$ per cent.; what difference will this make in his income if he now possesses £800 stock?

VIII. *Harrow School. Fifth Form.*

1. A yard of Cambridge butter weighs 1 lb.; what should be the length of one pennyworth at $17\frac{1}{2}$*d.* per lb.? and how many yards of butter does a man eat in a year whose consumption is at the rate of four pennyworth a day?

2. Find the contents of the smallest cask that can be filled by an exact number of any one of the following measures: $\frac{1}{2}$ pint, $\frac{1}{2}$ gallon, 3 gallons, 5 gallons, and 9 gallons.

3. Find the value of

(1) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}$.

(2) $\frac{3}{13}-\frac{1}{7}+\frac{8}{59}$.

Multiply the results together, and reduce the product to its lowest terms.

4. Find the sum of ·275 of £1 + 1·03125 of 1 guinea + ·00390625 of £1 „ 12*s.*, and reduce the first of these quantities to the decimal of the second.

5. A wine merchant mixes together 20 gallons at 12*s*., 30 gallons at 15*s*., 40 gallons at 20*s*., and 10 gallons of water; what must be the retail price so as to gain 10 per cent. profit?

6. A boat is propelled by 8 oars, which take 10 strokes per minute, and it goes at the rate of 9 miles an hour. Find the rate of a boat propelled by 6 oars, which take 8 strokes per minute, when 5 strokes of the latter are equivalent to 6 of the former.

7. Find the simple interest arising from £325 ,, 16*s*. ,, 8*d*. at $4\frac{1}{2}$ per cent. in $3\frac{1}{4}$ years: also the amount due at the end of 3 years if £350 be lent at 5 per cent. compound interest.

IX. *Harrow School. Fifth Form.*

1. Explain the terms *measure, common measure*, and *greatest common measure;* and prove that every common measure of dividend and divisor is a measure of the remainder.

Find the G.C.M. and L.C.M. of 1080, 672, and 756.

2. Find the greatest and least of the fractions $\frac{4}{15}, \frac{6}{17}, \frac{3}{11}$ by reducing them (1) to a common numerator; (2) to a common denominator; giving reasons for the several steps. Divide the difference of the first and second by that of the second and third.

3. In the International Exhibition is a mass of platinum, valued at £3840: given that platinum is worth 24*s*. per oz, and is 21·15 times as heavy as water, of which a cubic foot weighs 1000 oz., find the length of the ingot, its breadth and thickness being taken as 6 in. and 4 in. respectively.

Also find the length of an edge, supposing the ingot were a cube.

4. Reckoning the lunar month from full moon to full moon, an interval (on the average) of 29 d. ,, 12 h. ,, 44 m., find (by practice) the length of 705 lunar months, and shew that it is very nearly an exact number of years.

5. Express the height of a mountain, whose summit is ...00 feet above the sea level, both as a vulgar and as a deci-...action of the earth's diameter (7926 miles); and find ...eight would represent it on a globe 2 feet in diameter.

6. Distinguish between interest and discount, and find both the interest and discount on £5208 „ 16*s.* for 2 years „ 73 days, reckoning interest at $7\frac{1}{2}$ per cent.

7. A person borrows £130 on the 5th March, and pays back £132 „ 10*s.* „ 6*d.* on 18th October: find the rate of interest charged.

8. A capitalist in England, having invested £1000 in American railway shares at 70 dollars per share, wishes to sell out now that they are quoted at 75 dollars per share. In consequence of the suspension of specie payment, 100 dollars (gold) are now worth 110 dollars (paper): will he gain or lose on the whole, and how much?

X. *Neeld Medal Examination. Harrow School.*

1. What is the use of the ciphers in 1·006, 11001?

Write in words the value of 100, supposing eight instead of ten to be the radix of the scale of notation.

State and explain a common rule for dividing a number by 20.

2. When are two numbers said to be prime to each other? Shew that 8765, 13131 are so. What is their least common multiple?

3. A number diminished by one-tenth of itself is divided by 124, giving 144 as the quotient. What is the number?

4. State and explain the rule for reducing a vulgar fraction to a decimal fraction.

Find the values of $\frac{1}{4} \div \cdot 01001$ and of $10\cdot 01 \div \frac{1}{50}$.

Give the value of the first figure in each of the quotients

$$\frac{84}{\cdot 19},\ \frac{\cdot 31}{22},\ \frac{216\cdot 4}{193\cdot 2}.$$

5. The following rule is sometimes given to divide by 3·14159. "Multiply by 7, divide by 11, then by 2, and add one-eighth of one thousandth of the result." Find the error made in obtaining $10 \div 3\cdot 14159$ by this rule.

6. A square board contains 280·0209 square inches; find the length of a side?

Also find the length of the diagonal true at least to an eighth of an inch.

7. Compare $15\frac{5}{18}$ cubic feet and $2\frac{1}{4}$ cubic yards. How many cubic 3-inches do they together contain?

8. Distinguish between abstract and concrete numbers. Which of the two must a ratio be? What do you mean by a unit of measurement?

What is the unit of length, when a mile is 320?

9. What do you understand by inverse proportion?

A is inversely proportional to the square of *B*; *A* is 32 when *B* is 1, what is *A* when *B* is 60?

10. In gold ore from Columbia 97 parts are gold and 3 silver; whilst in gold ore from California $\frac{7}{8}$ths are gold and $\frac{1}{8}$th silver. Assuming gold to be worth 15 times as much as silver, find how much Columbian ore is worth 10000 oz. of Californian.

11. *A* and *B* are out partridge shooting, *A* fires 5 shots to *B*'s 3, but *A* kills only 1 in 3 shots, while *B* kills 1 in 2 shots. When *B* has missed 36 shots, which has killed most?

12. £180 is divided amongst 66 men, women and children. The sums of the men's shares, women's shares and children's shares are in the proportion of 5 : 4 : 3, but their individual shares are as 3 : 2 : 1 respectively. Find the number of men and of women and of children.

13. Find the amount of £62 „ 10*s.* in four years at 20 per cent. per annum compound interest.

Also the amount of £500 in $3\frac{1}{2}$ years at 4 per cent. compound interest.

14. The shares in a mine are £20 each. £2 „ 13*s.* „ 4*d.* is paid up and the shares are quoted at £46. The dividend is 15*s.* per share quarterly. *A* holds 100 original shares. Find what interest he makes per cent.; and what he would make, and how much per cent., if he sold out and invested in 4 per cent. stock at par.

15. The polar diameter of the earth is 7899·114 miles and its ratio to the equatorial diameter is $\frac{298\cdot 33}{299\cdot 33}$; determine the latter omitting fractions of a mile.

XI. *Ladies' College, Cheltenham.*

1. Explain the principle upon which numbers are represented by figures.

Subtract 4771213 from 6020600, and account for each step of the process.

2. What is meant by the Greatest Common Measure of two or more numbers? and how is it found?

Ex. Find the G. C. M. of

(1) 1236 and 1362.

(2) 4067 and 2573.

(3) 42237 and 75582.

3. Why is it necessary, before adding two or more vulgar fractions together, to reduce them to a common denominator? Find the value of

(1) $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$;

(2) $1\frac{1}{3}+4\frac{1}{6}+\frac{7}{8}+\frac{7}{9}$.

(3) $2\frac{3}{4}+3\frac{3}{8}$ of $2\frac{7}{8}$ of $\frac{3}{4}+\frac{1}{5}$ of $3\frac{1}{3}$.

4. What fraction

(*a*) of £1 „ 1*s*. „ 7*d*. is 12*s*. „ 4*d*.?

(*b*) of 4*s*. „ $7\frac{1}{2}$*d*. is £1 „ 2*s*. „ $4\frac{1}{2}$*d*.?

(*c*) of 2 acres „ 37 poles is 3 acres „ 2 roods „ 1 pole?

(*d*) of 6 days „ 1 hr. „ 48 min. „ 7 sec. is 13 hrs. „ 15 min. „ 17 sec.?

5. Simplify the following fractions:

(1) $\dfrac{17}{1\frac{2}{3}+\frac{3}{5}}$; (2) $\dfrac{1\frac{1}{2} \text{ of } 2\frac{2}{3}}{3\frac{3}{4} \text{ of } 4\frac{4}{5}}$; (3) $\dfrac{2\frac{2}{3}-1\frac{1}{2}}{2\frac{2}{3}+1\frac{1}{2}}$;

(4) $\left\{\frac{5}{7}\times\frac{2}{9}\times 13\frac{1}{2}\right\} \div \left\{\frac{1}{9}\times\frac{3}{7}+40\right\}$; (5) $2\frac{1}{2}\times\dfrac{1}{3\frac{1}{3}+\frac{1}{4\frac{1}{4}}}$.

6. *A* can mow a field of hay in 12 hours, and *B* in 16 hours; in how many hours will they mow it together?

7. Explain how a vulgar fraction is reduced to a decimal fraction; and under what circumstances the decimal will recur.

Reduce to decimals the following vulgar fractions, and prove the correctness of the results by reducing the corresponding decimals to vulgar fractions:

(1) $\frac{5}{16}$; (2) $3\frac{5}{8}$ of $\frac{1}{512}$; (3) $\frac{57}{240}$;

(4) $\frac{17}{30}$; (5) $\frac{16}{81}$; (6) $\frac{17}{99000}$.

8. Perform the following divisions, and explain clearly why, in the quotients, you place the points where you do: (1) 12·6 by ·0012. (2) ·00281 by 1·405. (3) ·02916 by ·0012. (4) $\dot{8}16$ by ·0004.

9. The imperial gallon contains 277·274 cubic inches. How many cubic inches are there in one pint? and what is the weight of one pint of water, if one cubic foot weighs 1000 ounces?

10. Find the value of (1) 898 things at 18*s.* „ $7\frac{3}{4}$*d.* each. (2) 421 things at £4 „ 2*s.* „ $6\frac{3}{4}$*d.* each. (3) 72 cwt. „ 3 qrs. „ 7 lbs. at £1 „ 4*s.* „ 6*d.* per cwt. (4) 225 acres „ 1 r. „ 19 p. at 13*s.* „ $2\frac{1}{2}$*d.* per acre.

11. (*a*) The French metre being 39·37 inches, how many yards is 3600 metres?

(*b*) The verst being equal to 2 miles, how long will it take a man to travel 2500 versts at the rate of 10 miles an hour?

12. Two trains are moving in the same direction on parallel lines of rails. The first is 75 yds. long, and moves at the rate of 40 miles an hour: the second is 120 yds. long, and moves at the rate of 16 miles per hour. How long will the first train take to pass the second from the time of first overlapping, till it is completely clear?

XII. *Ladies' College, Cheltenham. Midsummer*, 1869.

1. (i.) A certain number, composed of 6 digits, has the figure 2 in the place of units, and in that of hundred thousands; the figure 1 in the place of tens, and in that of ten thousands; a cipher in the place of hundreds, and in that of thousands. Write down the number in figures, and in words.

(ii.) What is the meaning of the expression 11110? On what principles is it that each of the first four figures in that expression, beginning from the left hand, has a different value?

(iii.) In the preceding example what would be the effect of placing a decimal point, (*a*) to the right of all the figures in the expression; (*b*) to the left of all of them; (*c*) between the second and third figures from the left hand?

(2) Explain carefully the meaning of the following terms—multiplication, integer, product, quotient, common denominator, multiple, measure, greatest common measure, decimal fraction.

3. Simplify the following:

(i.) $\frac{17}{12} + 2\frac{1}{5} + \frac{15}{9} + 3\frac{1}{2} + 4\frac{1}{6} + \frac{5}{3}$.

(ii.) $\frac{2}{3} \times \frac{3}{4} \times 4\frac{1}{3} \times \frac{8}{27}$.

(iii.) $\dfrac{3 - \left(\frac{1}{2} \text{ of } \frac{5}{9}\right)}{\frac{3\frac{1}{6}}{1} + \frac{1}{3\frac{1}{6}}}$.

4. Demonstrate, with examples, the following rules, and discuss them.

(i.) If both the numerator and the denominator of a fraction be multiplied by the same number, the value of the fraction is not altered.

(ii.) To multiply a fraction by a whole number, either multiply the numerator or divide the denominator of the fraction by the whole number.

(iii.) To divide by a fraction, multiply by its rociprocal.

5. (i.) To what number can I add $7\frac{3}{5}$ so as to make $24\frac{7}{12}$?

(ii.) What is the product of the sum and difference of $\frac{2}{7}$ and $\frac{5}{9}$; and what is the difference between their product and sum?

(iii.) What is the whole of which £5 „ 12 „ 6*d.* is three-eighths?

(iv.) A school is composed of 3 divisions. There are $\frac{12}{25}$ths of the whole number of girls in the first; $\frac{1}{5}$th in the

second; and the rest, 80 in number, are in the 3rd division. How many girls are there altogether ?

6. Suppose that it is required *to reduce* $\frac{5}{19}$ *to a decimal form:*

(i.) What does this mean ?

(ii.) Work the reduction true to 3 places of decimals.

(iii.) Explain the reason for each step in the process.

7. Divide 4·56 by 456, by 45·6, and by ·456 : and verify the results by vulgar fractions.

8. (i.) A clerk copied ·55 of £5 instead of 5·5 of £5. What was the amount of the error ?

(ii.) Mary owes Jane ·6 of what Jane owes Elizabeth; and Jane gives Mary five shillings to put the account between them all straight. What is Jane's debt to Elizabeth ?

(iii.) How much is 14·973 shillings of a ten-pound note ? and how much of a farthing ?

(iv.) It takes 87 yards of carpet, 1·25 yards wide, to cover a room. How many more yards will it take if the width be ·75 yard ?

XIII. *College of Preceptors*, 1867.

1. Calculate, by practice, the value of 4 cwt. „ 2 qrs. „ 10 lbs., at £2 „ 18*s*. „ 4*d*. per qr.

2. What fraction of $1\frac{3}{7}$ of £1 „ 2*s*. „ 9*d*. is $\frac{13}{14}$ of 5*s*. ?

3. A person having lost $\frac{2}{5}$ of his money, found that $\frac{1}{3}$ of $3\frac{4}{7}$ of what he then had was $2\frac{1}{4}$ of £51 „ 8*s*. „ $6\frac{6}{7}$*d*. How much money had he at first ?

4. If 9 women can do a certain piece of work in $11\frac{1}{7}$ days of $8\frac{1}{4}$ hours each, how many days of $9\frac{3}{4}$ hours each will it take 5 men who can do $\frac{4}{7}$ as much again as the women, to do $2\frac{2}{3}$ of the same work ?

5. Reduce $2\frac{1}{3}$*s*. to the decimal of half-a-guinea. Divide ·014616 by 7·2; anl 400·4 by ·0572.

6. Find the value of 1·75 of $3\frac{1}{3}-3\frac{2}{9}+\frac{\dot{5}}{7\frac{1}{2}}$; and of $\dot{1}4285\dot{7}$ of 1 fur. „ 18 po. „ 3 yds.

7. If by selling 8 oranges for $6\frac{2}{3}d.$, there be a profit of 10 per cent., at what price per dozen must they be sold to gain 21 per cent.?

8. If the discount on £249 be £9, simple interest being reckoned at 5 per cent., when is the same due?

9. What sum should be insured, at £2 „ 2*s.* „ 6*d.* per cent., on goods worth £783, that the owner may recover, in case of loss, the value of both goods and premium?

10. What is the price of the 4 per cent. stock, when a person gets the same interest for his money as if he invested it in the $4\frac{1}{2}$ per cents. at 90?

11. Extract the square root of $1{\cdot}7866\dot{7}$.

XIV. *Diocesan Training Colleges for National Schoolmasters.*

Section I.

1. Express in figures these numbers:—Seventy thousand and seven; nine thousand six hundred and eight; forty thousand nine hundred and seventy. Add them together, and take ninety-nine thousand and ninety-seven from the sum.

2. Make a receipted bill of the following articles in the proper form (as a model specimen for children):—7 ozs. at $9\frac{1}{2}d.$ an oz.; 13 lbs. at 1*s.* „ 2*d.* per lb.; $2\frac{1}{4}$ yds. at 11*d.* a yard; $4\frac{1}{2}$ doz. at 1*s.* „ 3*d.* per doz.; 2 cwt. „ 1 qr. at 9*s.* per cwt.

3. A bankrupt owes £78: his assets are £53; if the creditors only get 7*s.* „ 6*d.* in the pound, what was the amount of expenses incurred in settling the claims?

4. Write out the table of long measure. How many seconds are there in 74 days „ 19 hours „ 38 minutes?

5. The cost of covering a roof 18 yds. by 4 yds. amounted to £27; how much was that per square foot?

6. If a postmaster is allowed $1\frac{1}{4}$ per cent. profit on selling postage stamps, what is his gain on a sheet of 120 penny stamps?

Section II.

1. Multiply 31,472 by 974, divide the product by 583. What must be taken from the quotient to leave exactly 100?

2. Divide £16 among 18 persons. If the same sum were divided among 24 persons, what would be the difference of the share given to each?

3. Find the least common multiple of 12, 16, 20, and 30.

Section III.

1. Reduce to its simplest form:

$$\frac{\frac{3}{2}-\frac{2}{7}}{\frac{5}{6}-\frac{3}{5}} \div \frac{1\frac{5}{7}-\frac{13}{49}}{\frac{1}{3}-\frac{1}{11}} + \frac{2\frac{1}{4}}{\frac{3}{8}\times\frac{12}{35}}.$$

2. What fraction of 3 cwt. „ 1 qr. „ 17 lbs. „ 3 oz. is $\frac{4}{39}$ of a ton?

3. The price of gold in this country is £3 „ 17*s.* „ 10½*d.* per oz., what would be the price of ·0013 of an oz.?

Section IV.

1. Represent £3 „ 17*s.* „ 4*d.* as the decimal of £5 „ 14*s.* „ 6*d.*

2. In a school there are 17 children 6 years old, 26 aged 7½ years, 35 aged 9¼ years, 20 aged 10 years, and 8 aged 12¼ years; what is the average age of all the children?

3. Two clocks begin to strike 9 together; one strikes in 25 seconds, the other in 20 seconds; what decimal of a minute is there between their seventh strokes?

Section V.

1. How long will it take to walk round a square field containing 13 acres „ 81 yards at the rate of 3½ miles per hour?

2. If a man's income is reduced from £750 to £734 „ 7*s.* „ 6*d.* by income-tax, how much does he pay in the pound?

3. A house built for £664 is sold for £830; what is the gain per cent.? If it had been built for £830, and sold for £664, what would be the loss per cent.? Why do the rates differ?

Section VI.

1. Divide 3672965 by $2 \times 3 \times 6$ (short division). Explain every step of the process.

2. What is the weight of air in a room 5 metres long, 3 metres wide, 4 metres high, if 1 cube decimetre of air weighs ·0018 kilogrammes?

3. *A* and *B* fire at targets, having 35 cartridges each. *A* fires twice in 3 minutes, and *B* three times in 5 minutes. How many times will *B* have to fire after *A* has finished?

Section VII.

1. Find the simple interest on £742 „ 19*s.* „ 4*d.*, at 3½ per cent. for 3 years „ 5 months.

2. A person pays £550 for a bill of £605 due 2 years hence, what is the rate of interest?

3. If the three per cents are at 84, and a four per cent. stock at 98, which is the best investment? And if a man has £1000 to invest, what will be the difference in his income according as he purchases the one or the other?

XV. *Diocesan Training Colleges.*

General.

1. Explain fully the rule for division of one decimal fraction by another: and illustrate it in the following examples, (1) ·0054 ÷ 70, (2) ·054 ÷ ·00007, (3) 5·4 ÷ 700.

2. If a man pay £42 „ 5*s.* „ 10*d.* income-tax, when the tax is 7*d.* in the pound, what is his net income?

3. How many yards of carpet 28 inches wide will be required to cover a room 30 ft. „ 6 in. by 18 ft. „ 4 in., and what will it cost at 4*s.* „ 9*d.* per yard?

4. What wages are due to a servant engaged at £14 a year, who comes on March 11th and goes away on May 26th?

5. The 25942nd number of *The Times* was issued on Oct. 18, 1867; in what year was the first number issued, and on what day?

6. *A* and *B* run in a race; *B* is allowed 15 yards start, but loses by 3 seconds, *A* having done a mile in 5 minutes „ 35 seconds; in what time can *B* run a mile?

7. In a town of 15000 electors ·72 voted for one candidate and ·125 voted for another; how many electors did not vote at all?

8. *A*, *B*, *C*, can together do a piece of work in 20 days; after 6 days *A* gives up and is succeeded by *D*, who does half as much again as *A*, *B*, or *C* could do in a day; when will the work be finished?

XVI. *Diocesan Training Colleges.*

Section I.

1. Add the following numbers: eighty-four thousand three-hundred and one, nine hundred and thirty-three thousand; forty-seven million six thousand three hundred; and subtract from the result, two million eighty-one thousand and eighty.

2. How often will a cart wheel 16½ feet round revolve in going a mile?

3. Write the following account as a bill, and find the amount: 16 yards of broad cloth at 27*s.* „ 6*d.* per yard; 18 yards at 14*s.* „ 9*d.* per yard; 13 yards at 11*s.* „ 10*d.* per yard; 14½ yards at 24*s.* „ 4*d.* per yard; 62¾ yards at 9*s.* „ 8*d.* per yard.

4. If a bankrupt pays 13*s.* „ 7*d.* in the pound, what will he pay on a debt of £4572?

5. A piece of land is required which can be divided into allotments of 3 roods, 2 roods, 1 rood, 35 perches and 35 perches; there are to be 127 allotments of each size; how many acres must the land contain?

6. A farmer paid £780 for cows and sheep. Of this sum he paid £350 for 25 cows; if a cow cost seven times as much as a sheep, how many sheep did he buy with the rest of his money?

Section II.

1. What is the difference between the thousandth part of a million and the 25th part of 20,000?

2. What cash must be given with 24 yards of cloth at 8*s.* „ 3¾*d.* per yard, to pay for 6 cwt. of sugar at 38*s.* per cwt.?

3. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$, and $\frac{7}{10}$ to fractions, having the least common denominator.

Section III.

1. Find the difference between $\frac{3}{104}$ of $29\frac{1}{2}$, and $\frac{7}{26}$ of $\frac{5}{14}$ of $3\frac{1}{4}$.

2. Express in pecks $\frac{3}{5}$ of 1 qr. „ 3 bus. „ 2 pks.

3. When the oz. of gold costs £3·89, what is the cost of ·04 lb.?

Section IV.

1. Express as a simple decimal the difference between $\frac{3}{5}$ of $\frac{11}{22}$, and $\frac{7}{8}$ of $\frac{9}{14}$.

2. Reduce 118·37 feet (1) to the decimal of a mile, (2) to the decimal of a furlong, and (3) to the decimal of a yard.

3. If 3 bushels cost 1·1 of a pound, what will 33·4 qrs. cost at the same rate?

Section V.

1. A sum of £279 „ 10*s.* is to be levied by rate in a parish which yields a rental of £7850; what is the rate per pound?

2. If eggs be bought at 5 for three pence, at what price must they be sold to gain 20 per cent.?

3. If a family of 9 people spend £120 in 8 months, how much will serve a family of 24 people for 16 months at the same rate?

Section VI.

1. What will the paving of a court cost at $4\frac{3}{4}$*d.* per yard, the length being 58 feet „ 6 inches, and the breadth 54 feet „ 9 inches?

2. Explain the reasons of the following operations in simple proportion:

   (1) The reduction of the 1st and 2nd terms to the same name.

   (2) The multiplying the 2nd and 3rd terms together and dividing by the 1st.

3. The foot of a ladder 30 feet long is 14 feet from a house, and its top reaches the upper part of a circular window; when the foot is drawn away to a distance of 17 feet from the house, the top reaches the lower edge of the window; what is the diameter of the window?

Section VII.

1. What sum of money lent at $4\frac{1}{2}$ per cent. per annum for 7 months will yield interest £17 „ 0*s.* „ $9\frac{1}{4}$*d.*?

2. 17 lbs. of tea worth 4*s.* per lb. is mixed with 25 lbs. worth 4*s.* „ 8*d.* per lb.; the whole is sold at 5*s.* „ 4*d.* per lb.; what is the total gain, and what the profit per cent.?

3. One-fourth of an estate belongs to *A*; $\frac{2}{9}$ of it to *B*; and the remainder to *C*, which is worth £2,500 more than *A*'s share; what is the value of the whole?

XVII. *Diocesan Training Colleges.*

General.

1. Prove the Rule for dividing a fraction by an integer, and simplify the following expression:

$$\frac{4\frac{1}{13} \text{ of } \frac{8}{95} \text{ of } 7\frac{3}{7}}{12\frac{1}{35}-2\frac{3}{7}}+\frac{2\frac{1}{3}+1\frac{35}{48}}{9\frac{4}{7}-3\frac{39}{112}}.$$

2. If a model of the earth were made with a diameter of 20 yards, express as the fraction of an inch the height of the highest mountain, taking the earth's diameter as 7.900 miles, and the actual height of the highest mountain as 25,000 feet.

3. Find the cost of surrounding a bowling green 80 ft. by 46 ft. „ 2 in. with a paved walk a yard and a half wide, at 2*s.* „ 8*d.* per sq. foot.

4. Owing to leakage, the quantity of wine in a cask is less at the end of each year by $\frac{1}{100}$th part of what it was at the beginning of that year. How much will be lost from a cask which now contains 100 gallons, in the course of five years?

5. Find the difference between the simple and compound interest on £4296 „ 10*s.* for three years at four per cent. per annum.

6. A and B enter into partnership, A contributes £5500, and B £3500. The agreement is that £120 is to be put by as an insurance fund annually, and the remaining profits to be divided in proportion to the capital subscribed. At the end of the year, A gets £435 „ 8*s*. „ 4*d*. Find the per centage of the whole profit.

7. Assuming that the distance of the horizon from any station in miles is approximately equal to the square root of one and a half times the number of feet in the height of the station above the sea level, find in miles the distance of the horizon from a balloon 25,000 feet high to three places of decimals.

8. When the three per cents. were at 90, I found that by selling out and investing in Indian four per cents. at 95, I could improve my income by £24 „ 6*s*. What was the amount of my stock in the three per cents.?

9. A person near the sea shore sees the flash of a gun fired in a vessel steaming directly towards him, and hears the report in 15″. He then walks towards the ship at the rate of three miles an hour, and sees a second flash five minutes after the first, and immediately stops; the report follows in $10\frac{1}{2}$″. Find the rate of the ship, taking the velocity of sound at 1200 feet per second.

10. A person walks to a town at the rate of $3\frac{3}{4}$ miles per hour, and rides back at the rate of $7\frac{1}{2}$ miles per hour, after having rested half an hour. He then finds he has been absent 4 hours „ 10 min. What was the length of the walk?

11. At a school inspection $\frac{4}{5}$ of the numbers in average attendance were eligible for examination, but 25 per cent. of the average number were absent. Of those present and examined 8 per cent. failed in reading, and 14 per cent. in each of the other two subjects. The grant on examination was £52 „ 16*s*., at the rate of 2*s*. „ 8*d*. for each pass in each subject. Find the number examined, and the average for the year.

XVIII. *Civil Service Commissions.*

1. If by selling wine at 15*s*. a gallon I lose 10 per cent, at what price must I sell it to gain 15 per cent.?

2. Find the cube root of 134217728.

3. Multiply $\cdot 0\dot{0}2\dot{1}$ by $48\cdot 92\dot{6}$.

4. The content of a cistern is the sum of two cubes whose edges are 10 inches and 2 inches, and the area of its base is the difference of two squares whose sides are $1\frac{1}{5}$ and $1\frac{3}{5}$ feet. Find its depth.

5. If a man rows 10 miles in 2 hours and a half against a stream, the rate of which is 3 miles an hour, how long would he be in rowing 5 miles with the stream?

6. What must be the rate of interest in order that the discount on £1936 „ 18*s.* payable at the end of 3 years may be £207 „ 10*s.* „ 6*d.*?

7. If 48 pioneers, in 5 days of $12\frac{1}{2}$ hours long, can dig a trench 139·75 yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep; how many hours per day must 360 pioneers work during 42 days, in order to dig a trench $4910\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{3}$ yards deep?

XIX. *Civil Service Commissions.*

1. If a steamer makes the passage from New York to Liverpool (say 2760 miles) in 9 days „ 14 hours, and a train goes from London to Edinburgh (say 405 miles) in 18 hours: compare the rates of the steamer and the train.

2. Find the square root exactly of $2515\frac{294}{361}$.

3. Extract the cube root of 5·78 to three places of decimals.

4. Multiply by the method of duodecimals 3 ft. „ 1 in. „ 11 pts., by 2 ft. „ 6 in. „ 7 pts., and the product by 1 ft. „ 7 in.

5. Express the result of the last question in cubic feet, cubic inches, and a fraction of a cubic inch.

6. Divide $4\cdot 0\dot{3}$ by $\cdot 1\dot{4}0\dot{7}$.

7. Find the average of $21\frac{2}{3}$, $73\frac{4}{5}$, 0, 3·065, 82, $17\frac{3}{20}$, $5\frac{1}{4}$, $9\frac{5}{12}$; express the fractional part decimally.

8. A person sells as many 3 per cent. consols at $98\frac{5}{8}$ as produce £2000, and invests this sum in railway stock, paying $4\frac{1}{4}$ per cent., at $93\frac{3}{4}$. How is his income affected?

9. A person buys coffee at £5 „ 12*s.* „ 6*d.* per cwt. and chicory at £2 „ 5*s.* „ 5*d.* per cwt., and mixes them in the pro-

portion of two of chicory to five of coffee. He retails the mixture at 1*s.* „ 3*d.* per lb. What is his gain per cent.?

10. Find the true discount on £512 „ 15*s.* „ 3*d.* due 52 days hence at $2\frac{1}{2}d.$ per cent. a day.

11. If 5 men can perform a piece of work in 12 days of 10 hours each, how many men will perform a piece of work four times as large in a fifth part of the time, if they work the same number of hours in the day, supposing that 3 of the second set can do as much work in an hour as 2 of the first set.

12. A canal 10 miles long is 8 yards wide at the top, 6 yards wide at the bottom, and 5 feet deep. How soon would the excavation of it be completed by 800 men, each removing on an average 15 cubic yards per day?

13. The rate of a clock is ·0375 *per cent.* too fast. How much will the clock gain in a week?

14. A vessel whose speed was $9\frac{1}{4}$ miles per hour started at 8 o'clock to go a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 to 5, starting from the same place, arrived 5 minutes before the first. When did the second vessel start?

15. At a siege it was found that a certain length of trench could be dug by the soldiers and navvies in 4 days, but that when only half the navvies were present it required 7 days to dig the same length of trench. What proportion of the work was done by the soldiers?

XX. *Civil Service Commission.*

(Averages and Percentages.)

1. Find the average of $13\frac{12}{25}$, 21, $7\frac{3}{4}$, ·0023, $3\frac{1}{8}$, 0, $106\frac{1}{2}$, and $57\frac{7}{20}$; express the fractional part decimally.

2. If by selling wine at 15*s.* a gallon I lose 6 per cent., at what price must I sell it to gain $17\frac{1}{2}$ per cent.?

3. Of 32 selected candidates for the East Indian Civil Service in 1859, 3 were above 20 years of age when they went to India, 4 above 21, 12 above 22 and 23 respectively, and 1 above 24. From these data find what is the average age at which the men went to India.

4. A merchant has teas worth 4s. „ 6d. and 3s. „ 6d. per lb. respectively, which he mixes in the proportion of 3 lbs. of the former to 2 of the latter, and sells the mixture at 4s. „ 4d. per lb.; what does he gain or lose per cent.?

5. Between the years 1841 and 1851 the population of England increased 14·2 per cent. In the latter year it was 21,121290. What was it in the former year?

6. A person invests £5460 in the 3 per cents. at 91, he sells out £2000 stock when they have risen to 93½, and the remainder when they have fallen to 85; he then invests the produce in 4½ per cents. at 102. What is the difference in his income?

7. What must be the market value of 6 per cent. stock, in order that, after deducting the income-tax of 10d. in the pound, it may yield 6½ per cent. interest?

8. If the Roman Catholics are 3 to 1 of the population of Ireland, and the Protestant Dissenters bear the proportion of 2 to 3 to the members of the Established Church, find the proportion *per cent.* which the Protestant Dissenters bear to the Roman Catholics.

XXI. *Civil Service Commission.*

(Purchase of Stock, and Exchange.)

1. When a 3¼ per cent. stock is at 93, find what price a 4½ per cent. stock must bear, that an investment may be made with equal advantage in either stock.

2. A person sells Midland stock, paying 6½ per cent., at 128½, and invests in Great Western stock, paying 3 per cent., at 72½. By how much per cent. will the interest of his investment be altered?

3. A person invests £5000 in the new 6 per cent. Turkish Loan, issued at 68 per cent., at 2½ premium: how much stock will he have? and what rate of interest will the investment give?

4. What must be the market value of 3 per cent. stock, in order that, after deducting an income-tax of 10d. in the pound, it may yield 3½ per cent. interest?

5. What is meant by the *par of exchange* between two countries? When is the exchange said to be against a coun-

try? Explain briefly why the course of exchange between two countries varies.

6. If £3 = 20 Thalers; 25 Thalers = 93 Francs; 27 Francs = 5 Scudi; and 62 Scudi = 135 Gulden; how many Gulden = £1?

7. A trader in London owes a debt of 1,000 pistoles to one in Cadiz; find what he gains by sending it to him through France, the exchanges being £1 = 25·4 Francs; 19 Francs = 1 Spanish Pistole; 4 Spanish Pistoles = £3.

8. A person in London owes another in St Petersburgh 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 25·15 francs for £1, and between Paris and St Petersburgh 1·2 francs for 1 rouble. The remittance is delayed until the rates are 25·35 francs for £1 and 1·15 francs for 1 rouble. What does the broker gain or lose by the delay?

XXII. *Examination for Direct Commissions*, 1865.

1. How many times is £1 „ 11*s.* „ 2*d.* contained in £162 „ 1*s.* „ 4*d.*?

2. A man steps 2 feet „ 3 inches, how many steps does he take in walking 6 miles?

3. If 180 men can make a road in 15 days, in what time would 270 men make a road twice as long as the first?

4. If 7 fires burning 10 hours a day consume 4 tons „ 10 cwt. of coal in 30 days, how much coal will be consumed by 20 fires in 12 days burning for 14 hours a day?

5. Find the simple interest on £5656 „ 5*s.* for 6 years at $4\frac{1}{2}$ per cent. per annum.

6. Add together $\frac{7}{5}$ of $3\frac{1}{3}$, $\frac{5}{7}$ of $1\frac{10}{11}$, and $\frac{341}{363}$; divide the result obtained by $\frac{9}{11}$ of $\frac{10}{11}$.

7. It being given that $5\frac{1}{2}$ yards, linear measure, make one pole or perch, find the number of square yards in an acre.

A field is 55 yards long by 40 yards wide: express the area of the field as the fraction of an acre.

8. Multiply 21·56 by ·0035. Divide ·25 by 31·25. Verify the last result by vulgar fractions.

9. Find the value of ·125 of £88 „ 16*s.*; and the value of ·$\dot{3}$ of 5 guineas.

10. Determine by how much the square of 1·732 differs from 3: find the square root of 71 to three decimal places.

XXIII. *Examination for Direct Commissions*, 1865.

1. How many times is £17 „ 14*s.* „ 5*d.* contained in £655 „ 13*s.* „ 5*d.*?

2. Find the price of a mixture of 1 cwt. of black tea at 3*s.* „ 2*d.* a pound, with 20 pounds of green tea at 5*s.* „ 3*d.* a pound.

3. If 20 lamp-posts are required to light a road 1600 yards long, how many will be required for a road 4 miles long?

4. Find the cost of carpeting a room 34 ft. „ 6 in. long, by 18 ft. „ 4 in. wide, at 3*s.* „ 9*d.* a square yard.

5. Find the simple interest on £9062 „ 10*s.* for 6 years at $3\frac{3}{4}$ per cent. per annum.

6. Subtract $2\frac{13}{18}$ from $3\frac{1}{4}$; and divide $\frac{231}{535}$ by $\frac{77}{107}$, expressing the result in its lowest terms.

7. Multiply 24·35 by ·074; and divide 1·8019 by 243·5.

8. What fraction of £1 „ 2*s.* „ 6*d.* is $\frac{4}{3}$ of 2*s.* „ 6*d.*? Find the value of ·075 of £3 „ 5*s.*

9. Find $(\cdot03)^3$, and extract the square root of 484·176016.

XXIV. *Examination for Direct Commissions*, 1865.

1. Find the number of inches in two hundred thousand yards, and write the answer in words.

2. If 53 articles cost £10 „ 7*s.* „ 7*d.*, what is the price of each?

3. Find the value of 17 quarters „ 3 bushels „ 1 gallon of [illegible]t 1*s.* „ 4*d.* the peck.

4. In a piece of plate weighing 3 lbs. „ 9 ozs. „ 7 grs. there is alloy weighing 10 ozs. „ 8 dwts. „ 19 grs., what is the weight of the pure silver?

5. If the carriage of 8 cwt. for 128 miles be 24*s*., what weight can be carried 32 miles at the same rate for 18*s*.?

6. Reduce $\frac{1}{1001}$ to a decimal.

7. Find the value of ·3625 of £4.

8. Express 1 acre „ 3 roods „ 26 poles as the decimal of a square mile.

9. Find the simple interest upon £830 for 15 months at $3\frac{1}{2}$ per cent. per annum.

10. Extract the square root of 9042049.

XXV. *Direct Commissions. November*, 1868.

1. Multiply £12 „ 13*s*. „ 4*d*. by 1003; find the value of $\frac{1}{800}$ of £1020.

2. Express 132144 inches in miles, yards, and feet.

3. If the average step of a horse be $2\frac{3}{4}$ feet, and that of a man $2\frac{1}{2}$ feet, how many steps must a man take in the same distance that a horse takes 1200 steps, and what is that distance in yards?

4. If 16 fires consume 6 tons „ 6 cwt. of coal in 21 working days of 12 hours each, what weight of coal will be consumed by 24 fires in 7 working days of 15 hours each?

5. The simple interest on £132 „ 15*s*. for 4 years is £17 „ 14*s*.; find the rate per cent.

6. Subtract $\frac{88}{99}$ from $\frac{889}{999}$. Reduce to its simplest form the fraction $\dfrac{1+\frac{1}{6}-\frac{1}{4}}{\frac{11}{16}+\frac{1}{4}-\frac{1}{48}}$.

7. Express as a decimal $13+\frac{13}{10}+\frac{13}{100}+\frac{13}{1000}$; divide 818·118 by 6·04.

8. Find the difference in money value between ·85 of a guinea and ·87 of a pound sterling.

9. Taking the length of the year as 365·25 days, express 73·05 days as the decimal of a year.

10. Reduce $(\cdot 01)^2 \times (1\cdot 25)^2$. Extract the square root of 729·05940121.

XXVI. *Direct Commissions. May*, 1869.

1. Find the greatest number which will divide without any remainder each of the three numbers

2622, 2793, 2736.

2. Find the number of inches in 3 miles „ 1 furlong „ 7 yards.

3. Divide 64 tons „ 10 cwt. „ 5 lbs. „ 4 oz. by 73.

4. If the food of 123 men for 26 days cost £299 „ 16*s.* „ 3*d.*, what is the cost of the daily food of each man?

5. If a man performs a journey of 200 miles in 8 days when he travels 8 hours a day, how many hours a day must he travel at the same pace to perform a journey of 1125 miles in 60 days?

6. Add together the three quantities

$$4\tfrac{1}{3},\ \frac{3}{4} \text{ of } \frac{7}{8},\ \frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{5}{8}.$$

7. Find the value of $\frac{12}{7}$ of £1 „ 7*s.* „ $8\frac{1}{2}$*d.*, and express the result as a decimal of £10.

8. Find the number of square yards in 26·675 acres.

9. What is the principal sum on which the interest for 9 months at 4 per cent. per annum is £61 „ 4*s.*?

10. Extract the square root of 25908100.

XXVII. *Admission to the Staff College*, 1863.

1. Multiply £112 „ 5*s.* „ 8*d.* by 123.

2. A bale of cotton weighs 3 cwt. „ 2 qrs. „ 15 lbs.; 25 such bales were bought at $9\frac{1}{2}$*d.* per pound and sold at 1*s.* „ $0\frac{1}{2}$*d.* per pound: find the profit on the transaction.

3. A tower 103 feet high cast a shadow, the length of which was 79 feet „ 3 inches; find the length of the shadow cast at the same time by a tower whose height was 68 feet „ 8 inches.

4. In English gunpowder, 75 parts by weight are saltpetre, 10 parts sulphur, and 15 parts charcoal. How many pounds weight of each ingredient are used in the manufacture of 16 cwt. of gunpowder?

5. The simple interest on £19687 „ 10*s.* for five years is £5414 „ 1*s.* „ 3*d.*: find the rate per cent.

6. (A) has £10,000 stock in the 3 per cents; he sells out all his stock at $92\frac{1}{4}$; he then invests the purchase-money in railway shares of £20 each which pay 6 per cent. per annum, (A) paying £25 for each £20 share: find the change in his income.

7. Explain why dividing the numerator and denominator of a vulgar fraction by a whole number does not alter the numerical value of the fraction.

Reduce the fraction $\frac{3390}{17741}$ to its lowest terms.

8. Reduce to its simplest form

$$\frac{\dfrac{\frac{2}{5}}{1-\frac{1}{25}}+\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{7}\text{ of}\left(\dfrac{\frac{2}{5}}{1-\frac{1}{25}}+\frac{1}{3}\right)}$$

What fraction of 2*s.* „ 6*d.* is $\frac{1}{16}$ of $7\frac{1}{2}d.$

9. Express $\frac{63}{64}$ as a decimal fraction; point out how the denominator of this fraction indicates the number of decimal places in the result.

Find the value of $\cdot\dot{1}4285\dot{7}$; and multiply $\cdot\dot{1}4285\dot{7}$ by $\cdot\dot{6}\dot{3}$, expressing the product as a circulating decimal.

10. Divide $(\cdot 001)^3$ by $(\cdot 0002)^2$. Extract the square root of 257050·014001. Extract the square root of $5\frac{1}{2}$ to four decimal places; and write down the *true* value of the remainder when the four places have been obtained in the root.

XXVIII. *Oxford Local Examinations. Senior Students,* 1868.

1. How many pounds are there in 91200 farthings?

2. Divide £3050 „ 9*s.* „ $10\frac{1}{2}$*d.* by 81.

3. Find by Practice the value of 245 things at £3 „ 19*s.* „ $9\frac{3}{4}$*d.* each; and of 7 cwt. „ 3 qrs. „ 26 lbs. at £1 „ 10*s.* „ 4*d.* per cwt.

4. Simplify $\frac{261}{3103}$; and $\dfrac{3}{8-\dfrac{7}{2-\frac{3}{4}}}+\dfrac{5}{6-\dfrac{5}{2-\frac{5}{6}}}$.

5. Add together $\frac{2}{3}$ of a crown; $\frac{13}{126}$ of a guinea; $\cdot\dot{3}$ of 18*s.* „ 6*d.*; and $\cdot41\dot{6}$ of £1.

6. Divide ·024 by 60; 24 by ·006; and 2·4 by ·06.

7. Express as Vulgar Fractions ·375, ·0375, $\cdot01\dot{6}\dot{9}$.

8. What Vulgar Fraction and what decimal fraction is $1\frac{1}{2}$ feet of $\frac{1}{4}$ of a mile?

9. Extract the square root of 49196196 and ·0016.

10. Compare the simple and compound interest on £119 at the end of three years, at 4 per cent. per annum.

11. How many sovereigns are there in 80 lbs. of standard gold? an oz. of standard gold being worth £3 „ 17*s.* „ $10\frac{1}{2}$*d.*

12. How many planks, each $13\frac{1}{2}$ feet long and $10\frac{1}{2}$ inches wide, will be required for the construction of a platform 54 yards long and 21 yards broad? What will be the cost at $5\frac{1}{2}$*d.* per square foot?

13. If 5 horses eat 8 bushels „ $1\frac{3}{4}$ pecks of oats in 9 days, how long at the same rate will 66 bushels „ $3\frac{3}{4}$ pecks last 17 horses?

XXIX. *Oxford Local Examinations. Senior Students,* 1869.

1. Find the number of pounds (Troy weight) in two million three hundred and fifty thousand and eighty grains. Divide 10 shillings among sixty persons.

2. Divide one thousand five hundred and forty-nine pounds, nineteen shillings, and four pence one farthing, by thirty-one. Multiply one hundred weight twenty-seven pounds fifteen ounces by seventeen.

3. Find by practice the value of 309 things at £2 „ 18*s*. „ 4*d*. each; and the cost of 7 cwt. „ 2 qrs. „ 24 lbs. at £1 „ 7*s*. „ 5*d*. per cwt.

4. Simplify $\frac{377}{403}$; and $\dfrac{\frac{2}{9} \text{ of } 1\frac{3}{8} \text{ of } 2\frac{2}{7}}{\frac{7}{9}+\frac{5}{12}-\frac{3}{4}}$.

5. Add together $\frac{1}{3}$ of 2*s*. „ 6*d*.; $\frac{1}{6}$ of a guinea; $\frac{4}{15}$ of £1; and ·01$\dot{3}$ of £1 „ 5*s*.

6. Divide 1·44 by 1·2; 144 by ·6; and ·144 by ·016.

7. What part of a pound is 8*d*.? What decimal fraction is a scruple of a pound?

8. Cube 1·04, and extract the square root of 870014016.

9. Compare the simple and compound interest on £10 „ 8*s*. at the end of 3 years, reckoning money at 4 per cent. per annum.

10. Which is the better investment, the 4 per cents at 120, or the 2½ per cents at 75?

11. If £4 „ 14*s*. „ 10½*d*. buy 288 yards, how many can be bought for £1 „ 11*s*. „ 7½*d*.?

12. If 70 men dig 1 acre „ 2 roods „ 10 perches in 5 days, how many men will dig 2¼ acres in 28 days?

XXX. *Cambridge Local Examinations*, 1863.

1. Express in words and in figures, how much greater the value of one 5 is than the other, in the number 658457.

2. Multiply 129847 by 468. If in the process you shift all the figures resulting from the multiplication of the multiplicand by 4, two places farther to the left and then add, of what two numbers will the result be the product?

3. Divide 19094867 by 4009: hence write down at sight the quotients of 3058867 and 252567 when divided by 4009.

4. Give the rules for addition and subtraction of fractions. Add together 10, $7\frac{4}{9}$, $\frac{5}{27}$, $4\frac{2}{3}$, and $3\frac{1}{3}$ of $\frac{1}{9}$.

5. Compare the values of the fractions

$$\frac{11\times 4}{5\times 9},\ \frac{12\times 3}{4\times 10},\ \frac{10\times 5}{6\times 8},\ \text{and}\ \frac{11+4}{5+9}.$$

6. Reduce 3 ac. „ 1 ro. „ 20 po. to the fraction of 21 acres.

7. Multiply 35·85 by 2·09 and 3·585 by ·00209.

8. Divide ·005868 by ·036, and arrange the divisor, dividend, and quotient in order of magnitude.

9. Reduce $\frac{39}{40}$ to a decimal. What is the equivalent decimal fraction? What is done to $\frac{39}{40}$ to bring it into this form?

10. Reduce $\frac{12}{25}$ths of £20 to the decimal of £100.

11. Express the difference between $2\cdot\dot{5}3\dot{5}$ and 2·535 (1) by a circulating decimal, and (2) by its equivalent vulgar fraction.

12. If the area of a square field contain 824464 sq. yds., find the length of its side.

13. Define discount, and find the discount on a bill of £10 due at the end of a year at 10 per cent.

14. What sum of money is necessary to pay $\frac{1}{8}$ per cent. commission, and to purchase £5000 stock in the Funds when they are $92\frac{3}{4}$?

XXXI. *Cambridge Local Examinations. Senior Students,* 1868.

1. Subtract 34876 from 72093, and explain the process.

2. Find by how much the product of 567 and 809 is greater than their sum.

3. If $3\frac{1}{4}$ tons of coals are bought at 18*s.* per ton and sold at 1*s.* „ 2*d.* per cwt., what is gained by the transaction?

4. Simplify $2\frac{1}{3}$ of $5\frac{1}{6} - 1\frac{7}{8}$ of $6\frac{1}{15}$.

5. State the rule for converting any circulating decimal into a vulgar fraction, and explain it by means of the example $\cdot 5\dot{7}$.

Find the value of $\cdot 04\dot{8}$ of £1 „ 12*s.* „ $9\frac{3}{4}$*d.*

6. Divide ·482 by 37·164 to four places of decimals.

7. If 1 qr. „ 2 bus. „ 2 pks. of wheat be worth £3 „ 11*s.* „ 9*d.*, how much wheat can be bought for £85 „ 8*s.* „ 4*d.*?

8. Find by practice the value of a farm of 105 acres „ 3 roods „ 16 poles at £32 „ 1*s.* „ 8*d.* per acre.

9. A cubic foot of water weighs $62\frac{1}{2}$ lbs.; and a room 18 ft. „ 9 in. by 13 ft. „ 4 in. is flooded to a depth of 2 inches; what is the weight of water in the room?

10. In a village of 1000 people it is found that 515 are females. Find the percentage of males.

11. Assuming that three hectares contain 35881 square yards, and that one hectare contains 10000 square metres, find the length of a metre.

12. In coining shillings at the Mint a composition is used consisting of silver and copper in the ratio of 37 to 3. The price of silver being 5*s.* „ 2*d.* per ounce, and the weight of a shilling $\frac{2}{11}$ths of an ounce, what would be the exact value of the coin if it were all pure silver?

XXXII. *Oxford Responsions. Michaelmas Term*, 1862.

1. Find the value of $\dfrac{2\frac{3}{4} \text{ of } 3\frac{2}{3}}{2\frac{5}{6} + \frac{1}{2} + \frac{1}{3}}$ multiplied by $\frac{2}{33}$ of 6.

2. Define G.C.M. and L.C.M. Reduce $\frac{9019}{96721}$ to its lowest terms; and find the L.C.M. of 4, 9, 16, 28, 42.

3. Reduce 5*s.* „ 6*d.* to the decimal of $\frac{4}{7}$ of a guinea; and $\frac{3}{8}$ of a rood to the fraction of $\frac{1}{2}$ of an acre.

4. Multiply ·111 by ·011.

Divide 8·4 by ·7 and by ·0007; and $\cdot\dot{3}$ by $\cdot\dot{0}\dot{3}$.

5. Subtract ·625 of a crown from £1·375.

6. Reduce to decimals $\frac{1}{25}$ of $\frac{1}{2}$, and $8\frac{3}{5}$; and to fractions 1·75, and ·305.

7. If a mina = £4 „ 1*s.* „ 3*d.*, reduce to minæ £568 „ 15*s.*

8. How many yards of carpet $\frac{1}{2}$ yard wide will cover a floor 12 yards long by 6 yards „ 1 foot „ 6 inches broad?

9. If 9 men reap a field of 8 acres in 12 hours, how many men will reap a field of 28 acres in 18 hours?

10. Find the simple interest of £442 at 4 per cent. for 5 years; and the compound interest of £500 at 3 per cent. for 2 years.

11. If a man spends as much in 4 months as he gains in 3, how much will he lay by in a year from an income of £150?

12. Find the income produced by £26260, if invested in the $3\frac{1}{2}$ per cents at 91.

13. Divide £154 between 4 persons, in the proportion of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

XXXIII. *Oxford Responsions. Trinity Term*, 1865.

1. Find the G.C.M. of 48849 and 59133, and the L.C.M. of 3, 5, 7, 9, 15, 63.

2. Divide $1-\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{24}\right)$ by $1-\left(\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{24}\right)$; and find the value of $\dfrac{\frac{1}{6}+\frac{1}{8}}{1\frac{1}{2}+\frac{3}{14}}$.

3. Reduce 1 lb. Troy to the fraction of 1 lb. Avoirdupois.

How many minutes are there in $\frac{5}{73}$ of a year $+\frac{1}{56}$ of a week $+\frac{5}{12}$ of an hour?

4. Multiply ·000725 by 31·25, and divide the product successively by 6·25, 625, and ·0625.

5. Reduce to vulgar fractions ·0015625, ·0$\dot{5}$, ·03$\dot{1}\dot{8}$.

6. Find the value of 3·15625 of a £, and bring 3 lbs. „ 8 oz. to the decimal of a cwt.

7. Find the square root of 106929, and of 4·1209.

8. At the census of 1851 the population of Oxford was 27,843: at the census of 1861 the population was 27,560. What was the decrease per cent.?

(N.B. The answer need not be carried beyond 3 places of decimals.)

9. *A* can walk 5 miles while *B* is walking 4. Supposing *A* to walk 6 hours a day, and *B* 7 hours a day, how many days will *B* take in walking a distance, which *A* can accomplish in 14 days?

10. Find the simple interest on £354 „ 3*s.* „ 4*d.* for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.; and the compound interest on £266 „ 13*s.* „ 4*d.* for 2 years at 3 per cent.

11. A quadrangle is 50 feet long by 40 feet broad: it is crossed in each direction by a path 10 feet broad, and the remainder has to be turfed. How many strips of turf $1\frac{1}{2}$ feet long and 6 inches broad will be required for the purpose?

XXXIV. *Oxford Responsions. Michaelmas Term*, 1865.

1. Find the value of $\dfrac{\frac{7}{10}-\frac{2}{3}}{\frac{9}{12}+\frac{1}{5}} \div \dfrac{\frac{1}{3}}{9\frac{1}{2}}$ and of $\frac{1}{2}+\frac{2}{9}-\frac{4}{15}+\frac{5}{18}$.

2. £4500 is divided between *A*, *B*, *C*, *D*. *A* receives $\frac{1}{2}$; *B*, *C* each $\frac{2}{5}$ of the remainder. How much is left for *D*?

3. Reduce 1 lb. „ 12 oz. (avoirdupois weight) to the fraction of 1 cwt. „ 2 qrs.; and $\frac{5}{16}$ of 6*s.* „ 8*d.* to the fraction of a guinea.

4. Express as vulgar fractions ·001375, $\cdot\dot{0}\dot{3}$, $7\cdot193\dot{1}\dot{8}$; and divide ·175 by ·25, by 2·5, and by ·0025.

5. Find the value of $\cdot\dot{3}$ of a guinea + ·125 of a pound

+ ·208$\dot{3}$ of a shilling + ·5 of a penny; and bring 10 weeks „ 3 days to the decimal of a year.

6. Find the square root of 175·5625, and of 4080400.

7. *A* can walk 10 miles in $2\frac{1}{4}$ hours, *B* can walk 11 miles in $2\frac{1}{2}$ hours. They start to walk a match from London to Oxford a distance of 55 miles: which will arrive first, and by what amount of time will he win?

8. The profit made by selling beer at 1*d.* per pint above cost price is 50 per cent.: what is the gain per cent. made by selling it at 6*d.* a gallon above cost price?

9. Find the simple interest on £608 „ 6*s.* „ 8*d.* for 3 years at $4\frac{1}{8}$ per cent.; and the compound interest on £66 „ 13*s.* „ 4*d.* for 2 years at 9 per cent.

10. What will be the cost of papering a room 20 feet long, 12 feet broad, 10 feet high, with paper $2\frac{2}{3}$ feet broad, at $4\frac{1}{2}d.$ per yard, an extra $\frac{1}{2}d.$ per yard being charged for hanging?

11. A sovereign weighs $\frac{160}{623}$ of an oz. Troy: how many sovereigns may be coined out of a piece of metal weighing 1 lb. Troy? This metal being composed of $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy, what is the value (expressed in money) of an oz. of pure gold?

XXXV. *Cambridge Previous Examination. First Division A*, 1856.

1. *Explain by what method we are able with only the nine digits and a cipher (by our decimal system) to express any number however large.* Cf. § 11.

Multiply 2357 *by* 5, *explaining clearly each step of the process and its reason.* Cf. § 24 (*a*).

Define multiplication. Cf. § 22.

2. (*a*) *The income-tax of* £1 *is* 1*s.* „ 4*d.*; *what is it on* £100 „ 17*s.* „ 6*d.*?

(*β*) *A bankrupt pays* 17*s.* „ 6*d. in the pound; how much does he pay on* £267 „ 6*s.* „ 8*d.*?

N. B. (α) *and* (β) *are to be done by "Practice." To what class of examples is the rule applicable?* Cf. § 82.

$$
\begin{array}{lll|l}
(\alpha) & 1s. & \frac{1}{20} & \quad \pounds \quad s. \quad d. \\
 & & & 100 \,,, 17 \,,, 6 \\
 & 4d. & \frac{1}{3} & 5 \,,, 0 \,,, 10\frac{1}{2} = \text{tax at } 1s. \text{ in the pound} \\
 & & & 1 \,,, 13 \,,, 7\frac{1}{2} = \text{......} \ 4d. \ \text{...............} \\
\hline
 & & & \pounds 6 \,,, 14 \,,, 6 = \text{......} \ 1s. \,,, 4d. \ \text{.........}
\end{array}
$$

$$
\begin{array}{lll|r}
(\beta) & 2s.\ 6d. & \frac{1}{8} & \pounds \quad s. \quad d. \\
 & & & 267 \,,, 6 \,,, 8 \\
\hline
 & & & 33 \,,, 8 \,,, 4 \\
 & & & 7 \\
\hline
 & & & \pounds 233 \,,, 18 \,,, 4
\end{array}
$$

3. *Reduce* $\dfrac{3\frac{1}{2} - 2\frac{1}{6}}{\frac{1}{4} \text{ of } \left(\frac{1}{5} + \frac{1}{7}\right)} \div 15\frac{5}{9}$ *to its most simple form.*

$$\text{The fraction} = \frac{\frac{7}{2} - \frac{13}{6}}{\frac{1}{4} \times \frac{12}{35}} \div \frac{140}{9}$$

$$= \frac{8}{6} \times \frac{35}{3} \times \frac{9}{140}$$

$$= \frac{4 \times 7 \times 3}{3 \times 28}$$

$$= 1.$$

4. *The distance from Yarmouth to Norwich is* $20\frac{1}{2}$ *miles, and from Cambridge to London* $57\frac{1}{2}$; *and the third class fares are* 1s. „ 3d. *and* 8s. *respectively: how much would have to be deducted from the present third class fare per mile between Cambridge and London, so that it might be just double the third class fare per mile between Yarmouth and Norwich?*

Between Yarmouth and Norwich the fare per mile is $15d. \times \frac{2}{41}$.

Between Cambridge and London the fare per mile is $96 \times \frac{2}{115}$.

Now the *double* of $\frac{15\times 2}{41}$ is $\frac{60}{41}$.

Therefore $$\frac{192}{115}-\frac{60}{41}=Ans.$$

or $$Ans.=\frac{7872-6900}{4715}=\frac{972}{4715} \text{ of a penny.}$$

5. (1) *Multiply* £1875 „ 13*s.* „ $8\frac{3}{4}$*d. by* 21. (2) *Divide* £2 „ 12*s.* „ 3*d. by* 1*s.* „ $4\frac{1}{2}$*d.* (3) *Reduce* $\frac{1}{7}$ *of* £1 *to the fraction of* 19*s.* „ 6*d.* (4) *Find a sum of money which shall be the same fraction of* £61 „ 9*s.* „ 1*d., that* 2 cwt. „ 2 qrs. „ 10 lbs. *is of* 36 cwt. „ 1 qr.

(5) *Prove the rule for the division of two fractions, taking* $\frac{4}{5}\div\frac{3}{7}$ *as an example.*

(1)

£	s.	d.
1875 „	13 „	$8\frac{3}{4}$
		7
13129 „	16 „	$1\frac{1}{4}$
		3
39389 „	8 „	$3\frac{3}{4}$

(2) $$52\tfrac{1}{4}s.\div 1\tfrac{3}{8}s.=\frac{209}{4}\times\frac{8}{11}$$
$$=19\times 2$$
$$=38 \textit{ times.}$$

(3) $$\frac{1}{7}\text{ of } 20s.\div 19\tfrac{1}{2}s.=\frac{20}{7}\times\frac{2}{39}=\frac{40}{273}\ Ans.$$

(4) $$2\text{ cwt.} „ 2\text{ qrs.} „ 10\text{ lbs.}=\overset{\text{qrs.}}{10\tfrac{5}{14}},$$
$$36\text{ cwt.} „ 1\text{ qr.}=\overset{\text{qrs.}}{145};$$

therefore $$\frac{145}{14}\div 145=\frac{1}{14}.$$

Now to find the sum of money which is $\frac{1}{14}$ of 61 „ 9*s.* „ 1*d.*

$$
\begin{array}{l}
\quad\ \ \overset{\pounds}{}\quad \overset{s.}{}\quad \overset{d.}{}\quad \overset{\pounds}{}\quad \overset{s.}{}\quad \overset{d.}{}\\
14)\ 61\ ,,\ 9\ ,,\ 1\ (4\ ,,\ 7\ ,,\ 9\tfrac{1}{2}\\
\quad\ \ 5\\
\quad\ \ \underline{20}\\
\quad\ \ 109\\
\quad\ \ 11\\
\quad\ \ \underline{12}\\
\quad\ \ 133\\
\quad\ \ \ \ 7\\
\quad\ \ \ \ \underline{4}\\
\quad\ \ \underline{28}\\
\quad\ \ 00
\end{array}
$$

(5) For the division of Fractions, Cf. § 59.

6. *When are four quantities said to be in proportion? and shew by means of your definition, that*

6 *yds.* „ 3 *qrs.* : 73 *yds.* „ 2 *qrs.* :: 5*s.* „ 3*d.* : £2 „ 17*s.* „ 2*d.*

And deduce the method of solving the following question: "If 6 *yds.* „ 3 *qrs. cost* 5*s.* „ 3*d.*, *what will* 73 *yds.* „ 2 *qrs. cost?"*

For the definition of Proportion, Cf. § 88.

Since $\frac{6\frac{3}{4}}{73\frac{1}{2}} = \frac{9}{98}$, and $\frac{5\frac{1}{4}}{57\frac{1}{6}} = \frac{9}{98}$, we conclude that the four given quantities are proportionals.

To solve the given question, we write the proportion as follows;

$$\overset{\text{yds.}}{6\tfrac{3}{4}} : \overset{\text{yds.}}{73\tfrac{1}{2}} :: \overset{s.}{5\tfrac{1}{4}} : \overset{s.}{Ans.},$$

then, multiplying together extremes and means, we obtain the following equality:

$$Ans. \times \frac{27}{4} = \frac{147}{2} \times \frac{21}{4};$$

therefore
$$Ans. = \frac{147 \times 7}{2 \times 9} = \frac{343}{6}\,s.$$
$$= £2\ ,,\ 17s.\ ,,\ 2d.$$

7. (1) *Reduce* 12*s.* „ 6¾*d. to the decimal of* £1; *of* £1000; *and of* £·000001. (2) *Find the value of* ·790625 *of* £1.

(1)
$$
\begin{array}{r|l}
12 & 6\cdot75 \\ \hline
20 & 12\cdot5625 \\ \hline
 & \cdot628125 \text{ decimal of } £1.
\end{array}
$$

Therefore the decimal of £1000 is ·000628125. And the decimal of £·000001 is £628125.

(2) By inspection £·790625 = 15*s.* „ 9¾*d.*

8. *Divide* 1255 *by* 1·004; 12·55 *by* 1004; ·012550 *by* 1004000.

Reduce $17\frac{13}{1000}$, $\frac{123}{10}$, $\frac{4}{125}$, $\frac{5}{8}$, $\frac{5}{16}$ *to decimals, and then add them together.*

Reduce $\frac{5}{7}$ *of* ·375, *and* ·0458$\dot{3}$ *to vulgar fractions in their lowest terms.*

$$\begin{array}{l} 1\cdot004)\ 1255\cdot000\ (1250\ \textit{Ans.} \\ \qquad\qquad 251\,0 \\ \qquad\qquad\ 50\,20 \\ \qquad\qquad\ 00\,00 \end{array}$$

Whence the other quotients are ·0125,

and ·0000000125.

Also 17·013 + 12·3 + ·032 + ·625 + 3·3125 = 33·2825.

Again, $\frac{5}{7} \times \cdot375 = \frac{5}{7} \times \frac{3}{8} = \frac{15}{56}$.

And $\cdot0458\dot{3} = \frac{4125}{90000} = \frac{165}{3600} = \frac{11}{240}$ *Ans.*

9. *Shew that the fraction* $\frac{5}{7}$ *is not altered in value by multiplying* 3 *into the numerator and denominator. How is it that in a decimal fraction we do not alter its value by bringing down to the right hand of the last figure any number of ciphers?* (Cf. § 48 and § 66).

10. *What sum must A bequeath to B so that B may receive* £1000 *after a legacy duty of* 10 *per cent. has been deducted?*

If $\frac{9}{10}$ of legacy = 1000;

$$\text{legacy} = 1000 \times \frac{10}{9}$$

$$= 1111\tfrac{1}{9}$$

$$= £1111\ „\ 2s.\ „\ 2\tfrac{2}{3}d.$$

11. ***Find the simple and compound interest of* £625 *in* 2 *years* 4 *per cent.***

$$\begin{array}{r} 25)\,\underline{625} \\ \overline{25} \\ 25)\,\underline{650} \\ \overline{26} \end{array}$$

therefore in two years the simple interest $= 25 \times 2 =$ £50, and in two years compound interest $= 25 + 26 =$ £51.

12. ***In what time will* £2500 *double itself at* 4 *per cent. simple interest?***

$$\frac{100}{4} = 25 \text{ years } \textit{Ans.},$$

see this explained Chap. xiv., § 96, Ex. 11.

13. ***What must be the rate of interest in order that the discount on* £2573 *payable at the end of* 1 *year* „ 73 *days may be* £93?**

£93, which is discount on the debt, is the simple interest on the present worth, viz. £2480, for $1\frac{73}{365}$ years, or for $1\frac{1}{5}$ years.

Hence $\quad 100 \times 1 \;:\; 2480 \times \frac{6}{5} \;::\; \textit{Ans.} \;:\; 93,$

$$\textit{Ans.} \times 2480 \times \frac{6}{5} = 100 \times 93,$$

$$\textit{Ans.} = \frac{10 \times 31 \times 5}{248 \times 2}$$

$$= \frac{5 \times 5}{8}$$

$$= 3\tfrac{1}{8} \text{ rate of interest.}$$

14. ***Shew that the interest obtained by investing a sum of money in the* 3 *per cents. at* 82½ *is to the interest obtained by investing the same sum in the* 3½ *per cents. at* 93½ *as* 34 : 35.**

By "the interest obtained" is meant the ***true rate of interest per cent.*** which is obtained; and the rates in the two cases given may be compared thus;

$$\frac{100 \times 3 \div \frac{165}{2}}{100 \times 3\frac{1}{2} \div \frac{187}{2}}$$

$$= \frac{3 \times \frac{2}{165}}{\frac{7}{2} \times \frac{2}{187}} = \frac{6}{165} : \frac{7}{187} = \frac{2}{5} : \frac{7}{17} = 34 : 35.$$

15. *A gave 25s. for two tickets (a first and second class) from Norwich to Colchester; what did they cost him separately, if a first class ticket from Norwich to Diss cost 3s. „ 6d., and a second class cost 2s. „ 9d.? Of course the fares throughout the line are supposed to be always proportional to the distance.*

The 25s. must be divided into 2 parts, in the ratio of the sums paid for the first and second class tickets to Diss; *i. e.* in the ratio of $3\frac{1}{2} : 2\frac{3}{4}$.

First class fare $= \dfrac{3\frac{1}{2}}{6\frac{1}{4}}$ of $25 = \dfrac{7}{2} \times \dfrac{4}{25}$ of $25 = 14s.$,

and $25 - 14 = 11s.$, the second class fare.

16. *If in extracting the square root of* 0·2 *you had by mistake "pointed" thus,* $0\cdot\dot{2}0\dot{0}0\dot{0}$, *&c., and then proceeded with the operation, and if after marking off the decimal places in your result you had discovered your mistake, what quantity would you have to multiply the erroneous result by, in order to correct it, without extracting the root of* 0·2 *again? Find the first three places of decimals in this multiplier.*

The figures in the erroneous result would be 1414218, &c. Now if the decimal places in this result be marked off ·1414218, &c., this would be the approximate root of ·02; therefore you have been finding the root of ·02, when you were asked to find the root of ·2.

Now by the question

$$\sqrt{(\cdot 02)} \times x = \sqrt{(\cdot 2)},$$
$$x = \sqrt{(\cdot 2)} \times \frac{1}{\sqrt{(\cdot 02)}}$$
$$= \sqrt{(10)}$$
$$= 3\cdot 1626, \text{ \&c.}$$

First Division B, 1856.

1. Explain our decimal system of Arithmetic, and how it is that we are enabled with a few digits and a cipher to express any number however great. (Cf. § 11.)

Define "division." Divide 3472 by 5, explaining clearly the reason of each step of the process. (Cf. § 27.)

2. (α) What is the amount of income-tax paid on an annuity of 500 guineas, at 7*d.* in the £1?

(β) An article which cost 6*s.* „ 8*d.* is sold for 8*s.* „ $10\frac{2}{3}$*d.*, what is the profit on £100?

Apply the "Rule of Practice" to Examples (α) and (β). What is meant by "aliquot parts"?

3. Reduce $\dfrac{2\frac{3}{4}-1\frac{2}{9}}{\frac{19}{36}+\frac{3}{2}\text{ of }\frac{1}{4}} \div 1\frac{2}{13}$ to its simplest form.

If $1\frac{3}{5}$ of a sum of money $=\frac{3}{7}$ of 5*s.* „ 10*d.*, find the sum.

4. The distance from London to Cambridge is $57\frac{1}{2}$ miles; and from Yarmouth to Norwich $20\frac{1}{2}$. The second class fares between the same places are 11*s.* and 2*s.* respectively: what would have to be added to the present fare per mile (second class) between Cambridge and London, so as to make it exactly double the second class fare per mile between Yarmouth and Norwich?

5. Multiply £721 „ 0*s.* „ $5\frac{1}{4}$*d.* by 96; and divide 1283 cwt. „ 4 lbs. by 75. Reduce $\frac{3}{7}$ of £1 to the fraction of $1\frac{1}{2}$ of £3 „ 5*s.*

Prove the rule for the multiplication of two fractions, taking as an example $\frac{3}{5}\times\frac{4}{7}$. (Cf. § 56.)

6. When are four quantities said to be in proportion? Shew by means of your definition that

£191 „ 12*s.* „ 6*d.* : £31 „ 10*s.* :: 365 days : 60 days;

and *deduce* the method of working the following question: "If 3 workmen earn between them £191 „ 12*s.* „ 6*d.* in a year, in what time would they earn £31 „ 10*s.*?"

7. Reduce 2*s.* „ 6*d.* to the decimal of $\frac{5}{12}$ of £1; and of $\frac{5}{12}$ of £1000 respectively.

Find the value of ·875 of 15*s.* „ 6*d.*

8. Divide 12·55 by ·01004; 1255 by 10·04; and ·001255 by 1004.

Reduce $101\frac{3}{8}$, $\frac{7}{25}$, $4\frac{1}{2000}$ to decimals, and then add them together.

Reduce $\frac{3}{7}$ of 1·375 and $\cdot\dot{2}8571\dot{4}$ to vulgar fractions in their lowest terms.

9. Shew that the fraction $\frac{3}{4}$ is not altered in value by multiplying 5 into numerator and denominator. How is it that we do not alter the value of a *decimal* fraction by bringing down any number of ciphers to the right hand of the last figure? (Cf. § 48 and § 65.)

10. After paying an income-tax of 10 per cent., a person has £1250 a year; what was his entire income?

11. Find the difference between the simple and compound interest of £3300 at $3\frac{1}{3}$ per cent. for 2 years.

12. In what time will £537 „ 16*s.* „ 8*d.* amount to £591 „ 12*s.* „ 4*d.* at $2\frac{1}{2}$ per cent. simple interest?

13. What must be the rate of interest in order that the discount on £387 „ 7*s.* „ $7\frac{1}{2}$*d.* payable at the end of 3 years may be £41 „ 10*s.* „ $1\frac{1}{2}$*d.*?

14. At what price must the $3\frac{1}{2}$ per cents. be, in order that a person may obtain an equal rate of interest by investing in them, as he would by investing in the 3 per cents. at 72?

15. A person taking two tickets (a 1st and a 2nd class) from Norwich to Stowmarket receives 7*s.* „ 6*d.* change out of a sovereign, how much had he to pay for each ticket separately, supposing that the 1st and 2nd class fares from Norwich to Diss are 3*s.* „ 6*d.* and 2*s.* „ 9*d.* respectively? Of course the fares throughout are supposed proportional to the distance.

16. In extracting the square root of 0·003 you have by mistake "pointed" thus $0\cdot\dot{0}0\dot{3}0\dot{0}$, &c.; and proceeded with the operation and marked off the decimals accordingly. Without extracting the root of 0·003 over again, there is a certain quantity which if *multiplied* into your erroneous result, will give a correct value of $\sqrt{(\cdot 003)}$; find the first three decimal places of this multiplier.

Second Division A, 1856.

1. Prove that 5 times 27 = 27 times 5; and that

$$\frac{1}{5} \text{ of } 3 = \frac{3}{5} \text{ of } 1.$$

2. (α) What is the dividend on £2045 „ 15*s.* „ 9*d.* at 5*s.* „ 11½*d.* in the £?

(β) Find the value of 9 yds. „ 2 ft. , 10 in. at 5*s.* „ 7½*d.* per yard.

N.B. (α) and (β) both by "Practice." To what class of examples is the Rule of Practice applicable? what is the meaning of an aliquot part?

3. Easter-day is always the Sunday directly *following* the *first* full moon which falls *after* March 20th. There will be a full moon on March 21st, 1856 (a Friday), February in 1856 has 29 days, being a leap year. Find from these data when Easter Sunday fell in 1854.

4. Find the area of a room 12 ft. „ 4 in. long by 10 ft. „ 5 in. broad, by duodecimals or cross multiplication. If, in this example, the room were not supposed to be a *rectangular* parallelogram, how would the answer have to be interpreted?

5. Add together

$$\frac{1}{2} \text{ of } 2s. \text{ „ } 6\tfrac{3}{4}d. + \frac{1}{3} \text{ of } £3 \text{ „ } 2s. \text{ „ } 6\tfrac{1}{4}d. + \frac{1}{6} \text{ of } £5 \text{ „ } 7s. \text{ „ } 3\tfrac{1}{4}d.,$$

and reduce to its simplest form

$$\left\{2\tfrac{3}{4} + \frac{5}{2} \text{ of } \frac{7}{3\frac{4}{5}} - \frac{1\frac{3}{8}}{2\frac{1}{2}}\right\} \therefore 1\tfrac{71}{328}.$$

6. What fraction is 1*s.* „ 6½*d.* of 2*s.* „ 5*d.*? and 5¼ of 4⅓?

If A be 2⅔ of B, B 1⅜ of C, and D be 7⅛ of C, what fraction is A of D?

What is meant by "reducing one quantity to the fraction of another"?

7. A person rows from A to B (a distance of a mile and a half) and back again in an hour; how long would it have taken him if he had "pulled" equally hard, and there had been a stream of 1½ miles an hour flowing from A towards B?

8. Divide 2·0271 by 1000, 20·21 by ·001, 23·0142 by 121, 23014200 by ·0121, and 2301·420 by 0·0012100. Prove the foregoing results by vulgar fractions, and reduce

$$\left(\frac{3}{25} \text{ of } 2{\cdot}45 - \frac{1}{100} \text{ of } {\cdot}02\right) \div 1000$$

to a decimal.

9. Find the value of ·375 of a guinea; and reduce 4*s.* „ 7½*d.* to the decimal of 0·01 of £1, and likewise to that of £0·01.

10. When are four quantities said to be in proportion?

The four quantities, 1 lb. „ 4 oz., £23 „ 16s. „ 3d., £19 „ 1s., and 1 lb. „ 9 oz. taken in a certain order are in proportion, prove that they are so by means of your definition. What are *concrete* quantities? Can 1 lb. „ 4 oz. be multiplied by £19 „ 1s.?

11. If $2\frac{2}{3}$ of $B = 1\frac{1}{2}$ of $(A + \frac{3}{4}$ of $A)$, find two whole numbers which shall bear to each other the ratio of A to B.

12. If a certain number of workmen can do a piece of work in 25 days, in what time will $1\frac{2}{3}$ of that number of men do a piece of work twice as great, supposing that 2 of the first set can do as much work in an hour as 3 of the second set can in $1\frac{1}{2}$ hours, and that the second set work half as long a day as the first set?

13. A person investing in the 4 per cents. receives $4\frac{4}{9}$ per cent. interest for his money; what is the price of stock?

14. How much stock at $92\frac{3}{4}$ must be sold out to pay a bill of £715 „ 17s. due 9 months hence at 4 per cent. simple interest?

15. (a) Given that the square of 15334 = 235131556; find that of 153347, without going through the operation of squaring.

(β) Given that the square root of 1038361 is 1019; find the square root of 103876864.

(γ) Extract the cube root of 0·01 to 3 places of decimals.

Second Division B, 1856.

1. Prove that 29 multiplied by 15 = 15 multiplied by 29. Likewise that $\frac{3}{11}$ of $1 = \frac{1}{11}$ of 3.

2. (a) If a person's estate be worth £1384 „ 16s. a year, and the land be assessed at 2s. „ $9\frac{1}{2}d.$ per £, what is his clear annual income?

(β) What is the cost of 39 cwt. „ 3 qrs. „ 26 lbs. at £4 „ 17s. „ 10d. per cwt.?

N.B. (a) and (β) both by "Practice." To what class of

examples does the "rule of Practice" apply, and why is it so called?

What is the meaning of an "aliquot part"?

3. Easter Sunday is always the Sunday directly *following* the *first* full moon which falls *after* March 20th: there are $29\frac{1}{2}$ days between any two consecutive full moons: February 1852 (being a "leap" year) had 29 days, and there was a full moon on April 18th, 1848 (a *Tuesday*).

From these data, find when Easter fell in 1855.

4. Find the area of a room 8 ft. „ 4 in. long, by 12 ft. „ 2 in. broad, by duodecimals or cross multiplication. If in this example the room were not supposed to be a *rectangular* parallelogram, how would the answer have to be interpreted?

5. Add together

$$\frac{1}{2} \text{ of } 16s. \text{ „ } 6\tfrac{1}{4}d. + \frac{1}{3} \text{ of } 12s. \text{ „ } 10\tfrac{3}{4}d. + \frac{1}{6} \text{ of } £2 \text{ „ } 4s. \text{ „ } 8\tfrac{3}{4}d.$$

Reduce to its lowest terms

$$\left(\frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \times 3\frac{1}{3} + \frac{13}{36}} - \frac{1}{2\frac{1}{2}}\right) \div \frac{1}{1\frac{2}{3}}.$$

6. What fraction is $1s.$ „ $5d.$ of $5\frac{3}{4}d.$? and $2\frac{1}{2}$ of $3\frac{1}{3}$?

If A be $\frac{1}{2}$ of $2\frac{2}{3}$ of B, and C be $1\frac{1}{3}$ of B, what fraction is A of C?

What is *meant* by "reducing one quantity to the fraction of another"?

7. A person rows a distance of $1\frac{1}{2}$ miles *down* a stream in 20 minutes; but without the aid of the stream, it would have taken him half an hour; what is the rate of the stream per hour? and how long would it take him to return *against* it?

8. Divide ·01 by 1000; 202 by ·01; and 13099·52 by ·0011008; and prove your results by vulgar fractions.

Reduce $\left(\frac{5}{80} \text{ of } 11{\cdot}02 - \frac{3}{50} \text{ of } 11{\cdot}8\right) \div 0{\cdot}1$ to a decimal.

9. Reduce 18s. „ $4\frac{1}{2}d.$ to the decimal of £1, and likewise to that of £1000.

Find the value of ·785 of £10.

10. When are four quantities said to be in proportion? and apply your definition to ascertain whether the four quantities 3 lb. „ 2 oz.; 1s. „ $1\frac{3}{4}d.$; 1s. „ $7\frac{1}{2}d.$; 4 lb. „ 2 oz. can be so arranged as to form a proportion. Can pounds and ounces be multiplied into shillings and pence?

11. If $1\frac{3}{5}$ of $\left(A-\frac{2}{5} \text{ of } A\right)=2\frac{1}{2}$ of $\left(B+\frac{B}{4}\right)$; find two whole numbers which shall be to each other in the ratio of A to B.

12. If 20 men can perform a piece of work in 12 days, how many men will perform a piece of work half as large again in a fifth part of the time, if they work the same number of hours a day; supposing that 2 of the second set can do as much work in an hour as 3 of the first set?

13. A person investing in the 4 per cents. receives 5 per cent. for his money; what is the price of stock?

14. When the 3 per cents. are at 80, how much stock must be sold out to pay a bill of £690 „ 3s. „ 9d. due 9 months hence at 3 per cent. simple interest?

15. (α) Given that the square of 10129 is 102596641; find the square of 101293 without going through the operation of squaring.

(β) Given that the square root of 105625 is 325, find that of 10573009.

Extract the cube root of 0·5 to 3 places of decimals.

October, 1856.

ABSTRACT NUMBERS.

1. What *two* definitions are given of any fractional symbol, as for instance of $\frac{3}{7}$? Shew that the one definition involves the other.

And prove, without *assuming* any property of fractions,

$$\text{that } \frac{2}{5} \text{ of } \frac{3}{7}=\frac{2\times 3}{5\times 7};$$

$$\text{likewise that } 5\div\frac{2}{3}=\frac{5\times 3}{2}.$$

2. What fraction of $5\frac{1}{4}$ is $4\frac{1}{3}$? and show from your definition of a fraction the correctness of your result.

If $A=1\frac{1}{6}$ of B, and $C=2\frac{1}{6}$ of B, what is the ratio of A to C?

3. (α) Reduce $\frac{3}{27}, \frac{8}{20}, \frac{2}{25}$, and $\frac{1}{3}$ to their *least* common denominator.

(β) Reduce to their simplest forms

$$\frac{\frac{3}{5}}{\frac{8}{9}}+\frac{\frac{5}{7}}{7\frac{1}{2} \text{ of } \frac{11}{12}}; \quad \left(\frac{3\frac{3}{4}}{4\frac{2}{7}}-\frac{3\frac{2}{5}}{4\frac{1}{4}}+\frac{\frac{4}{7}}{2\frac{1}{2}}\right)\div 4\frac{6}{7}.$$

4. State your rule for the division of one decimal by another, and apply it to the two following examples:

$$10{\cdot}836 \div 5{\cdot}16; \quad 1083{\cdot}6 \div 5{\cdot}16:$$

and prove the truth of each result by vulgar fractions.

5. Perform the following operation in decimals:

$$\left(7\frac{1}{2} \text{ of } \frac{1}{5}+\frac{17}{25}-{\cdot}02\right)\div{\cdot}005.$$

Likewise find the value of $\frac{2}{7}$ of ·03, determining the recurring period.

CONCRETE NUMBERS.

6. Without reducing the whole sum to farthings, find the correct value (to the fraction of a farthing) of $\frac{2}{7}$ of £365 „ 4*s.* „ $7\frac{1}{2}$*d.*; and divide the result by 11.

7. Reduce to its simplest form, *i.e.* to days, hours, &c., the following expressions:

$$1\frac{1}{2} \text{ of } \frac{4\frac{1}{5}}{5\frac{1}{4}} \text{ of } \frac{18s. \text{ „ } 6\frac{1}{4}d.}{\pounds 1} \text{ of 3 days „ 2 hours.}$$

8. Do the following example by "Practice."

What is the tax on £1234 „ 15*s.* at 3*s.* „ $7\frac{1}{2}$*d.* in the pound?

9. If £1 sterling = 10 florins = 100 cents = 1000 mils; how many florins, cents, &c. is £25 „ 10*s.* „ $7\frac{1}{2}$*d.* equal to? find the exact value with the decimal remainder, if there be any.

Likewise express the result in the form of the decimal of a florin.

10. Apply the common rules for the multiplication and division of decimals to the two following examples:

(*a*) Multiply £360 „ 7 florins „ 4 cents „ 3 mils by 230.

(*β*) Divide £45 „ 3 florins „ 3 cents „ 3 mils by £36 „ 5 florins.

11. In France (where the different tables are all adapted to decimal computation), the unit of weight is a "gramme."

A kilogramme = 10 hectogrammes = 100 decagrammes = 1000 grammes.

If we had the same table of weights as in France, and had pounds, florins, cents, and mils, as defined above in example 9, how should we find the price of 57 kilogr. „ 8 decagr. „ 4 gr. of any article which cost £17 „ 5 florins „ 7 cents per kilogramme? find the exact result in florins, cents, &c. by means of decimal fractions.

12. The exchange between London and Paris is 25·5 francs per pound sterling; between Paris and Amsterdam is 117 francs for 55 florins; between Amsterdam and Hamburgh is 11 florins for 13 marks; what is the exchange between London and Hamburgh? (*i. e.* how many marks is £1 sterling worth?)

13. Find the difference between the simple and compound interest on £416 „ 13*s.* „ 4*d.* for two years at $2\frac{1}{4}$ per cent.

14. At what rate per cent. simple interest will £936 „ 13*s.* „ 4*d.* amount to £1157 „ 7*s.* „ $4\frac{1}{2}$*d.* in $4\frac{7}{8}$ years?

15. A person buys £500 stock at $98\frac{3}{4}$ and sells out at £103; what does he gain by the transaction?

16. At what rate per cent. will a person receive interest, who invests in the three per cents. when they are at 91?

First Division A, 1857.

1. *Find the value of* 17 *cwt.* „ 3 *qrs.* „ 21 *lbs. at* £1 „ 6*s.* „ 4*d. per cwt.*

		£	s.	d.
qrs. 2	$\frac{1}{2}$	1 „	6 „	4
				17
		22 „	7 „	8
			13 „	2
qr. 1	$\frac{1}{2}$		6 „	7
lbs. 14	$\frac{1}{2}$		3 „	$3\frac{1}{2}$
lbs. 7	$\frac{1}{2}$		1 „	$7\frac{3}{4}$
		23 „	12 „	$4\frac{1}{4}$

2. *What is the least number of dollars at 4s. „ 2d. each, which is equal to an exact number of sovereigns?*

A dollar, being 50 pence, is $\overset{\pounds}{\frac{5}{24}}$;

therefore $\qquad$ 24 dollars $= 24 \times \overset{\pounds}{\frac{5}{24}} = \pounds 5.$

3. *Prove that the fraction* $\frac{5+6}{6+7}$ *is greater than* $\frac{5}{6}$ *and less than* $\frac{6}{7}$.

i. e. Compare $\frac{5}{6}, \frac{11}{13}, \frac{6}{7}$.

These fractions become respectively

$$\frac{455}{6 \times 13 \times 7}, \quad \frac{462}{6 \times 13 \times 7}, \quad \frac{468}{6 \times 13 \times 7}.$$

Therefore $\frac{11}{13}$ is greater than $\frac{5}{6}$ but less than $\frac{6}{7}$.

4. *Reduce* $\dfrac{2\frac{1}{2} - \frac{5}{6}}{2\frac{1}{2} + \frac{5}{6}} + \frac{7}{12}$ *of* $\frac{9 \times 10}{14 \times 3} - \frac{22\frac{1}{2}}{30}$ *to its simplest form.*

$$\frac{\frac{5}{2} - \frac{5}{6}}{\frac{5}{2} + \frac{5}{6}} + \frac{\overset{1}{\cancel{7}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{5}{\cancel{10}}}{\cancel{3}} - \frac{45}{60}$$

$$= \frac{10}{6} \times \frac{6}{20} + \frac{5}{4} - \frac{3}{4}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1 \text{ } \textit{Ans.}$$

5. *A man contracts to perform a piece of work in 30 days, and immediately employs 15 men upon it; at the end of 24 days the work is only half done; required the additional number of men necessary to fulfil the contract.*

i. e. If 15 men do $\frac{1}{2}$ the work in 24 days, how many men will do $\frac{1}{2}$ the work in 6 days?

Therefore $\quad 15 \times 24 : x \times 6 :: \frac{1}{2} : \frac{1}{2},$

$$x \times 6 = 15 \times 24,$$

$$x = 60.$$

Therefore the whole number required to work at it for the remaining 6 days would be 60; and as there were only 15 originally, therefore 45 *additional* hands must be set on.

6. *Multiply* ·025 *by* 10000; *and divide* 10000 *by* ·025.

$$\cdot 025 \times 1000 = 250.$$

$$2{\cdot}5)\ 1000000\ (400000$$

7. *Convert* $\frac{2}{5}$ *of a florin and* $\frac{3}{10}$ *of half-a-crown into decimals of £5.*

$\frac{2}{5}$, or ·4, of a *florin*, divided by 10, and by 5, will be brought into the denomination of £5.

$$\begin{array}{r|l} 10 & \cdot 4 \\ \hline 5 & \cdot 04 \\ \hline & \cdot 008\ \textit{Ans.} \end{array}$$

$\frac{3}{10}$ of half-a-crown, divided by 8, and by 5, will also be brought into the denomination of £5.

$$\begin{array}{r|l} 8 & \cdot 3 \\ \hline 5 & \cdot 0375 \\ \hline & \cdot 0075\ \textit{Ans.} \end{array}$$

8. *Extract the square root of the product of* ·004 *and* 15·625. *Of what number is* ·1 *the square root?*

$$\begin{array}{rl} & 15{\cdot}625 \\ & \cdot 004 \\ \hline & \cdot 0\dot{6}2\dot{5}00\ (\cdot 25 \\ & 4 \\ \hline 45) & 225 \\ & 225 \\ \hline \end{array}$$

since $\quad \cdot 1 \times \cdot 1 = \cdot 01;$

therefore ·01 is the number of which ·1 is square root.

9. *If the tenth, the hundredth, and the thousandth part of a pound sterling be called a florin, a cent, and a mil respectively, and a man's weekly wages are £2 „ 1 florin „*

2 cents „ 5 mils, on which he pays an income-tax of 5 cents in the pound, find his net yearly income, and convert the result into pounds, shillings, and pence.

$$\begin{array}{lllll} \text{Weekly wages are} & & & \overset{\pounds}{2{\cdot}125} & \\ & & & \underline{52} & \\ & & & 4\ 250 & \\ & & & \underline{106\ 25} & \\ \text{But} & 5 \text{ cents} & \frac{1}{20} & 110{\cdot}500 & \text{yearly wages} \\ & & & \underline{5{\cdot}525} & \text{tax} \\ & & & 104{\cdot}975 & \text{net income} \end{array}$$

therefore, by inspection, £104 „ 19*s.* „ 6*d. Ans.*

10. *Find the compound interest on £200 in 3 years at 5 per cent. per annum.*

What sum will amount to £2315 „ 5s. in 3 years at 5 per cent. compound interest?

$$\begin{array}{r|ll} \frac{1}{20} & 200 & \\ & \underline{10} & \\ \frac{1}{20} & 210 & \text{principal of second year} \\ & \underline{10{\cdot}5} & \\ \frac{1}{20} & 220{\cdot}5 & \text{principal for third year} \\ & \underline{11{\cdot}025} & \\ & 231{\cdot}525 & \text{amount at end of 3 years} \end{array}$$

therefore £31 „ 10*s.* „ 6*d. Ans.*

Also knowing from this the amount of £200 in 3 years at 5 per cent., we state

$$200 : x :: 231\tfrac{21}{40} : 2315\tfrac{1}{4},$$

$$x \times \frac{9261}{40} = 200 \times \frac{9261}{4},$$

$$x = 2000 \ \textit{Ans.}$$

11. *Find the present value of £415 „ 8s. „ 8d. due 9 months hence, allowing 4 per cent. per annum interest.*

$$\frac{3}{4} \text{ of } 4 = 3,$$

$$103 : 415\tfrac{13}{30} :: 100 : \overset{\text{p. w.}}{\textit{Ans.}}$$

$$103 \times \textit{Ans.} = \frac{12463}{30} \times 100,$$

$$\textit{Ans.} = \frac{1210}{3} = \pounds 403 \text{ „ } 6s. \text{ „ } 8d.$$

12. *A fixed rent of £780 per annum is converted into a corn rent of one-half wheat at 48s. per quarter, and the other half barley at 30s. per quarter. What will be the rent when wheat has advanced to 56s. and barley to 32s. per quarter?*

The ambiguous expression "a fixed rent is converted into a corn rent of one-half wheat and the other half barley," may be taken to mean either that the sum of £780 was paid in *equal* quantities of wheat and of barley; or that *half* of the rent was paid in wheat, and *half* in barley.

Assuming first, that a certain fixed number of quarters of wheat were always to be paid, and the *same* number of quarters of barley, (the number of quarters is not required, but would be found on trial to be 200), we should have

wheat at 48	wheat at 56
barley at 30	barley at 32
78	88

therefore 78 : 88 :: 780 : *Ans.*

£880 *Ans.*

But if half the *value* of the fixed rent be paid in wheat and half in barley, then $\frac{780}{2}$ or £390 is value of the wheat and £390 the value of the barley. Also £390 when wheat is at 48s. per quarter gives

$$\frac{\overset{65}{\cancel{390}} \times \overset{5}{\cancel{20}}}{\underset{\underset{2}{\cancel{12}}}{\cancel{48}}} = \frac{65 \times 5}{2} = \frac{325}{2} \text{ qrs.} = 162\tfrac{1}{2} \text{ qrs.}$$

And £390 when barley is at 30s. per quarter gives

$$\frac{390 \times 2\cancel{0}}{3\cancel{0}} = 130 \times 2 = 260 \text{ qrs.}$$

And assuming that the value of $162\frac{1}{2}$ quarters of wheat and of 260 quarters of barley were always paid, we have

value of wheat.

$$\frac{325}{2} \text{ qrs.} \times 56s. = \frac{\overset{65}{\cancel{325}} \times \overset{7}{\cancel{56}}}{\cancel{2} \times \underset{\cancel{4}}{\cancel{20}}} = £455$$

value of barley.

and $$\frac{26\cancel{0} \times 32}{2\cancel{0}} = 13 \times 32 = 416$$

and $$455 + 416 = 871 \quad \textit{Ans.}$$

13. *A person invested £4410 in 3 per cents. consols at 90; at the end of the year he sold out at $93\frac{1}{2}$ and invested the proceeds in Russian $4\frac{1}{2}$ per cent. stock at 98. What addition is thereby made to his income?*

$$90 : 4410 :: 100 : x,$$

$$\cancel{90}x = 100 \times \overset{49}{\cancel{4410}},$$

$$x = £4900, \text{ the stock originally held,}$$

from which £147 was the income obtained.

He now transfers his stock; therefore

$$93\tfrac{1}{2} : 98 :: x : 4900,$$

$$98 \times x = \frac{187}{2} \times 4900,$$

$$x = 4675 \text{ Russian stock,}$$

from which £210 „ 7*s.* „ 6*d.* was the income.

Hence (£210 „ 7*s.* „ 6*d.*) − £147 = £63 „ 7*s.* „ 6*d.* the addition required.

14. *If the estimated annual value of the property in a certain parish consist of the yearly rent paid to the Landlord together with the rates, and the rates be calculated upon the rent after a reduction of 30 per cent., find the rateable value of a tithe rent charge, the estimated value of which is £663 per annum, when rates are 3s. in the pound.*

Estimated value = rent + rates.

Rates are 3*s.* in the pound, or are $\frac{3}{20}$ of rateable property; therefore estimated value = rent + $\frac{3}{20}$ of rateable property. But rates are calculated on rent *less* 30 *per cent.*, *i.e.* on rent less $\frac{3}{10}$ of rent, *i.e.* on $\frac{7}{10}$ rent; therefore rates are $\frac{3}{20}$ of $\frac{7}{10}$ of rent, or are $\frac{21}{200}$ rent.

But estimated value = rent + rates

$$= \text{rent} + \frac{21}{200}\,\text{rent}$$

$$= \frac{221}{200}\,\text{rent};$$

therefore $663 = \dfrac{221}{200}$ rent,

$$\overset{3}{\cancel{663}} \times 200 = \overset{\pounds}{\cancel{221}} \times \text{rent},$$

$$600 = \text{rent},$$

take off 180, which is 30 per cent. of this,

and there remains 420, rateable value.

First Division B, 1857.

1. Find the value of 35 cwt. „ 3 qrs. „ 14 lbs. at £1 „ 19*s.* „ 6*d.* per cwt.

2. What is the least number of dollars at 4*s.* „ 3*d.* each, which is equal to an exact number of sovereigns?

3. Prove that the fraction $\dfrac{6+7}{7+8}$ is greater than $\dfrac{6}{7}$ and less than $\dfrac{7}{8}$.

4. Reduce to its simplest form

$$\frac{1\frac{1}{4} - \frac{5}{12}}{1\frac{1}{4} + \frac{5}{12}} + \frac{6}{7} \text{ of } \frac{9 \times 5}{14 \times 3} - \frac{11\frac{1}{4}}{15}.$$

5. A man contracts to perform a piece of work in 60 days, and immediately employs upon it 30 men: at the end of 48 days the work was only half done; required the additional men necessary to fulfil the contract.

6. Multiply ·075 by 10000, and divide 10000 by ·075.

7. Convert $\frac{1}{5}$ of a florin and $\frac{3}{20}$ of half-a-crown into decimals of £5.

8. Extract the square root of the product of ·001 and ·625. Of what number is ·01 the square root?

9. If the tenth, hundredth, and thousandth part of a pound be called a florin, a cent, and a mil respectively, and a man's weekly wages are £2 „ 9 florins „ 7 cents „ 5 mils, upon which he pays an income-tax of 5 cents in the pound, find his net yearly income, and convert the result into pounds, shillings, and pence.

10. Find the compound interest of £600 in 3 years at 5 per cent. per annum.

What sum will amount to £6945 „ 15*s*. in 3 years at 5 per cent. compound interest?

11. Find the present value of £428 „ 15*s*. due 5 months hence, allowing 5 per cent. per annum interest.

12. A fixed rent of £1170 is converted into a corn rent of one-half wheat at 48*s*. per quarter, and the other half barley at 30*s*. per quarter. What will the rent be when wheat has advanced to 56*s*. and barley to 32*s*. per quarter?

13. A person invested £2205 in the 3 per cent. consols at 90. At the end of a year he sold out at 93½, and invested the proceeds in Russian 4½ per cent. stock at 98. What addition is thereby made to his income?

14. If the estimated value of the property in a parish consist of the yearly rent paid to the landlord together with the rates, and the rates be calculated upon the rent after a reduction of 30 per cent., find the rateable value of a tithe rent charge, the estimated annual value of which is £884 per annum, when the rates amount to 3*s*. in the pound.

Second Division A, 1857.

1. How many pounds of tea at 4*s*. „ 2*d*. per pound can be bought for £12 „ 10*s*.?

2. If 14 men can do a piece of work in 18 days, in how many days will 24 men do it?

3. Add together $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, and subtract the sum from 1½.

4. What fraction of £58 „ 5*s*. „ 6*d*. is $\frac{36}{37}$ of £17 „ 2*s*. „ 3*d*.?

5. The net rental of an estate, after deducting 7*d*. in the pound for income-tax and 5 per cent. on the remainder for the expenses of collecting, is £479 „ 11*s*. „ 10*d*., what is the gross rental?

6. Multiply 1·075 by ·0101, and divide the product by ·43.

7. Add together 2·095 hours, ·07 days, and ·05 weeks, and express the sum as a decimal of 365·25 days.

8. The surface of a cube is 86·64 square feet, what is the length of an edge?

9. A bankrupt has book-debts equal in amount to his liabilities, but on £3000 of them he can only recover 6*s*. „ 8*d*. in the pound, and the expenses of the bankruptcy are 5 per cent. on the book-debts; if he pays 11*s*. in the pound, what is the amount of his liabilities?

10. What will £360 amount to in 4 years and 2 months at £3 „ 6*s*. „ 8*d*. per cent. per annum, simple interest?

In what time will a sum double itself at the above rate?

11. Find the discount on £31 „ 13*s*. „ 4*d*. due 4 months hence at 4 per cent. per annum.

12. If a cubic foot of marble weighs 2·716 times as much as a cubic foot of water, find the weight of a block of marble 6 ft. „ 4 in. long, 1 ft. „ 6 in. broad, 1 ft. thick, supposing a cubic foot of water to weigh 1000 oz.

13. A tithe-rent of £385 per annum is commuted in equal parts into a corn-rent consisting of wheat at 56*s*. per quarter, barley at 32*s*. per quarter, and oats at 22*s*. per quarter; find its value when wheat is at 64*s*. per quarter, barley at 44*s*. per quarter, and oats at 24*s*. per quarter.

14. The receipts of a railway company are apportioned in the following manner; 48 per cent. for the working expenses, 10 per cent. for the reserve fund, a guaranteed dividend of 5 per cent. on one-fifth of the capital, and the remainder, £32000, for division amongst the holders of the rest of the stock, being a dividend at the rate of 4 per cent. per annum; find the capital and the receipts.

Second Division B, 1857.

1. How many pounds of tea at 4*s*. „ 2*d*. per lb. can be bought for £37 „ 10*s*.?

2. If 12 men can do a piece of work in 20 days, in how many days will 15 men do it?

3. Add together $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$, and subtract the sum from $2\frac{2}{3}$.

4. What fraction of £174 „ 16*s.* „ 6*d.* is $\frac{36}{37}$ of £34 „ 4*s.* „ 6*d.*?

5. The net rental of an estate, after deducting 7*d.* in the pound for income-tax and 5 per cent. on the remainder for the expenses of collecting, is £959 „ 3*s.* „ 8*d.*; what is the gross rental?

6. Multiply 3·225 by ·0101, and divide the product by ·215.

7. Add together 12·57 hours, ·42 days, and ·3 weeks, and express the sum as a decimal of 365·25 days.

8. The surface of a cube is 346·56 square feet, what is the length of an edge?

9. A bankrupt has book-debts equal in amount to his liabilities, but on £6000 of them he can only recover 13*s.* „ 4*d.* in the pound, and the expenses of the bankruptcy are 5 per cent. on the book-debts; if he pays 13*s.* in the pound, what is the amount of his liabilities?

10. What will £480 amount to in 3 years and 3 months at £4 „ 3*s.* „ 4*d.* per cent. per annum, simple interest?

In what time will a sum double itself at the above rate?

11. Find the discount on £158 „ 6*s.* „ 8*d.* due 4 months hence at 4 per cent. per annum.

12. If a cubic foot of marble weighs 2·716 times as much as a cubic foot of water, find the weight of a block of marble 9 ft. „ 6 in. long, 2 ft. „ 3 in. broad, 2 ft. thick, supposing a cubic foot of water to weigh 1000 oz.

13. A tithe-rent of £310 per annum is commuted in equal parts into a corn-rent consisting of wheat at 56*s.* per quarter, barley at 32*s.* per quarter, and oats at 22*s.* per quarter; find its value when wheat is at 64*s.* per quarter, barley at 44*s.* per quarter, and oats at 24*s.* per quarter.

14. The receipts of a railway company are apportioned in the following manner; 48 per cent. for the working expenses, 10 per cent. for the reserve fund, a guaranteed dividend of 5 per cent. on one-fifth of the capital, and the remainder, £48000,

for division amongst the holders of the rest of the stock, being a dividend at the rate of 4 per cent. por annum; find the capital and the receipts.

October, 1857, (*A*).

1. Which is the more valuable crop, wheat yielding 5 quarters the acro and selling at 6*s*. „ 9*d*. per bushel, or barley yielding 6 quarters „ 6 bushels the acre, and selling at 4*s*. „ 10*d*. per bushel?

2. A tradesman by selling an article for 5*s*. gains 20 per cent., what was the cost price?

3. Find the difference between

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}} \text{ and } \frac{\frac{1}{4}-\frac{1}{6}}{\frac{1}{4}+\frac{1}{6}}-\frac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}.$$

4. From $\frac{1}{2}$ of $\frac{1}{3}$ of a penny subtract $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of a shilling.

5. If standard gold worth £3 „ 17*s*. „ 10½*d*. per ounce be so far alloyed as to be worth only £3 „ 16*s*. „ 1½*d*. per ounce; find the least number of sovereigns made of the alloyed gold which shall be equal to an exact number made of standard gold.

6. Divide 14·4 by ·0012, and also by 1200.

7. Add together 16·75 yards, 1·3125 feet, and 11·25 inches, and convert the sum into the decimal of a mile.

8. If the diameter of the fore-wheel of a carriage be 2 feet „ 3 inches, and that of the hind-wheel be 3 feet „ 6 inches, find how many times oftener the one will revolve than the other in a distance of ten miles, having given that the circumference of a circle is to the diameter as 3·1416 to 1.

9. A person invests a sum of money in the 3 per cent. consols at 88, and at the end of 4½ months, after receiving one half-year's dividend, sells out at 87⅞. At what rate per cent. per annum does he receive interest for his capital?

10. Find the compound interest of £800 for 2 years at 5 per cent. per annum.

What difference will it make if the interest be charged half-yearly instead of yearly?

11. What is the present worth of £257 „ 10*s.* due 8 months hence, allowing $4\frac{1}{2}$ per cent. per annum interest?

12. Extract the square root of 65537 and of ·65537, each to three places of decimals.

13. Gunter's chain consists of 100 links, and a rectangular area 10 chains long by one chain broad contains an acre; find the area of a rectangular field whose sides are 56 chains „ 25 links, and 25 chains „ 20 links, respectively.

14. The governors of Queen Anne's bounty advance £845 on mortgage of a living on the following conditions; the principal to be repaid in 30 years by equal annual instalments, and interest at the rate of $3\frac{1}{2}$ per cent. to be charged on the part unpaid. If the sum due in any particular year be £43 „ 18*s.* „ $9\frac{3}{5}$*d.*, find how many previous annual payments have been made.

October, 1857, (*B*).

1. Which is the more valuable crop, wheat yielding 4 quarters „ 4 bushels the acre and selling at 6*s.* „ 3*d.* per bushel, or barley yielding 6 quarters the acre and selling at 4*s.* „ 8*d.* per bushel?

2. A tradesman by selling an article for 6*s.* gains 20 per cent., what was the cost price?

3. Find the difference between

$$\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}+\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}} \text{ and } \frac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}-\frac{\frac{1}{8}-\frac{1}{10}}{\frac{1}{8}+\frac{1}{10}}.$$

4. From $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of a shilling subtract $\frac{1}{2}$ of $\frac{1}{3}$ of a penny.

5. If standard gold worth £3 „ 17*s.* „ $10\frac{1}{2}$*d.* per oz. be so far alloyed as to be worth only £3 „ 12*s.* „ $3\frac{3}{4}$*d.* per oz.: find the least number of sovereigns made of the alloyed gold which shall be equal to an exact number made of standard gold.

6. Divide 12·1 by ·0011, and also by 1100.

7. Add together £16·75, 1·3125 shillings, and 11·25 pence, and convert the result into the decimal of £25.

8. If the diameter of the fore-wheel of a carriage be 3 ft. and that of the hind-wheel be 4 ft. „ 6 in., find how many times oftener the one will revolve than the other in a distance of 5 miles, having given that the circumference of a circle is to the diameter as 3·1416 to 1.

9. A person invests a sum of money in the 3 per cent. consols at 90, and at the end of 3 months, after receiving one half-year's dividend, sells out at 89½. At what rate per cent. per annum does he receive interest for his capital?

10. Find the compound interest of £400 for two years at 5 per cent. per annum.

What difference will it make if the interest be charged half-yearly instead of yearly?

11. What is the present worth of £257 „ 10*s.* due 9 months hence, allowing 4 per cent. per annum interest?

12. Extract the square root of 65535 and of ·65535, each to three places of decimals.

13. Gunter's chain consists of 100 links, and a rectangular area 10 chains long by 1 chain broad contains an acre; find the area of a rectangular field whose sides are 67 chains „ 50 links, and 30 chains „ 25 links respectively.

14. The governors of Queen Anne's bounty advance £725 on mortgage of a living on the following conditions: the principal to be repaid in 30 years by equal annual instalments, and interest at the rate of 3½ per cent. to be charged on the part unpaid. If the sum due in any particular year be £37 „ 14*s.*, find how many previous annual payments have been made.

First Division A, 1858.

1. *What is meant by Numeration?*

Express in words the number 817001014.

Explain the principle upon which you have obtained your result.

Cf. § 9, § 11, and § 12.

2. *Reduce* $1\frac{3}{4}d.$ *to the fraction of a florin; and find the value of* ·25 *of* 3*s*. „ 6*d*.

$$\frac{1\frac{3}{4}}{24}=\frac{7}{96}.$$

Again,

$$\cdot 25 \text{ of } 3s. \text{ „ } 6d.$$

$$=\frac{1}{4}\times 42d.$$

$$=10\tfrac{1}{2}d.$$

3. *A florin being the tenth part of a pound, a cent the hundredth, and a mil the thousandth; express* £32 „ 16*s*. „ $5\frac{1}{4}d.$ *in pounds, florins, cents, and mils.*

By inspection, £32 „ 8 florins „ 2 cents „ 2 mils.

4. *Add together* ·0016, 1·7, ·02, *and subtract the result from* 2.

·0016	
1·7	2·0
·02	1·7216
1·7216	·2784

5. *What fraction of* 5 *m*. „ 5 *fgs*. „ 7 *p*. „ 0 *yds*. „ 0 *ft*. „ $11\frac{11}{17}$ *in. is* $\frac{8}{17}$ *of a league?*

5	miles	= 8800	yards
5	furg.	= 1100	......
7	poles	= $38\frac{1}{2}$	......
$11\frac{11}{17}$	inch	= $\frac{11}{34}$	......
		$9938\frac{14}{17}$	......

therefore fraction required

$$=\left(\frac{8}{17}\times 3\times 1760\right)\div 9938\tfrac{14}{17}$$

$$=\frac{8\times 3\times \overset{22}{\cancel{1760}}}{17}\times\frac{17}{\underset{2112}{\cancel{168960}}}$$

$$=\frac{8\times 3\times 2}{192}$$

$$=\frac{1}{4}.$$

6. *If* 100 *men can make an embankment* 2 *miles long in* 20 *days, how much over-time must* 120 *men work in order to*

finish an embankment 3 miles long in 24 days? Twelve hours is supposed to be a regular day's work.

$$100 \times 20 \times 12 : 120 \times 24 \times x :: 2 : 3,$$
$$6 \times 2 \times x = 5 \times 10 \times 3.$$
$$2x = 25,$$
$$x = 12\tfrac{1}{2} \text{ hours};$$

therefore they must work *half-an-hour* over-time.

7. *Divide* 76·57 *by* ·0019; *and multiply the quotient by* $\frac{3}{7}$ *of* ·0008568.

```
1·9) 76570 (40300
        57
        00

   7) ·0008568
      ·0001224
             3
      ·0003672
         40300
       1101600
      146880
      14·79186 Ans.
```

8. *A Landlord has an estate that brings him in £3000 a year, but this gross income is liable to deductions for rates and repairs to the extent of* 12 *per cent. He sells it at* 24 *years' purchase on the gross income, and invests the produce of the sale in* 3 *per cents. at* 97½. *What difference is caused in his income?*

	£
From gross income.........	3000
Deduct rates and repairs	360
	2640 net income.

He sells it for 24 × 3000, by investing which the income obtained

$$= 24 \times 3000 \times 3 \times \frac{2}{195}$$
$$= \frac{28800}{13}$$
$$= £2215 \text{ ,, } 7s. \text{ ,, } 8\tfrac{4}{13}d.$$

therefore 2640 − £2215 ,, 7*s.* ,, $8\frac{4}{13}d.$ = £424 ,, 12*s.* ,, $3\frac{9}{13}d.$ which is the diminution of his income caused by the sale.

9. *A wine merchant buys 3 kinds of wine and mixes them together in this proportion; 1 cask of the first, the price of which is £80 a cask, 3 casks of the second, the price of which is £90 a cask, 2 of the third kind. He keeps this mixture for 12 months, and then sells it for £104 " 10s. a cask, clearing 10 per cent. after allowing 4 per cent. for interest of capital. What was the original price of the third kind of wine?*

1 cask at 80 cost 80
3 casks at 90 cost 270

therefore 4 casks out of the 6 cost 350.

But 6 casks were sold for $6 \times 104\frac{1}{2} = 627$.

Now 627 is the *amount* of original price of all 6 casks, put to simple interest for 1 year at 14 per cent.; therefore

$$100 : x :: 114 : 627,$$

$$x = \frac{627 \times 100}{114} = \frac{\overset{11}{\cancel{627}} \times 50}{\cancel{57}} = 550,$$

deduct 350, the price of 4 casks, and we have £200 as the price of the remaining 2 casks, which therefore cost £100 each.

10. *Explain the difference between simple and compound interest. Find the compound interest on £25000 for 3 years at 4 per cent., supposing interest to be made capital at the end of each year.*

25)	25000	
	1000	first year's interest
25)	26000	capital for second year
	1040	second year's interest
25)	27040	capital for third year
	$1081\frac{1}{2}\frac{3}{5}$	third year's interest,

therefore £3121 " 12*s*. compound interest.

11. *A room is 14 feet " 3 in. high, 20 feet wide, 24 feet long. What will it cost to paper it with a paper 2 feet " 6 in. wide, whose price is $11\frac{1}{4}$d. per yard? Allow 8 feet by 5 feet " 3 in. for each of 4 doors; 10 feet by 6 feet " 8 in. for each of two windows, and 6 feet " 6 inches by 5 feet for a fire place.*

[To obtain the area of the 4 walls, add length and breadth, multiply by the height, and double the result.]

Writing the quantities in the *duodecimal* scale, we have

20	$(5{\cdot}3) \times 4 = 19$
18	8
38	120, area of 4 doors
123	$6{\cdot}8 \times 2 = 11{\cdot}4$
e 0	*t*
74	*e*1·4, area of 2 windows
38	6·6
443·0	5
2	28·6, area of fire place
886, area of 4 walls	

Deduct 239·*t*, area of doors, windows, and fire place

648·2
12

76
12

$920\frac{1}{6}$, area to be papered.

$$3 \times 2\tfrac{1}{2} : 920\tfrac{1}{6} :: 11\tfrac{1}{4} : x \text{ pence.}$$

$$3 \times \frac{5}{2} \times x = \frac{5521}{6} \times \frac{45}{4}$$

$$x = \frac{5521}{4} \text{ pence}$$

$$= £5 \text{ ,, } 15 \text{ ,, } 0\tfrac{1}{4}d.$$

12. *Explain the advantages of a decimal system of coinage and accounts.*

Do you apprehend any disadvantages as likely to arise from the introduction of the system into England? (Cf. § 43, p. 59.)

First Division B, 1858.

1. What is meant by *Numeration?* Express in words the number 127800021.

Explain the principle upon which you have obtained your result.

2. Reduce $2\frac{1}{2}d.$ to the fraction of 15 shillings; and find the value of ·05 of 1*s.* ,, 8*d.*

3. A florin being the tenth part of a pound, a cent the hundredth, and a mil the thousandth, express £6 „ 14*s.* „ $2\frac{1}{2}$*d.* in pounds, florins, cents, and mils.

4. Add together the following: ·172, ·06, 1·004, and multiply the result by ·04.

5. What fraction of 6 m. 2 fgs. 7 p. 11 y. 1 ft. 6 in. is $\frac{7}{15}$ of a league?

6. If 50 men can make an embankment 3 miles long in 60 days, working 12 hours a day, how many hours a day must 80 men work in order to finish an embankment 4 miles long in 40 days?

7. Divide 73·8 by ·0018 and multiply the quotient by $\frac{3}{19}$ of ·0009747.

8. A landlord has an estate that brings him in £4000 a year, but this gross income is liable to deductions for rates and repairs to the extent of 15 per cent. He sells the estate at 24 years' purchase on the gross income, and invests the price in the 3 per cents. at $97\frac{1}{2}$. What difference is caused in his income?

9. A wine merchant buys 3 kinds of wine and mixes them in the following proportions; 2 casks of the first kind the price of which is £80 a cask, 1 of the second kind the price of which is £90, and 2 of the third kind. He keeps the mixture 6 months, and then sells it for £99 a cask, clearing thereby 8 per cent. allowing interest on capital at the rate of 4 per cent. per annum. What was the original price of the third kind?

10. Explain the difference between simple and compound interest, and find the compound interest on £24000 for 3 years at 5 per cent., supposing interest to be made capital at the end of each year.

11. A room is 14 ft. „ 6 in. high, 20 ft. wide, and 22 ft. long. What will it cost to paper it with a paper 2 ft. „ 6 in. wide, whose price is $10\frac{1}{2}$*d.* a yard? Allow 8 ft. by 5 ft. „ 3 in. for each of 2 doors, 6 ft. „ 6 in. by 6 ft. for a fire place, and 12 ft. by 5 ft. „ 7 in. for one window.

12. Explain the advantages of a decimal system of coinage and accounts.

Do you apprehend any disadvantages as likely to arise from the introduction of the system into England?

Second Division A, 1858.

1. Express in figures two hundred and thirty-two million three thousand and fourteen.

Explain the principle upon which your figures represent the number.

2. Reduce 1*s*. „ 9*d*. to the fraction of a crown; and find the value of ·075 of a pound.

3. Add together the following: ·064, 12·4, ·006, and divide the result by ·02.

4. A florin being the tenth part of a pound, a cent the hundredth, and a mil the thousandth, express £18 „ 12*s*. „ $6\frac{1}{4}d$. in pounds, florins, cents, and mils.

5. Express $\dfrac{\frac{3}{4}-\frac{2}{5}}{\frac{2}{3}-\frac{1}{12}} \div$ by $\dfrac{\frac{3}{7}-\frac{1}{3}}{\frac{11}{21}-\frac{1}{6}}$ in its simplest form, and square your result.

6. The price of gold in this country is £3 „ 17*s*. „ $10\frac{1}{2}d$. per oz. What ought 100 sovereigns to weigh, supposing that $\frac{5}{6}$ of each sovereign is pure gold, and that the value of the sovereign is that of the gold which it contains?

7. If a rupee be worth 2*s*. „ 4*d*., what decimal fraction is it of 9*s*. „ 4*d*.? Express £6·944 in rupees and decimal parts of a rupee.

8. What does 5 cwt. „ 2 qrs. „ 6 lbs. of bread cost at 1*s*. „ 9*d*. a stone?

9. Suppose that £1 exchanges for 24·8 francs, and that the French 3 per cents. are selling for 70·2 francs. What amount of such stock will £589 buy?

10. Find the fourth root of ·00028561.

11. Find the discount on £50 „ 3*s*. due six months hence, allowing 4 per cent. interest for money.

12. Explain the advantages of a decimal system of coinage and accounts.

Do you apprehend any disadvantages as likely to arise from the introduction of the system into England?

October, 1858, (*A*).

1. Express in figures three hundred and eighty-one million two hundred and seventy-four thousand nine hundred and fifty-four.

Explain the principle upon which you have proceeded.

2. If the mean diameter of the earth be 504,979,200 inches in length, express its length in feet, yards, poles, furlongs, and miles.

3. Reduce $4\frac{3}{4}d.$ to the fraction of half-a-crown: and find the value of ·04 of £1 „ 5*s.*

4. Add $\frac{3}{4}$ to $\frac{9}{17}$. Square the sum and subtract it from 2.

5. Add together the following: ·185, ·0185 and 1·85. Divide the result by ·02.

6. Express 1*s.* „ 9*d.* as a decimal of £1.

If a dollar be worth 4*s.* „ 10*d.*, how many dollars and decimal parts of a dollar are worth £1 „ 1*s.* „ 9*d.*?

7. Multiply together 73·8 and ·0058, and divide the product by $\frac{1}{7}$ of ·00812.

8. If 1000 men can excavate a basin 1600 yards long, 500 broad, 40 deep in 8 months, how many men will be required to excavate a basin 2000 yards long, 400 wide, 50 deep in 10 months?

9. A room is 20 feet long and 16 feet wide, what must be its height in order that the area of the floor and ceiling together may be equal to the area of the walls?

10. Find the discount on £164 „ 2*s.* „ 6*d.* due 3 months hence, at 4 per cent. per annum.

11. *A*, *B*, *C* enter into business together and embark £3000, £4000, and £5000 respectively. At the end of 12 months they have made a gross profit of £1380, but the expenses of their concern have been $7\frac{1}{2}$ per cent. on its capital; find how much each of them would have lost if, instead of entering into business, he had invested his money in the 3 per cents. at 90.

12. A railway train has a journey of 65 miles to perform, and ought to perform it in 3 hours; if its starting be delayed by a quarter of an hour, how many miles per hour must it increase its speed so as to arrive at the proper time?

First Division A, 1859.

1. *What number subtracted from* 670194 *will leave* 3825?

The product of two numbers is 36865365: *one of them is* 365. *What is the other?*

$$\begin{array}{r} 670194 \\ 3825 \\ \hline 666369 \end{array} \qquad \begin{array}{l} 365)\ 36865365\ (101001 \\ \quad\quad 365 \\ \quad\quad\quad 365 \\ \quad\quad\quad 000 \end{array}$$

2. *How many bricks are there in a wall which is* 120 *bricks long*, 15 *bricks high, and* 2 *bricks thick?*

$$120 \times 15 \times 2 = 120 \times 30 = 3600.$$

3. *Find the cost of* 250 *lbs. of tea at* 3*s.* „ $11\frac{1}{2}$ *per lb. If* 10 *lbs. be spoiled, what will the merchant gain by selling the remainder at* 4*s.* „ 6*d. per lb.?*

Find the cost of 250 lbs. at 4*s.*, *minus* the cost at $\frac{1}{2}d.$

$$\begin{array}{r|r|r} \frac{1}{2}d. & \frac{1}{24} & 250 \\ & & 4 \\ \hline & & 1000 \\ & \text{deduct} & 10 „ 5 \\ \hline & 20) & 989 „ 7 \\ \hline & & 49 „ 9 „ 7 \end{array}$$

also by selling 240 lbs. at 4*s.* „ 6*d.* he obtains

$$\begin{array}{r} 240 \\ 4{\cdot}5 \\ \hline 1200 \\ 960 \\ \hline 20)\ 1080 \\ \hline £54 \end{array} \qquad \begin{array}{lr} & £\quad s.\quad d. \\ \text{therefore} & 54 „ 0 „ 0 \\ & 49 „ 9 „ 7 \\ \hline & 4 „ 10 „ 5 \text{ gain} \end{array}$$

4. *Six dollars, four florins, and four half-crowns amount to £2 „ 3s. What is the value of a dollar?*

4 florins and 4 half-crowns = 18*s.*,

(£2 „ 3*s.*) − 18*s.* = £1 „ 5*s.* = 6 dollars;

therefore $\quad$ one dollar $=\frac{25}{6}=4\frac{1}{6}s.=4s.$ „ $2d.$

5. *If 24 men can reap 76 acres in 6 days, how many men can reap 114 acres in 9 days?*

$$\overset{\text{men}}{24}\times 6 : x\times 9 :: 76 : 114,$$

$$x\times 9\times 76 = 24\times 6\times 114,$$

$$x=\frac{24\times 6\times 114}{9\times 76}$$

$$=24 \text{ men.}$$

6. *Add together* $6\frac{1}{8}$, $23\frac{1}{2}$, 464, *and* 6·375.

$$\begin{array}{r} 6{\cdot}125 \\ 23{\cdot}5 \\ 464{\cdot} \\ 6{\cdot}375 \\ \hline 500{\cdot}000 \end{array}$$

Reduce the following fractions to their lowest terms:

(1)
$$\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\div\frac{1+\frac{3}{5}}{1-\frac{3}{5}}$$

$$=\left(\frac{4}{3}\times\frac{3}{2}\right)\div\left(\frac{8}{5}\times\frac{5}{2}\right)$$

$$=2\div 4$$

$$=\frac{1}{2}.$$

(2)
$$\frac{1-\frac{4}{9}}{3\frac{1}{4}+1\frac{1}{2}+\frac{1\frac{2}{3}}{6\frac{2}{3}}}\times\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)^2$$

$$=\frac{\frac{5}{9}}{4\frac{3}{4}+\frac{\cancel{5}}{\cancel{3}}\times\frac{\cancel{3}}{\underset{4}{\cancel{20}}}}\times\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right)^2$$

$$=\frac{\frac{5}{9}}{\frac{19}{4}+\frac{1}{4}}\times\left(\frac{3}{2}\times\frac{2}{1}\right)^2$$

$$=\frac{5}{9}\div 5\times 3^2$$

$$=1.$$

7. *Multiply* 39·39 *by* 7·878.

Divide 55·6591 *by* 1·813 *and* 230 *by* ·016.

```
      39·39
      7·878
     ------
      31512
     2 7573
    31 512
   275 73
   --------
   310·31442 Ans.
```

```
1·813) 55·6591 (30·7 Ans.          1·6) 23000·0
        1 2691                          -------
        0 0000                           14375
```

8. *What fraction of* 10*s.* *is* 4*s.* „ 6*d.*? *Reduce the result to a decimal.*

i.e. bring 4*s.* „ 6*d.* to a fraction of 10*s.*

$$\frac{4{\cdot}5}{10}={\cdot}45.$$

9. *Extract the square root of* 46090521 *and of* 136966·6081.

```
    46090521 (6789 Ans.              136966·6081 (370·09 Ans.
    36                                9
   ----                             ----
127) 1009                        67) 469
     889                             469
    -----                          --------
1348) 12005                   74009)   666081
      10784                            666081
      ------                           ------
13569) 122121                          .........
       122121
       ------
       .........
```

10. *A boat's crew row down from Searle's boat house to the locks at Baitsbite in half an hour, and they row back in three quarters of an hour. If they are* $7\frac{1}{2}$ *hours rowing to Ely and back, how long were they going down?*

Their time of going *down* is to their time of coming *up*, as $2:3$.

Therefore divide $7\frac{1}{2}$ hours into two parts, which are to each other in the ratio of $2:3$.

Therefore the time of going down

$$=\frac{2}{5} \text{ of } 7\tfrac{1}{2}$$

$$=\frac{2}{5}\times\frac{15}{2}$$

$$=3 \text{ hours.}$$

11. *A woman buys a certain number of apples for 3 a penny and the same number at 2 a penny. How much does she gain or lose per cent. by selling them all at 5 for twopence?*

For one apple of each kind she gave respectively $\frac{1}{3}$ and $\frac{1}{2}$ of a penny;

therefore for every 2 apples she gave $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}d.$,

and for every 2 apples she got $2\times\frac{2}{5}=\frac{4}{5}$;

therefore $\qquad \frac{5}{6}-\frac{4}{5}=\frac{25-24}{30}=\frac{1}{30}$ her loss.

Now, what fraction of her outlay was that? *i.e.* what fraction is $\frac{1}{30}$ of $\frac{5}{6}$?

$\frac{1}{30}\times\frac{6}{5}=\frac{1}{25}$; therefore she lost $\frac{1}{25}$ of her outlay.

But $\frac{1}{25}$ of 100 is 4; therefore she lost 4 per cent.

12. *A person has a number of oranges to dispose of: he sells half of what he has and one more to one person, half of the remainder and one more to a second person, half of the remainder and one more to a third person, and half of the remainder and one more to a fourth person: by which time he has disposed of all he had. How many had he at first?*

Let x = number of oranges he had at first.

If to the *first* he had sold *half*, he would have had

$$\frac{x}{2} \text{ remaining.}$$

But he had less than $\frac{x}{2}$ by 1, *i.e.* had $\frac{x}{2}-1$, or $\frac{x-2}{2}$ remaining.

If to the *second* he had sold *half*, he would have had

$$\frac{x-2}{4} \text{ remaining.}$$

But he had $\frac{x-2}{4}-1$ left, *i.e.* he had $\frac{x-6}{4}$ remaining.

If to the *third* he had sold *half*, he would have had

$$\frac{x-6}{8} \text{ remaining.}$$

But he had $\frac{x-6}{8}-1$ left, *i.e.* had $\frac{x-14}{8}$ remaining.

If to the *fourth* he had then sold *half*, he would have had

$$\frac{x-14}{16} \text{ remaining.}$$

But he had $\frac{x-14}{16}-1$ left, *i.e.* had $\frac{x-30}{16}$ remaining.

Now $$\frac{x-30}{16}=0,$$

$$x-30=0,$$

$$x=30.$$

13. *Explain the meaning of the terms interest and discount; pointing out the difference between them.* (Cf. § 99, p. 216.)

What is the discount upon £399 „ 1*s.* „ 6*d.* *due* 13 *months hence, interest being at* 5 *per cent.?*

$$\frac{13}{12} \text{ of } 5=5{\cdot}41\dot{6}$$

therefore $105{\cdot}41\dot{6} : 399{\cdot}075 :: 5{\cdot}41\dot{6} : \text{discount}.$

$$\text{discount} \times 105{\cdot}41\dot{6} = 399{\cdot}075 \times 5{\cdot}41\dot{6}$$

therefore $$\text{discount}=\frac{399{\cdot}075 \times 5{\cdot}4166}{105{\cdot}41\dot{6}}.$$

Hence, (inverting the multiplier, and using contracted multiplication and division,)

$$\begin{array}{r}
\cdot 03990750 \\
66145 \\
\hline
19953750 \\
1596300 \\
39907 \\
23945 \\
2394 \\
\hline
\end{array}$$

$$1\cdot 05416)\ 21\cdot 616296\ (20\cdot 505$$
$$\begin{array}{r}
5330 \\
59 \\
6
\end{array}$$

therefore £20 „ 10*s.* „ $1\frac{1}{4}$*d.* is the discount.

14. *In what time will* £158 „ 6*s.* „ 8*d.* *amount to* £176 „ 14*s.* „ 8*d.* *at* 3 *per cent. simple interest?*

	£	s.	d.	
From the amount ...	176 „	14 „	8	
Deduct the principal	158 „	6 „	8	
	18 „	8 „	0	interest gained

therefore $158\frac{1}{3} \times \overset{\text{years.}}{x} : 100 \times 1 :: 18\frac{2}{5} : 3,$

$$\frac{\overset{95}{\cancel{475}}}{\cancel{3}} \times x \times \cancel{3} = \overset{4}{\cancel{\overset{20}{\cancel{100}}}} \times \frac{92}{\cancel{5}},$$

$$x = \frac{368}{95} = 3\tfrac{83}{95} \text{ years.}$$

15. *A person invests* £4095 *in the* 3 *per cents. at* 91; *he sells out* £3000 *stock when they have risen to* $93\frac{1}{2}$ *and the remainder when they have fallen to* 85. *How much does he gain or lose by the transaction? If he invest the produce in the* $4\frac{1}{2}$ *per cent. stock at* 102, *what is the difference in his income?*

$$91 : 4095 :: 100 : \overset{\text{stock.}}{x},$$

$$x = \frac{\overset{45}{\cancel{4095}} \times 100}{\cancel{91}} = 5400 \text{ stock},$$

from which his income is £135.

He sells 3000 stock at $93\frac{1}{2}$; therefore he receives

$$30 \times 93\tfrac{1}{2},$$

or $$15 \times 187,$$

or $$2805 \text{ cash.}$$

He then sells 1500 at 85; therefore he receives

$$15 \times 85,$$

or $$1275 \text{ cash};$$

	£
therefore	2805
	1275
	4080 cash realized
but	4095 cash invested
therefore	15 loss

Next he invests 4080 in the $4\frac{1}{2}$ per cents., at 102; from which his income

$$= 4080 \times \frac{9}{2} \times \frac{1}{102}$$

$$= \frac{1020 \times 9}{51}$$

$$= 20 \times 9$$

$$= 180$$

therefore £180 − £135 = £45, the increase in income.

First Division B, 1859.

1. What number subtracted from 850967 will leave 3946?

The 365th part of a number is 101001, what is the number?

2. How many yards of cloth are there in 27 bales, each containing 15 pieces, and each piece 15 yards?

3. Find the cost of 20 dozen at 4*s*. $11\frac{1}{2}$*d*. per bottle, and if 3 bottles are spoiled, what will the merchant gain by selling the remainder at 5*s*. „ 4*d*. per bottle?

4. Four thalers, six half-crowns, and 8 florins amount to £2. What is the value of a thaler?

5. If 16 men can reap 76 acres in 4 days, how many men will reap 114 acres in 6 days?

6. Add together $3\frac{5}{8}$, $17\frac{1}{4}$, 476, and 3·125.

Reduce the following fractions:

$$\frac{1+\frac{1}{5}}{1-\frac{1}{5}} \div \frac{1+\frac{5}{13}}{1-\frac{5}{13}},$$

$$\frac{1-\frac{18}{25}}{3\frac{1}{3}+\frac{\frac{2\frac{3}{4}}{3}}{4}} \times \left(\frac{1+\frac{2}{3}}{1-\frac{2}{3}}\right)^2.$$

7. Multiply 237·07 by 4·567.

Divide 140·02564 by 1·871 and 406·8 by ·018.

8. What fraction of 5*s*. is 1*s*. „ $4\frac{1}{2}$*d*.?

Reduce the result to a decimal.

9. Extract the square root of 10004569 and of 240168·6049.

10. An ordinary train on the Eastern Counties Railway is 1 hour „ 57 minutes in travelling between Wymondham and Ely, and the express trains take 54 minutes less. If an express train leave Cambridge at 9 a.m. and arrive in London just as an ordinary train is leaving, which arrives in Cambridge at 2 p.m., find how long the express is in going to London?

11. A woman buys a certain number of eggs at 21 a shilling and the same number at 19 a shilling; she mixes them together and sells them at 20 a shilling; how much does she gain or lose per cent. by the transaction?

12. A man has a certain number of apples: he sells half the number and one more to one person, half the remainder and one more to a second person, half the remainder and one more to a third person, and half the remainder and one more to a fourth person, by which time he has disposed of all that he had. How many had he?

13. Explain the meaning of the terms interest and discount, pointing out the difference between them. Find the present worth of £396 „ 10*s*. „ 6*d*. due 11 months hence at 4 per cent.

14. In what time will £229 „ 10*s*. amount to £258 „ 3*s*. „ 9*d*. at 5 per cent. per annum at simple interest?

15. A person invests £6825 in the 3 per cents. at 91; he sells out £5000 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85. How much does he gain or lose by the transaction? If he invests the produce in $4\frac{1}{2}$ per cent. stock at par, what is the difference in his income?

Second Division A, 1859.

1. Find the sum, difference, and product of 12345678 and 28814412. The last may be found by only 3 lines of multiplication.

2. Thirty years ago a man was 3 times as old as his son, whose present age is 45. How old is the father?

3. When will a number divide by 8, 9, or 11?

Reduce $\frac{217800}{245025}$ to its simplest form.

4. A number may be divided by 25 by multiplying it by 4, and marking off the last two digits in the result as decimals.

Explain the reason for this; and divide 5335 by 25.

5. Add together $\frac{2\frac{1}{4}+3\frac{1}{28}}{3\frac{12}{35}+12\frac{2}{5}}$ and $\frac{6\frac{2}{3}+3\frac{19}{39}}{12\frac{19}{26}+2\frac{1}{2}}$. Find continued product of $24\frac{2}{3}$, $\frac{1}{4}$, $11\frac{3}{5}$, $\frac{3}{29}$, and $\frac{5}{37}$.

6. Reduce $\frac{3\frac{1\frac{3}{14}}{4}}{20}$ of a ton to cwts. qrs. lbs. &c.

7. A man has £3000 in hand, having lost a quarter of his property in speculation, and purchased a partnership in business with three quarters of the remainder. What was he worth at first?

8. Find the value of 157 tons at

(1) £7 „ 7*s.* (2) £2 „ 16*s.* „ 8*d.* (3) £4 „ 11*s.* „ 8*d.* per ton. Each result may be obtained by Practice, by making use of one aliquot part only.

9. Extract the square root of 9030025, of ·0144, and of $\cdot\dot{1}$.

10. Define simple and compound interest. Find simple interest on £1127 „ 18*s.* „ 4*d.* for $1\frac{1}{2}$ years, at 3 per cent. per annum?

11. Which is the better interest, 5 per cent. payable quarterly, or $5\frac{1}{8}$ per cent. payable yearly?

12. What is discount?

A bill due 3 months hence is discounted at 4 per cent. and its present value is £1225. What is the amount of the bill?

13. An estate is bought at 20 years purchase for £20,000, three quarters of the purchase-money remaining on mortgage at 4 per cent. The cost of repairs averages £150 per annum. What interest does the purchaser make of his investment?

14. A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses are 20 per cent.: the price of flour rises 50 per cent., and trade expenses are thereby increased 25 per cent. What advance must he make in the price of a fivepenny loaf, that he may still realise the same amount of profit from it?

15. Two houses are built: the first is twice as long in building as the second: half as many men again are employed in building the first; their wages per hour are one-third higher, and they work 10 hours a day and 6 days a week, whilst the others work only 8 hours a day and 5 days a week; the cost of the second in workmen's wages was £1000. What was that of the first?

October, 1859, (*A*).

1. Find the sum, difference, and product of 25435 and 34256.

2. The digits in the units and millions places of a number are 4 and 6 respectively. What will be the digits in the same places, when 999999 is added to the number?

3. If the excise duty on hops be 2*d.* per lb., and the whole duty average £234,000 per annum; what is the average growth in this country?

4. What is the freight on 480 bales of cotton weighing 4 cwt. „ 4 lbs. each, at $\frac{3}{4}$*d.* per lb., and 5 per cent. additional?

5. Define interest and discount, and find the interest on £5325 for 4 months at $4\frac{1}{2}$ per cent. per annum.

6. What is the discount on £479 „ 15*s.* for 3 months, at 4 per cent. per annum?

7. Reduce $\frac{3}{4}$ of $\frac{8}{9}$ of a £, to the decimal of $\frac{1}{4}$ of $\frac{4}{9}$ of £30.

8. When will a number divide by 3 or 8? Simplify $\frac{4608}{5184}$.

9. Find the value of $\dfrac{2\frac{3\frac{3}{4}}{12}}{20}$ of £2.

10. £935 is invested in the 3 per cents. at $93\frac{1}{2}$. What income is derived from the investment?

11. A steamer makes a voyage in 72 days, sailing on the average 9 knots per hour. How long will another be in making the same voyage, whose average rate of sailing is 8 knots per hour?

12. What is the value of 147 bullocks; one-third of them being sold at £18 „ 14*s.* „ 6*d.*; one-third at £20; and the remainder at £21 „ 5*s.* „ 6*d.* each?

13. Find the continued product of ·01, ·001, and 1·01.

14. Divide £325 amongst 4 persons in the proportion of 1, 2, 4, 6.

15. The pattern of a carpet is a yard long, and its width 2 feet „ 3 inches. How much must be bought to cover a room $20\frac{1}{4}$ feet square?

16. According to the Carlisle tables, the probable duration of the lives of persons of the ages of 10, 30, 50, 70, and 90 respectively, will be 48·82, 36·34, 21·11, 9·18, and 3·28 years respectively. If the premium for the whole life insured at the age of 10, be £1 „ 12*s.* per cent., construct a table of corresponding premiums for the other given ages.

First Division A, 1860.

1. *Explain the common system of notation, and point out its advantages.*

From 527 *take* 398, *explaining the reasons for the process.*

Cf. § 11, and § 19.

2. *Define a vulgar fraction, and shew that a fraction is not altered in value, if the numerator and denominator be multiplied by the same number. In what operations on fractions is this change necessary?*

Cf. § 44, and § 48.

A has twice as much money as B. They play together for a certain stake. At the end of the first game B wins from A one-third of A's money. What fraction of the sum B now has must A win back in the second game, that they may have exactly equal sums?

$$A \text{ has } 2, \text{ while } B \text{ has } 1,$$

$$A \text{ loses } \frac{1}{3} \text{ of } 2,\ B \text{ wins } \frac{1}{3} \text{ of } 2,$$

$$A \text{ has } \frac{4}{3},\ B \text{ has } \frac{5}{3}.$$

Now take $\frac{1}{3}$ from $\frac{5}{3}$, and there would be $\frac{4}{3}$ left; *i.e.* take away $\frac{1}{3}$ from B's money, there would be left the same sum that A has. But $\frac{1}{3}$ is one-fifth of $\frac{5}{3}$, so that by taking from B one-fifth of his money, he would have the same sum that A has.

If now one-half of that one-fifth, or one-tenth of B's money, be given to each, they would then have exactly equal sums.

Therefore *one-tenth* is the fractional part of B's money, which A must win back.

3. *Define a decimal fraction, and taking* ·4568 *as an example, shew from your definition that* $\cdot 4568 = \frac{4568}{10000}$. Cf. § 65.

Express as decimals $\frac{2^5}{10^8}$ *and* $\frac{3^4}{10^5}$, *and the sum, and the product of these quantities.*

$$\frac{2^5}{10^8} = \frac{32}{100000000} = \cdot 00000032,$$

$$\frac{3^4}{10^5} = \frac{81}{100000} = \cdot 00081.$$

Therefore the *sum* of these quantities is ·00081032, and the product ·0000000002592.

4. *Express* $\frac{5}{8}$ *of* 17*s.* „ 6*d.* + ·125 *of* 16*s.* − ·5$\dot{2}\dot{7}$ *of* 13*s.* „ 9*d.* *as a decimal fraction of* £5.

$$\frac{5}{8} \times 17s. \text{ „ } 6d. = 5 \times (2s. \text{ „ } 2\tfrac{1}{4}d.) = 10s. \text{ „ } 11\tfrac{1}{4}d.,$$

$$\cdot125 \times 16s. = 2\cdot000 = 2s.,$$

$$\cdot5\dot{2}\dot{7} = \frac{522}{900} = \frac{261}{495},$$

and $$\frac{261}{495} \text{ of } 165d. = \frac{261}{3} = 87d. = 7s. \text{ „ } 3d.;$$

therefore $$10s. \text{ „ } 11\tfrac{1}{4}d. + 2s. - 7s. \text{ „ } 3d. = 5s. \text{ „ } 8\tfrac{1}{4}d.$$

12	8·25
100	5·6875
	·056875 *Ans.*

5. *Divide* 1028·5 *by* ·0000017, *and* $\frac{2\frac{3}{5}}{3\frac{1}{4}}$ *by* ·000$\dot{6}$; *and multiply the difference of the quotients by* ·00025.

$$1\cdot7)\ \underline{1028500000}$$
$$605000000$$

Again, $$\frac{13}{5} \times \frac{4}{13} = \frac{4}{5},$$

$$\cdot000\dot{6} = \frac{6}{9000} = \frac{1}{1500},$$

therefore $$\frac{4}{5} \times \frac{1500}{1} = 1200.$$

Hence the *difference* between the two quotients is 604998800; and the *product* is 151249·7.

6. *A farmer rents a farm of* 800 *acres on the following terms: he pays a fixed rent of* 5*s. per acre, and a corn rent of* 200 *quarters of wheat,* 150 *quarters of barley, and* 120 *quarters of oats. The price of wheat, barley, and oats being* respectively 49*s.* „ 6*d.*, 30*s.* „ 8*d.*, *and* 19*s.* „ 2*d. per quarter,* find *his rent per acre.*

$$200 \times 49\tfrac{1}{2}s. = 200 \times \frac{99}{2} = 9900 \text{ shillings},$$

$$150 \times 30\tfrac{2}{3}s. = 150 \times \frac{92}{3} = 4600 \text{ shillings},$$

$$120 \times 19\tfrac{1}{6}s. = 120 \times \frac{115}{6} = 2300 \text{ shillings},$$

$$\text{total } 16800 \text{ shillings},$$

now $$\frac{16800}{800} = 21 \text{ shillings},$$

add the fixed rent of five shillings, and the rent per acre is 26*s.*

7. *A and B contract to execute a certain order for* £1245. *A employs* 100 *children for* 3 *months*, 80 *women for* 2 *months, and* 40 *men for* 1 *month; B employs* 120 *children for* 2 *months*, 60 *women for* $1\frac{1}{2}$ *months, and* 80 *men for* $2\frac{1}{2}$ *months. If the work done in the same time by a child, a woman, and a man be in the ratio of* 1 : 2 : 3, *find the sum of money which A and B must each receive.*

80 women do the work of 160 children,
40 men .... 120 ....,

so that *A* altogether employs what is equivalent to the labour of 100 children for 3 months, or 300 children for 1 month; and of 160 children for 2 months, or 320 children for 1 month; and of 120 children for 1 month: total 740 children for one month.

Also 60 women do the work of 120 children,
80 men........................ 240 ....,

so that *B* employs what is equivalent to the labour of 120 children for 2 months, or 240 children for 1 month; and of 120 children for $1\frac{1}{2}$ months, or 180 children for 1 month; and of 240 children for $2\frac{1}{2}$ months, or 600 children for 1 month: total 1020 children for one month.

The money must consequently be divided between *A* and *B* in the ratio of

740 : 1020, or 37 : 51;

therefore *A*'s share is $\frac{37}{88}$ of £1245, or £523 „ 9*s.* „ $3\frac{9}{11}d.$

B's share is £721 „ 10*s.* „ $8\frac{2}{11}d.$

8. *A man allows to his agent* 5 *per cent. on his gross income for the expense of collecting his rents. He spends* $\frac{1}{7}$ *th of his net income in assuring his life, and this part of his income is in consequence exempt from income-tax. The income-tax being* 10*d. in the pound, and his income-tax amounting to* £38 „ 19*s.; find his gross income.*

He allows his agent 5 per cent., or $\frac{1}{20}$ th of his income, and has $\frac{19}{20}$ as his net income. Now as $\frac{1}{7}$ of this net income is not taxed, only $\frac{6}{7}$ of $\frac{19}{20}$, or $\frac{57}{70}$ of his gross income is taxed, and this at 10*d.* in the pound pays £38 „ 19*s.*; therefore

$$\frac{10}{240} : 38\tfrac{19}{20} :: 1 : x,$$

$$\frac{1}{24} \times x = \frac{779}{20},$$

$$x = \frac{779}{20} \times 24$$

$$= \frac{779 \times 6}{5}$$

i.e. $$£\frac{779 \times 6}{5} = \frac{57}{70} \text{ of his gross income.}$$

Hence his gross income $= \frac{70}{57} \times \frac{779 \times 6}{5}$

$$= \frac{14 \times 779 \times 2}{19}$$

$$= 28 \times 41$$

$$= 1148.$$

9. *A young lady desires to paper her room with postage stamps, but being herself unable to calculate the number which will be required, she supplies the following data: her room is* 14*ft.* „ 9 *in. long,* 9*ft.* „ 3 *in. broad, and* 10*ft.* „ 6 *in. high; it contains two windows, each* $5\frac{1}{2}$ *ft. by* 4 *ft., and* 3 *doors, each* 6*ft. by* 3*ft.; a postage stamp is* $\frac{15}{16}$ *in. long, and* $\frac{3}{4}$ *in. broad. Make the calculation for her.*

Writing the quantities in the duodecimal scale,

$$\begin{array}{lr@{\qquad\qquad}l}
\text{length} & 12{\cdot}9 & 2\times 5\tfrac{1}{2} = e \\
\text{breadth} & \underline{9{\cdot}3} & \underline{4} \\
 & 20 & 38,\ \text{area of windows} \\
\text{multiply by height} & \underline{t{\cdot}6} & \\
 & 100 & 3\times 3 = 9 \\
 & 180 & \underline{6} \\
 & \overline{190{\cdot}0} & 46,\ \text{area of doors} \\
 & \underline{2} & \\
 & 360,\ \text{area of 4 walls} & \\
 & \underline{82},\ \text{area to be deducted} & \\
 & 29t,\ \text{area to be papered} & \\
 & \underline{12} & \\
 & 33 & \\
 & \underline{12} & \\
 & 406,\ \text{square feet to be papered.} &
\end{array}$$

But $\frac{15}{16}\times\frac{3}{4}$ is the fraction of a square inch covered by each stamp; therefore

$$\frac{15}{16}\times\frac{3}{4}\times x = 406\times 144$$

$$x = 406\times 144\times\frac{16}{15}\times\frac{4}{3}$$

$$= 406\times 16\times\frac{16}{5}\times 4$$

$$= \frac{415744}{5}$$

$$= 83148\tfrac{4}{5} \text{ stamps.}$$

10. *The area of the coal field of South Wales is* 1000 *square miles, and the average thickness of the coal is* 60 *feet. If a cubic yard of coal weigh* 1 *ton, and the annual consumption of coal in Great Britain be* 70,000,000 *tons; find the number of years for which this coal field alone would supply Great Britain with coal at the present rate of consumption.*

If the coal annually consumed in this country were piled up into a pyramid having for base the great court of Trinity College, the dimensions of which are 110 *by* 90 *yards; find the height of the pyramid.*

N.B. *The volume of a pyramid is equal to the area of the base multiplied into one-third of the height.*

Each square mile contains 1760×1760 square yards; and as the thickness of the coal is 20 yards, the content of coal field is

$$1000 \times 1760 \times 1760 \times 20 \text{ cubic yards,}$$

and this weighs $1000 \times 1760 \times 1760 \times 20$ tons.

Hence number of years

$$= \frac{1000 \times 1760 \times 1760 \times 20}{70000000}$$
$$= \frac{176 \times 176 \times 2}{70}$$
$$= 885\tfrac{1}{35} \text{ years.}$$

Also, volume of required pyramid is 70,000,000 cub. yds.

$$70000000 = \frac{1}{3} \times \text{height} \times 110 \times 90,$$
$$= \text{height} \times 110 \times 30,$$

$$\text{therefore height} = \frac{70000000}{110 \times 30} \text{ yards}$$
$$= \frac{700000}{33}$$
$$= 21212\tfrac{4}{33} \text{ yards}$$
$$= 12 \text{ miles ,, } 92\tfrac{4}{33} \text{ yards.}$$

11. *Define discount and present worth.*

Find the present worth of a bill of £283 ,, 10*s. due* $4\frac{1}{2}$ *months hence at* 3 *per cent.*

Distinguish between the mathematical and the mercantile discount, and find their difference in the above example.

$$\frac{9}{2} \times \frac{1}{12} \times 3 = \frac{9}{8} = 1{\cdot}125$$

$$101{\cdot}125 : 283{\cdot}5 :: 1{\cdot}125 : \text{discount.}$$

$$\text{discount} \times 20{\cdot}225 = 283{\cdot}5 \times {\cdot}225$$

$$\text{discount} = \frac{283{\cdot}5 \times {\cdot}009}{{\cdot}809}.$$

```
8·09) 25·515 (3·1539
         1 245
           4360
            325
             83
             11
```

Hence present worth $= 283{\cdot}5 - 3{\cdot}1539 = 280{\cdot}3461 = £280$ „ $6s.$ „ $11d.$

But the *mercantile* discount is only the simple interest on £283·5 for $4\frac{1}{2}$ months; and $283{\cdot}5 \times 1{\cdot}125 \times \frac{1}{100} = 2{\cdot}835 \times 1{\cdot}125$; and, by contracted multiplication,

$$\begin{array}{r} \cdot0028350 \\ 5211 \\ \hline 28350 \\ 2835 \\ 567 \\ 142 \\ \hline 3{\cdot}1894 \end{array}$$

Hence $\begin{array}{r} 3{\cdot}1894 \\ 3{\cdot}1539 \\ \hline £{\cdot}0355 \end{array}$ is the difference, and this, by inspection, is $8\frac{1}{2}d.$

12. *A man invests £4297 „ 10s. in the 3 per cents. at* $95\frac{1}{2}$. *He sells out one-third of his stock when the funds have fallen to 94, £1600 stock when they have risen to* $96\frac{1}{4}$, *and the remainder at par. What sum does he gain? And, if he invests the proceeds in the French 3 per cents. at 67·50, what is the difference in his income?*

$$95\tfrac{1}{2} : 4297\tfrac{1}{2} :: 100 : x,$$

$$\frac{191}{2} \times x = \frac{8595}{2} \times 100,$$

$$x = \frac{8595}{2} \times 100 \times \frac{2}{191}$$

$$= 4500 \text{ stock.}$$

And as he held this stock in the 3 per cents., the income he obtained was £135.

Now one-third of this, or 1500 is sold out at 94, 1600 at $96\frac{1}{4}$, 1400 at 100.

Therefore he receives

$$15 \times 94 + 16 \times 96\tfrac{1}{4} + 1400,$$

$$\text{or } 1410 + 1540 + 1400, \text{ or } 4350 \text{ cash.}$$

Therefore the *sum* he gains is £52 „ 10s.

By investing in the French 3 per cents., we have

$$67{\cdot}5 : 4350 :: 3 : \textit{Ans.}$$

$$Ans. = \frac{43500 \times 3}{675} = \frac{8700}{45}$$
$$= \frac{580}{3}$$
$$= £193 \text{ ,, } 6s. \text{ ,, } 8d.;$$

therefore £193 ,, 6*s*. ,, 8*d*. − 135 = £58 ,, 6*s*. ,, 8*d*., the gain in his income.

First Division B, 1860.

1. Explain the common system of notation and point out its advantages. From 613 take 49 explaining the reasons for the process.

2. Define a vulgar fraction, and shew that a fraction is not altered in value if the numerator and denominator be multiplied by the same quantity. In what operations on fractions is this change necessary? *A* has three times as much money as *B*. They play together for a stake, and at the end of the 1st game *B* wins from *A* $\frac{3}{8}$ths of *A*'s money. What fraction of the sum *B* now has must *A* win back in the second game, that they may have exactly equal sums?

3. Define a decimal fraction, and taking ·7256 as an example, shew from your definition that $\cdot 7256 = \frac{7256}{10000}$.

Express as decimals $\frac{2^4}{10^7}$ and $\frac{3^5}{10^8}$, and the sum, and the product of these quantities.

4. Express $\frac{3}{8}$ of 7*s*. ,, 6*d*. + ·625 of 10*s*. − $\cdot 5\dot{4}\dot{5}$ of 9*s*. ,, 2*d*. as a decimal fraction of £10.

5. Divide 9·614 by ·0000019, and $\frac{2\frac{1}{5}}{5\frac{1}{2}}$ by $\cdot 000\dot{3}$ and multiply the sum of the quotients by ·0005.

6. A farmer rents a farm of 800 acres on the following terms He pays a fixed rent of 4*s*. ,, 6*d*. per acre, and a corn rent of 250 quarters of wheat, 150 quarters of barley, and 100 quarters of oats. The price of wheat, barley, and oats being respectively 45*s*., 29*s*. ,, 4*d*. and 19*s*. ,, 6*d*. per quarter, find his rent per acre.

7. A and B rent a field for £60. A puts in 10 horses for $1\frac{1}{2}$ months, 30 oxen for 2 months and 100 sheep for $3\frac{1}{4}$ months; B puts in 20 horses for 1 month, 40 oxen for $1\frac{1}{2}$ months and 200 sheep for 4 months. If the food consumed in the same time by a horse, an ox, and a sheep be in the ratio $3:2:1$; find the portion of the rent of the field which each must pay.

8. A man allows to his agent 5 per cent. on his gross income for the expense of collecting his rents. He spends $\frac{1}{7}$th of his net income in assuring his own life, and this portion of his income is in consequence exempt from income tax. The income tax being 10*d.* in the pound and his income tax amounting to £39 „ 18*s.*; find his gross income.

9. The daily issue of the Times is 60,000 copies. Three days of the week it consists of 3 sheets, and for the remaining three of 4 sheets. If a sheet be 3 ft. long and 2 ft. broad; find the number of acres, which the weekly issue of the Times would cover.

10. The area of the Yorkshire coal field is $937\frac{1}{2}$ square miles, and the average thickness of the coal is 70 feet. If a cubic yard of coal weigh 1 ton, and the annual consumption of coal in Great Britain be 70,000,000 tons; find the number of years for which this coal field alone would supply Great Britain with coal, at the present rate of consumption.

If the coal annually consumed in this country were piled up into a rectangular stack having for base the great court of Trinity College, the dimensions of which are 110 yards by 90 yards; find the height of the stack.

11. Define discount and present worth.

A Jew discounts a bill of £180 drawn at 4 months, at 60 per cent. per annum, and insists on giving in part payment 5 dozen of wine which he charges at 4 guineas a dozen, and a picture which he charges at £19. How much ready money does he pay? If the cost to the Jew of the wine and the picture be only $\frac{1}{4}$th of the sum he has charged for them, what is the real interest the Jew has been charging?

12. A man invests £7620 in the 3 per cents. at $95\frac{1}{4}$. He sells out $\frac{1}{4}$th of his stock, when the funds have fallen to $93\frac{1}{2}$;

£3600 stock when they have risen to 96, and the remainder at par. What sum does he gain?

If he invest the proceeds in the Russian $4\frac{1}{2}$ per cents. at 97; what is the difference in his income?

Second Division A, 1860.

1. Find the sum, difference, product and quotient of 9765625 and 78125.

2. To 479 add $1\frac{804}{905}$, and repeat the addition six times.

3. Find the sum, difference, product and two quotients of 10·01 and ·0091.

4. There are three quantities, (i) £5, (ii) 8*s*. (iii) 75 gallons. Multiply one of these by the quotient of the other two.

State accurately the result of the operation, and perform it in as many different ways as possible.

5. Explain the statement of a question by "the rule of three." In how many different orders may the three terms be placed? And give a reason for preferring one order to another.

What is the value of 95 tons „ 17 cwt. of coals at £1 „ 15*s*. per load of $1\frac{3}{4}$ tons?

6. Upon what principle does the method of "practice" depend? Find the value of

(i) 44 things at £23 for every 40,

(ii) 23 things at £16 „ 10*s*. for every 11,

adopting the method of "rule of three," or "practice," whichever is the more convenient, in each example.

7. The solution of questions in "practice" may often be simplified by taking proportional parts of the multiplied instead of the original quantity; or by subtracting proportional parts instead of adding them. The values of the following may thus be found, by the aid of one proportional part only.

(i) 26 things at £11 „ 19*s*.

(ii) 59 things at £5 „ 12*s*. „ 6*d*.

(iii) 78 things at £6 „ 8*s*. „ 4*d*.

8. Define "discount."

What is the discount on £328 „ 13*s.* „ 5*d.* due 3 months hence at 4 per cent. per annum ?

9. Any sum of money may be expressed in pounds, twelfths of a pound, and a proper fraction of a twelfth; and five per cent. on the same may be immediately obtained by considering the pounds as shillings, the twelfths as pence, and the fraction of a twelfth as the same fraction of a penny.

(i) Explain the reason of this; and

(ii) Hence find 5 per cent. on £621 „ 13*s.* „ 8*d.*

(iii) Deduce $4\frac{3}{4}$ per cent. on the same amount.

10. Which is the better investment, bank stock paying 10 per cent. at 319, or 3 per cent. consols at 96 ?

11. An American dollar at par of exchange is worth 4*s.* 6*d.* of our money. What is the value of 642 dollars when the exchange is 7 per cent. in favour of England ?

12. A room is 60 feet long, by 29 feet wide; how many people can be seated in it on chairs $1\frac{1}{2}$ feet wide, and placed two feet apart from back to back; allowing a clear passage 3 feet wide down the middle of the room, and a space 15 feet deep at one end ?

13. The paper duty was $1\frac{1}{2}$*d.* per lb., and the weight of a certain book $1\frac{1}{2}$ lbs. The paper manufacturer realised 10 per cent. on his sale, and the publisher 20 per cent. on his outlay. What reduction might be made in the price of the book on the abolition of the paper duty, allowing to each tradesman the same rate of profit as before ?

October, 1860, (*A*).

1. Find the sum, difference, product, and quotient of 1653125 and 13225.

2. Find the square, and square root of ·007569.

3. There are three quantities: (1) 4 miles, (2) 4 furlongs, (3) £2. Multiply one of these by the quotient of the other two; state accurately the result of the operation, and perform it in as many different ways as possible.

4. Multiply $99\frac{4}{4}\frac{2}{2}\frac{3}{4}$ by 324; and find the value of $\dfrac{\dfrac{6\,11\frac{13}{60}}{12}}{14}$ of a week in days, hours, &c.

5. Find the value of $\cdot51875$ of a £; and $\cdot10\dot{7}1428\dot{5}$ of a cwt.

6. State the tests of divisibility of numbers by 4, 9, and 11; and apply them to the number 71016.

7. What is the value of a cargo of tallow, weighing 515 tons, at 51*s.* „ 3*d.* per cwt.?

8. Five per cent. on a given sum amounts to £25 „ 13*s.* „ 4*d.* Find $4\frac{3}{4}$ and $4\frac{1}{4}$ per cent. on the same sum.

9. Define interest and discount. What is the discount on £429 „ 5*s.* due 3 months hence at 4 per cent. per annum?

10. Two bills for £456 „ 5*s.* and £274 „ 2*s.* „ 6*d.* are due on the 1st and 30th June respectively. What is their value on the 20th June, interest being reckoned at the rate of 5 per cent. per annum?

11. Divide £3920 amongst 4 persons in the proportions of 2, 4, 6, 8.

12. A speculator sells at a profit of 50 per cent.; but his purchaser fails, and only pays 10*s.* in the £. How much per cent. does the speculator gain or lose by his venture?

13. *A* and *B* run a race. *A* starts at the rate of 400 yards a minute, but in every successive minute increases his pace by a yard a minute: *B* diminishes his pace by the same, and is overtaken by *A* in 4 minutes. What was *B*'s pace at starting?

First Division A, 1861.

1. *Add* 375 *and* 493; *and explain the process.*

Cf. § 15.

2. *Employ short division in dividing* 663072 *by* 5760. *Write down the remainder, and compare the process by which* 663072 *grains may be reduced to lbs., oz., and dwts., Troy.*

Since 24 grs. make 1 dwt., 20 dwts. 1 oz., 12 oz. 1 lb., and since

$$24 \times 20 \times 12 = 5760,$$

by dividing by these factors we shall be able to obtain both results by a single process.

$$\begin{array}{l|l} \begin{cases} 2 \\ 12 \end{cases} & \begin{array}{r} 663072 \\ \hline 331536 \end{array} \\ \hline 20 & \quad 27628 \\ \hline 12 & \quad\; 1381,8 \\ \hline & \quad\;\; 115,1 \end{array}$$

Here in abstract numbers the remainder is

$$8 \times 24 + 1 \times 20 \times 24, \text{ i.e. is } 672;$$

so that the quotient is 115, with a remainder 672.

But if the dividend be 663072 grs., the quotient 27628 is in the denomination dwts., the quotient 1381 is in the denomination oz. with a remainder of 8 dwts., the quotient 115 is in the denomination lbs. with a remainder 1 oz.; so that 115 lbs. „ 1 oz. „ 8 dwts. is *Ans.*

3. *Add* $\frac{1}{7}$, $1\frac{1}{4}$, $\frac{2}{3}$; *and divide the sum by* $\frac{1}{2} + \frac{1}{3}\left(\frac{1}{2} - \frac{1}{7}\right)$.

$$1 + \frac{1}{7} + \frac{1}{4} + \frac{2}{5} = 1 + \frac{20 + 35 + 56}{140}$$

$$= 1\tfrac{111}{140},$$

$$\frac{1}{2} + \frac{1}{3} \times \frac{5}{14} = \frac{1}{2} + \frac{5}{42} = \frac{13}{21}.$$

$$\frac{251}{140} \times \frac{21}{13} = \frac{753}{260} = 2\frac{233}{260}.$$

4. *What fraction of* 2 *cwt.* „ 14 *lbs. is* $\frac{1}{3}$ *of* 2 *qrs.* „ 14 *lbs.?*

That is to say, bring $\frac{1}{3}$ of $2\frac{1}{2}$ qrs. to the fraction of $2\frac{1}{8}$ cwt.

$$\frac{1}{3} \times 2\tfrac{1}{2} \div (2\tfrac{1}{8} \times 4)$$

$$= \frac{1}{3} \times \frac{5}{2} \times \frac{8}{17 \times 4}$$

$$= \frac{5}{51}.$$

Find the rent of 225 *ac.* „ 1 *rd.* „ 19 *p. at* 13*s.* „ $2\frac{1}{2}$*d. per rood.*

Expressing 225 ac. „ 1 rd. „ 19 p. as roods and a decimal of a rood, and 13*s.* „ $2\frac{1}{2}$*d.* as decimal of £1, we have 901·475 roods, at £·66041$\dot{6}$ per rood: hence by contracted multiplication,

$$\begin{array}{r} \cdot000901475 \\ 614066 \\ \hline 5408850 \\ 540885 \\ 3606 \\ 90 \\ 54 \\ \hline \pounds 595\cdot3485 \end{array}$$

whence, by inspection, £595 „ 6*s.* „ $11\frac{3}{4}$*d.* is the rent.

5. *Reduce* £2 „ 17*s.* „ $4\frac{1}{2}$*d. to the decimal of* £7. *Also add* ·275 *of a bushel to* ·725 *of a quarter, and find the value at* 6*s.* „ 8*d. per bushel.*

$$\begin{array}{r|l} 12 & 4\cdot5 \\ 20 & 17\cdot375 \\ 7 & 2\cdot86875 \\ \hline & \cdot40982\dot{1}4285\dot{7} \end{array}$$

Also

$$\begin{array}{l} \cdot725 \text{ of a quarter} \\ \quad 8 \\ \hline 5\cdot800 \text{ bushels} \\ \cdot275 \text{ bushel} \end{array}$$

$$6s. „ 8d. \;\Big|\; \tfrac{1}{3} \;\Big|\; \underline{6\cdot075} \qquad \pounds 2\cdot025$$

Therefore £2 „ 0*s.* „ 6*d.* *Ans.*

6. *Extract the square root of* 17424 *and of* 175·250564.

$$\sqrt{(17424)} = 132,$$

$$\sqrt{(175\cdot250564)} = 13\cdot2382, \text{ \&c.}$$

N.B. If the given number had been 175·350564 the exact root would have been 13·242.

The length of a rectangle is three times its breadth, and its area is 5808 *yards. What is the length in feet?*

$$3 \text{ breadth} \times \text{breadth} = 5808 \text{ square yards},$$

$$(\text{breadth})^2 = \frac{5808 \times 9}{3} \text{ square feet},$$

$$\begin{aligned} \text{breadth} &= \surd(5808 \times 3) \text{ square feet} \\ &= \surd(17424) \text{ square feet} \\ &= 132 \text{ linear feet}, \end{aligned}$$

whence $\text{length} = 396$ feet.

7. *If* 12 *Carlini be worth* 4*s.* „ 1*d.*, *and a Napoleon be worth* 16*s.*, *how many Carlini ought to be received for* 15 *Napoleons?*

$$12 \text{ Carlini} = 49 \text{ pence},$$

$$1 \text{ Carlino} = \frac{49}{12} \text{ pence}.$$

In 15 Napoleons there are $15 \times 16 \times 12$ pence; therefore

$$\begin{aligned} (15 \times 16 \times 12) \div \frac{49}{12} & \\ &= 2880 \times \frac{12}{49} \\ &= \frac{34560}{49} \\ &= 705\tfrac{15}{49} \text{ Carlini}. \end{aligned}$$

8. *If* 5 *men with* 7 *women earn* £7 „ 13*s.* *in* 6 *days, and* 2 *men with* 3 *women earn* 3 *guineas in the same time; in what time will* 6 *men with* 12 *women earn* £60?

$$5 \text{ men} + 7 \text{ women in 6 days earn } 153^{s.},$$

$$5 \text{ men} + 7 \text{ women in 1 day earn } \frac{153}{6},$$

(*a*) $$10 \text{ men} + 14 \text{ women in 1 day earn } \frac{153}{3}.$$

Again $$2 \text{ men} + 3 \text{ women in 6 days earn } 63,$$

$$2 \text{ men} + 3 \text{ women in 1 day earn } \frac{63}{6},$$

(β) $$10 \text{ men} + 15 \text{ women in 1 day earn } \frac{21 \times 5}{2}.$$

But, from (*a*), 10 men + 14 women in 1 day earn 51;

Therefore subtracting (α) from (β)

$$1 \text{ woman in } 1 \text{ day earns } 1\tfrac{1}{2}^{s.}$$

Also since $\frac{63}{6}s.$, or $10\frac{1}{2}s.$ is earned by 2 men and 3 women daily, and the 3 women earn $4\frac{1}{2}s.$ of this, the 2 men earn $6s.$, or 1 man in 1 day earns $3s.$ Hence 6 men with 12 women earn $36s.$ daily. Therefore they will earn 60×20 shillings in $\frac{60 \times 20}{36}$, or in $\frac{100}{3}$, or in $33\frac{1}{3}$ days.

9. *What is meant by interest and discount?*

Find the interest on £474 „ 13s. „ 4d. at 4 per cent. per annum for $3\frac{1}{2}$ years, simple interest.

$$\frac{1}{25} \times \frac{7}{2} = \frac{7}{50}.$$

$$\begin{array}{r} 474 \text{ „ } 13 \text{ „ } 4 \\ 7 \\ \hline 50)\ 3322 \text{ „ } 13 \text{ „ } 4 \\ \hline 66 \text{ „ } 9 \text{ „ } 0 \text{ „ } 3\frac{1}{5} \end{array}$$

10. *A tradesman who is ready to allow 5 per cent. per annum, compound interest, for ready money, is asked to give credit for two years. If he charge £27 „ 11s. „ 3d. in his bill, what ought the ready money price to have been?*

In other words, what is the present worth of £27 „ 11s. „ 3d. due 2 years hence, allowing *compound* interest at 5 per cent.?

£$10\frac{1}{4}$ is the compound interest of £100 in 2 years.

$$110\tfrac{1}{4} : 27\tfrac{9}{16} :: 100 : Ans.,$$

$$\frac{441}{4} \times Ans. = \frac{441}{16} \times 100,$$

$$Ans. = \frac{441}{16} \times 100 \times \frac{4}{441}$$

$$= 25.$$

11. *A person invests £2000 „ 16s. „ 1d. in the 3 per cents. at $9\frac{1}{8}$. What is the income derived by his investment?*

$$90{\cdot}5 : 2000{\cdot}8041\dot{6} :: 3 : \text{inc.}$$

$$\text{inc.} = \frac{6002{\cdot}41250}{90{\cdot}5}$$

$$9{\cdot}05\ )\ 600{\cdot}24125\ (\ 66{\cdot}325$$

$$\begin{array}{r} 57\,24 \\ 2\,941 \\ 226 \\ 45 \\ 00 \end{array}$$

whence the income = £66 „ 6*s.* „ 6*d.*

A person invests in the 3 per cents. so as to obtain 3 per cent. clear on his investment when there is an income-tax of 7d. in the pound. What per centage clear does he obtain if the tax be doubled?

In £3 there are 720 pence, which by paying a tax of 21 pence (7*d.* in the pound) are reduced to 699 pence; and which, when the tax is 14*d.* in the pound, are reduced to 678 pence. But when every 100 pound stock pays him 699 pence, he is making 3 per cent. clear; what is he making clear when every 100 stock pays 678 pence?

$$699 : 678 :: 3 : Ans.$$

$$699 \times Ans. = 678 \times 3,$$

$$Ans. = \frac{678 \times 3}{699}$$

$$= \frac{678}{233}$$

$$= 2\tfrac{212}{233} \text{ per cent.}$$

12. *If the price of barley be 6s. „ 1d. per bushel, and the cost of malting a quarter of barley be 2s. „ 2d., how much malt is made from 621 quarters of barley, supposing the maltster to pay 24s. „ 2d. tax per quarter of malt and gain 5 per cent. on the whole of his outlay by selling malt at 77s. „ 1½d. per quarter?*

Since by selling malt at 77*s.* „ 1½*d.* per quarter, he gains 5 per cent., the cost price of the malt was $\frac{20}{21}$ of 77*s.* „ 1½*d.*, or was 73*s.* „ 5$\frac{3}{7}$*d.*

Of this outlay, 24*s.* „ 2*d.* was the tax; hence outlay for buying and malting each quarter of *malt* is 49*s.* „ $3\frac{3}{7}$*d.*

But price of barley being 48*s.* „ 8*d.* per quarter, and the cost of malting 2*s.* „ 2*d.*, the outlay for buying and malting each quarter of *barley* is 50*s.* „ 10*d.*

Let x quarters of *malt* = 621 quarters of *barley*.

Then $$x \times 49\tfrac{2}{7} = 621 \times 50\tfrac{5}{6},$$

$$x = 621 \times \frac{50\frac{5}{6}}{49\frac{2}{7}}$$

$$= 621 \times \frac{305}{6} \times \frac{7}{345}$$

$$= \frac{1281}{2}$$

$$= 640\tfrac{1}{2} \text{ quarters.}$$

First Division B, 1861.

1. From 1861 take 1423 and explain the process.

2. Employ short division in dividing 195477 by 7920. Write down the remainder and compare the process by which 195477 inches may be reduced to furlongs, yards, feet, and inches.

3. Add $\frac{1}{5}$, $2\frac{1}{3}$, $\frac{1}{9}$, and $\frac{5}{21}$, and divide the sum by

$$\frac{1}{3} + \frac{1}{4}\left(\frac{1}{3} - \frac{1}{8}\right).$$

4. What fraction of 2 sq. yds. 7 ft. is $\frac{1}{4}$ of 2 sq. yds. 5 ft.?

Find the value of 72 cwt. „ 3 qrs. „ 17 lbs. at £1 „ 4*s.* „ 6*d.* per cwt.

5. Reduce £3 „ 15*s.* „ $9\frac{1}{2}$*d.* to the decimal of £9.

Also add 1·275 of a yard to 3·75 of a foot, and find the value at 3*s.* „ 4*d.* per foot.

6. Extract the square root of 21025 and 210·358669.

The length of a room is twice its breadth and the area is 1152 feet, what is its length?

7. If 10 scudi be worth 52·5 francs, and 16 shillings are worth 20 francs; how much in English money will be equivalent to 45 scudi?

8. If 3 men with 4 boys earn £5 „ 16*s*. in 8 days, and 2 men with 3 boys earn £4 in the same time; in what time will 6 men and 7 boys earn 20 guineas?

9. State the meaning of interest and discount.

Find the sum which will produce £146 „ 11*s*. „ $1\frac{1}{2}$*d*. interest in $4\frac{1}{2}$ years at 3 per cent. per annum, simple interest.

10. A tradesman who is ready to allow 4 per cent. per annum, compound interest, for ready money, is asked to give credit for two years. If he charge £22 „ 10*s*. „ 8*d*. in his bill; what ought the ready money price to have been?

11. A person invests £1839 „ 18*s*. „ 3*d*. in the 3 per cents. at $91\frac{1}{2}$. What is his income derived from the investment?

A person invests in the 3 per cents. so as to receive 3 per cent. clear on his investment when there is an income-tax of 9*d*. in the pound. What per centage does he receive if the tax be increased to 1*s*. in the pound?

12. If the price of barley be 6*s*. per bushel and the cost of malting a quarter of barley be 2*s*. „ 10*d*.; how much malt is made from 621 quarters of barley, provided the maltster pay 25*s*. tax per quarter of malt and obtain 5 per cent. on the whole of his outlay by selling malt at 78*s*. per quarter?

Second Division B, 1861.

1. What number must be added to sixty-nine thousand, four hundred and twenty-seven, to produce three hundred and twenty-five million, seven thousand and twenty-one?

2. Define a vulgar fraction, and prove that a fraction is not altered in value if the numerator and denominator be multiplied by the same quantity.

Arrange in order of magnitude the fractions $\frac{8}{33}$, $\frac{39}{161}$ and $\frac{47}{194}$, and express the difference of the first two as a fraction of the difference of the last two.

3. State and prove the rule for the multiplication of decimal fractions. Multiply $\cdot01385$ by $61\cdot3\dot{7}$; and divide the result by $2\cdot77$.

4. Find the value of

$$\frac{3}{4} \text{ of } \frac{1}{9\frac{1}{2}} \text{ of } £1 \,,,\, 18s. + \frac{2}{3} \text{ of } \cdot375 \text{ of } 15s. + \frac{2}{5} \text{ of } \cdot4\dot{2}\dot{9} \text{ of } 8s. \,,,\, 3d.$$

and express the result as a decimal fraction of £5.

5. The examination for mathematical honors commences each year at 9 o'clock on the 1st Tuesday in January.

In 1861, the examination commenced on January 1st. Find the number of seconds which will have elapsed from the commencement of the examination in 1861 till its commencement in 1862.

6. A man purchases a bale of cloth containing 80 yards at £1 „ 12*s.* per yard. He sells half of it at an advance of 25 per cent.; two-fifths of it at an advance of 4*s.* per yard, and the remainder, which is injured, at half the cost price; find his total gain, and his gain per cent.

7. Explain the mode of stating a question in the "double rule of three."

If the penny-loaf weigh 6 oz. when wheat is at 5*s.* per bushel, what should be the price of a loaf weighing $4\frac{1}{2}$ lbs. when wheat is at 7*s.* „ 6*d.* per bushel?

8. A cubic foot of gold is extended by hammering, so as to cover an area of 6 acres. Find the thickness of the gold in decimals of an inch, correct to the first two significant figures.

9. Find the interest of £808 „ 6*s.* „ 8*d.* from the 1st of January, 1861, to May 27th, 1861, at $4\frac{1}{2}$ per cent. per annum.

10. What is discount? and what is the present worth of a bill?

Find the discount on a bill of £461 „ 15*s.* „ $10\frac{1}{2}d.$ due three months hence, and discounted at $7\frac{1}{2}$ per cent. per annum.

11. If 6 per cent. be gained by selling a horse for £79 „ 10*s.*; how much is lost per cent. by selling him for £69?

12. In the University boat-race of 1860, the Cambridge crew rowed 39 strokes per minute, and the Oxford crew 41; but 19 strokes of the former were equal to 20 of the latter. The Cambridge crew rowed over the course in 25 minutes, and

the length of the course was 4 miles. Find the number of feet and the number of seconds by which the race was won.

13. A man invests £8063 in the 3 per cents. at $91\frac{1}{2}$, the brokerage being $\frac{1}{8}$ per cent.; what will be his clear income, after an income-tax of 10*d.* in the pound is deducted?

October, 1861, (*A*).

1. From one thousand and eighty-nine million seven hundred and four, subtract eighty thousand five hundred and forty-two; and divide the remainder by one hundred and thirty-nine.

2. An Englishman going abroad takes with him 50 guineas; during the 28 days he is abroad his average expenditure is 30 francs per day: his travelling expenses out and home amount amount to £5 extra. If 25 francs be equivalent to £1, how much money does he bring home with him?

3. Express in their simplest form,

$$(1)\quad \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)\div\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}\right).$$

$$(2)\quad \left(\frac{4\frac{1}{2}+5\frac{3}{4}}{5\frac{7}{8}-2\frac{1}{4}}\right)\div\left(2-\frac{3}{5\frac{1}{8}}\right).$$

4. Reduce to their equivalent vulgar fractions in their lowest terms the decimal fractions,

$$\cdot 03125;\ \cdot\dot{7}2\dot{9},\ \text{and}\ \cdot 7\dot{2}\dot{9},$$

and find the value of

$$\cdot 03125 \text{ of } £20 + \cdot\dot{7}2\dot{9} \text{ of } 6s.\ ,,\ 2d. + \cdot 7\dot{2}\dot{9} \text{ of } £2\ ,,\ 1s.\ ,,\ 3d.$$

5. If 4 men working 12 hours a day can reap a field 400 yds. long by 60 broad in 3 days; in how many days will 8 men working 10 hours a day reap a field 1000 yds. long by 200 broad?

6. A factory has 120 windows; 80 of which contain 16 panes, each 15 inches by 12; the remainder contain 12 panes each 1 foot square; find the cost of glazing the whole at 1*s.* „ 6*d.* per square foot.

7. An excursion train a quarter of a mile long leaves a

station at 8 h. „ 22 m.; and travels at the rate of 40 miles an hour; the ordinary train which travels at the rate of 66 feet per second, leaves the station at 8 h. „ 26 m., and follows the other.

How soon afterwards may a collision be expected?

8. £550 „ 10*s.* is borrowed on the 1st of January, 1861, at the rate of 5 per cent. per annum.

What sum will repay the debt on the 20th October, 1861?

9. Distinguish between interest and discount.

What is the present worth of a bill?

Find the present worth of a bill for £804 „ 13*s.* „ 4*d.* discounted 2 months before it is due, at $3\frac{1}{2}$ per cent. per annum.

10. Find the amount of £2500 at the end of 3 years, reckoning compound interest at 4 per cent. per annum.

11. A person invests 1000 guineas in the 3 per cents. at $92\frac{3}{4}$, paying $\frac{1}{8}$th per cent. for brokerage; what income does he derive from his investment? If he sell out when the funds have risen to 96 (brokerage as before), what does he gain by the transaction?

First Division A, 1862.

1. *Write in figures one million ten thousand and one.*

1010001.

Subtract 397 *from* 1862 *and explain the process.*

Cf. § 19.

A number augmented by one-fourth of itself is multiplied by 219 *and the product is* 3417495. *What is the number?*

Since $\frac{5}{4}$ of the number $\times 219 = 3417495$,

$$\text{the number} = \frac{3417495}{219} \times \frac{4}{5}$$

$$= \frac{683499 \times 4}{219}$$

$$= 12484.$$

2. *Divide 347923 by 35 by short division, and explain the rule for obtaining the remainder.*

Since $35 = 5 \times 7$, we have

$$\begin{array}{r|l} 5 & 347923 \\ \hline 7 & 69584,\ 3 \\ \hline & 9940,\ 4 \end{array}$$

Therefore $4 \times 5 + 3$, or 23 is remainder. See this explained § 29, p. 30.

3. *The regulations respecting Exhibition tickets from the opening on Thursday, May 1, to Saturday, October 18, are as follows: three-guinea season-tickets alone admit to the opening. £1 will be charged on May 2 and 3, and on three exceptional days (not in May, nor shilling days). From May 5 to 17 the charge will be 5s., and for the rest of the month 2s. „ 6d., except one day in each week when the charge is to be 5s. After May the charge for admission will be 1s. on four days of the week, and probably 2s. „ 6d. on the remaining days. On this supposition estimate the saving, by taking a season-ticket, of a person who proposes to be a daily visitor.*

To the daily visitor, if not a season-ticket holder, the charge would be as follows :

	£. s. d.
For May 2 and 3	2 „ 0 „ 0
May 5 to 17, 12 days, inclusive of one Sunday, at 5s. per day	3 „ 0 „ 0
May 19 to 31, ten days at 2s. „ 6d., and two days at 5s.	1 „ 15 „ 0
From 1 June (first of June being Sunday) to Saturday 18 October, there are 20 weeks at 9s. each	9 „ 0 „ 0
Add extra charge for 3 exceptional days on half-crown days; i. e. add 3 × 17s. „ 6d.	2 „ 12 „ 6
Total	18 „ 7 „ 6
Deduct price of a season-ticket	3 „ 3 „ 0
	15 „ 4 „ 6

4. *How many grains are there in a pound of gold?*

The gold procured from Australia in 6 months in 1851 amounted to 209,096 ounces. In 1861 the New Zealand gold

fields yielded 228,292 *ounces in the same time. What is the excess in weight and value (at £3 „ 17s. „ $10\frac{1}{2}$d. per ounce) of the average monthly return from New Zealand over that from Australia?*

Gold is weighed by *Troy* weight, in which 5760 grs. = 1 lb.

$$\begin{array}{lrl} \text{Next, from New Zealand} & 228292 & \\ \text{from Australia} & \underline{209096} & \\ 6\,)\!\! & \underline{19196} & \text{excess in 6 months} \\ & 3199\tfrac{1}{3} & \text{excess in ounces in 1 month.} \end{array}$$

To find the value of $3199\frac{1}{3}$ ounces at £3 „ 17s. „ $10\frac{1}{2}$d., is to multiply $3199\cdot\dot{3}$ by $3\cdot89375$: whence, by contracted multiplication,

$$\begin{array}{r} \cdot0319933333 \\ \underline{573983} \\ 95980000 \\ 25594667 \\ 2879400 \\ 95980 \\ 22395 \\ \underline{1600} \\ 12457\cdot4042 \end{array}$$

Therefore £12457 „ 8s. „ 1d. is, by inspection, the value.

5. *State the rule for the multiplication of Vulgar Fractions, and deduce a meaning for the operation.*

Reduce to simplest forms

$$\left(\frac{8}{17}+\frac{3}{5}\text{ of }7\tfrac{1}{2}\right)\div\frac{9}{11};\text{ and }\frac{\frac{3}{5}}{\frac{8}{9}}+\frac{\frac{5}{7}}{7\frac{1}{3}+\frac{11}{12}}.$$

Here first $$\left(\frac{8}{17}+\frac{3}{5}\times\frac{15}{2}\right)\div\frac{9}{11}$$

$$=\left(\frac{8}{17}+\frac{9}{2}\right)\times\frac{11}{9}=\frac{169}{34}\times\frac{11}{9}$$

$$=\frac{1859}{306}=6\frac{23}{306}.$$

Also
$$\frac{\frac{3}{5}}{\frac{8}{9}}+\frac{\frac{5}{7}}{7\frac{1}{2}+\frac{11}{12}}$$
$$=\frac{27}{40}+\frac{5}{7}\div\frac{101}{12}$$
$$=\frac{27}{40}+\frac{60}{707}$$
$$=\frac{21489}{28280}.$$

6. *State and explain, from an example or otherwise, the rule for converting a Vulgar Fraction into a decimal.*

Cf. § 75.

Find the value of

(1) $(3{\cdot}71-1{\cdot}908)\times 7{\cdot}03$.

```
  3·71
  1·908
 ------
  1·802
   7·03
 ------
  5406
126140
-------
12·66806
```

(2) $620{\cdot}5\div{\cdot}025$.

```
2·5 ) 62050 ( 24820
       120
        205
          50
          00
```

7. *Find by Practice the value of*

(1) 1032 *articles at* £1 „ 11*s.* „ $5\frac{1}{2}$*d. each.*

(2) 6 *tons* „ 7 *cwt.* „ 2 *qrs.* „ 17 *lbs. at* £3 „ 10*s.* „ 7*d.* *per cwt.*

1032 articles at £1 each would cost £1032.

		£	s.
10*s.*	$\frac{1}{2}$	1032 „	0
1	$\frac{1}{10}$	516 „	0
4	$\frac{1}{3}$	51 „	12
1	$\frac{1}{4}$	17 „	4
$\frac{1}{2}$	$\frac{1}{2}$	4 „	6
		2 „	3
		1623 „	5

again £3 „ 10*s.* „ 7*d.* = £$3{\cdot}5291\dot{6}$,

and 6 tons „ 7 cwt. „ 2 qrs. „ 17 lbs. = $127{\cdot}651785$ cwt.

```
·00127651785
      619253
  ----------
     3829554
      638259
       25530
       11489
         127
          77
  ----------
    450·5036
```

therefore, determining the value of the decimal by inspection, £450 „ 10*s.* „ 1*d.* is the cost.

8. *What is the value of* ·3375 *of an acre?*

Reduce £1 „ 15*s.* „ 4*d. to the decimal of* 2 *guineas.*

```
 ·3375
     4
 ------
 1·3500
     40
 -------
 14·0000
```

therefore 1 rood „ 14 poles *Ans.*

$$4\cdot2)\ 3\cdot5\dot{3}\ (\cdot\dot{8}4126\dot{9}$$

```
1 73
  53
 113
  293
   413
    353
```

9. *Distinguish between interest and discount.*

Shew that there is 15*s. difference between the interest and discount of* £82 „ 10*s. for two years at* 5 *per cent.*

$$82\cdot5 \times \frac{1}{20} \times 2$$

$= 8\cdot25$, the interest

110 : 82·5 :: 10 : *Dis.*,

110 *Dis.* = 825,

Dis. = 7·5,

8·25 interest for 2 years
7·5 discount for 2 years

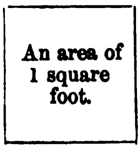
·75

therefore 1·5s. is difference.

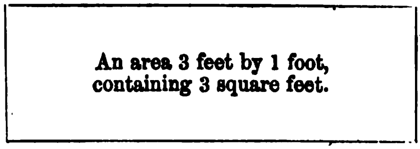
10. *Draw the shapes, and name, as descriptive of magnitude, the following products:*

(1) 1 *foot* × 1 *foot*. (2) 1 *yard* × 1 *foot*. (3) 1 *inch* × $\frac{1}{12}$ *inch*.

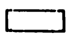
(1)



(2)



(3)



An area containing $\frac{1}{12}$ of a square inch.

How many bricks, of which length, breadth, and thickness are 12, 9, 6 *inches respectively, will be required to build a wall, whereof the length, height, and thickness are* 64, 9, *and* $1\frac{1}{2}$ *feet?*

Each brick being 1 foot long, $\frac{3}{4}$ of a foot wide and $\frac{1}{2}$ of a foot thick,

$$1 \times \frac{3}{4} \times \frac{1}{2} \times x = 64 \times 9 \times \frac{3}{2},$$

$$x = 64 \times 9 \times 4,$$

$$x = 2304 \text{ bricks.}$$

11. *A person sells out of the* $3\frac{1}{2}$ *per cents. at* $92\frac{3}{4}$ *and realizes* £18550; *if he invest one-fifth of the produce in the* 4 *per cents. at* 96, *and the remainder in the* 3 *per cents. at* 90, *find the alteration in his income.*

$$92\tfrac{3}{4} : 18550 :: 3\tfrac{1}{2} : x,$$

$$\frac{371}{4} \times x = 18550 \times 3\tfrac{1}{2},$$

$$x = 18550 \times \frac{7}{2} \times \frac{4}{371}$$
$$= 50 \times 7 \times 2$$
$= 700$, income originally obtained from $3\frac{1}{2}$ per cents.

He invests 3710 in the 4 per cents. at 96,

From which his income $= \dfrac{3710 \times \cancel{4}}{\underset{24}{\cancel{96}}} = £154$ „ $11s.$ „ $8d.$

He next invests 14840 in the 3 per cents. at 90,

From which his income $= \dfrac{1484 \times \cancel{3}}{\underset{3}{\cancel{9}}}$

$= £494$ „ $13s.$ „ $4d.$

Therefore £649 „ $5s.$ entire income,

deduct this from original income of 700, and the difference is £50 „ $15s.$, his loss in annual income.

12. *Find the square root of* 998001 *and of* 3·14159 *to three places of decimals.*

```
      998001 (999              3·141590 (1·772, &c.
      81                       1
189) 1880                  27) 214
     1701                      189
1989) 17901               347)  2515
      17901                     2429
      .......            3542)   8690
                                 7084
```

13. *If* 5 *pumps, each having a length of stroke of* 3 *feet, working* 15 *hours a day for* 5 *days empty the water out of a mine, how many pumps, with a length of stroke* $2\frac{1}{2}$ *feet, working* 10 *hours a day for* 12 *days, will be required to empty the same mine, the strokes of the former set of pumps being performed four times as fast as those of the latter?*

$$5 \times 3 \times 15 \times 5 \times 4 : Ans. \times \frac{5}{2} \times 10 \times 12 :: 1 : 1,$$

$$Ans. \times \frac{5}{\cancel{2}} \times \overset{5}{\cancel{10}} \times 12 = 5 \times 3 \times 15 \times 5 \times 4,$$

$$Ans. = \frac{\cancel{5} \times \cancel{3} \times 15 \times \cancel{5} \times \cancel{4}}{\cancel{5} \times \cancel{5} \times \cancel{12}}$$
$$= 15 \text{ pumps.}$$

First Division B, 1862.

1. Write in figures *ten millions one thousand and one.*

Add 397 to 1862 and explain the process.

A number diminished by one-fourth of itself is multiplied by 219 and the product is 2050497. What is the number?

2. Divide 329744 by 55 by short division, and explain the rule for obtaining the remainder.

3. The regulations respecting exhibition tickets from the opening on Thursday, May 1, to Saturday, October 18, are as follows:

Three guinea season tickets alone admit to the opening. £1 will be charged on May 2 and 3, and on three exceptional days (not in May, nor shilling days). From May 5 to 17 the charge will be 5*s.*, and for the rest of the month 2*s.* 6*d.*, except one day in each week when the charge is to be 5*s.* After May the charge for admission will be 1*s.* on four days of the week. If of the remaining days 18 should be 5*s.* days and the rest half-crown days, estimate the saving, by taking a season ticket, of a person who proposes to be a daily visitor.

4. How many lbs. are there in 97920 grains of gold?

The gold procured from Australia in nine months in 1851 amounted to 313644 ounces. In 1861 the New Zealand gold-fields yielded 342438 ounces in the same time. What is the excess in weight and value (at £3 „ 17*s.* „ $10\frac{1}{2}d.$ per ounce) of the average monthly return from New Zealand over that from Australia?

5. State what is meant by multiplication of fractions, and hence deduce the rule for the operation.

Reduce to simplest forms

$$\left(\frac{3}{5} \text{ of } 7\tfrac{1}{2} - \frac{8}{17}\right) \div 1\tfrac{2}{3}\text{; and } \frac{\frac{3}{5}}{\frac{8}{9}} - \frac{\frac{5}{7}}{7\frac{1}{2} - \frac{11}{12}}.$$

6. State and explain, from an example or otherwise, the rule for converting a vulgar fraction into a decimal.

Find the value of

(1) $(37\cdot1-19\cdot08)\times\cdot703$. (2) $62\cdot05\div\cdot0125$.

7. Find by practice the value of

(1) 2157 articles at £2 „ 7*s.* „ $4\frac{1}{2}d.$ each.

(2) 25 acres „ 3 roods „ 16 poles at £3 „ 12*s.* „ 6*d.* per acre.

8. What is the value of ·3375 of a ton?

Reduce 14*s.* „ $9\frac{3}{4}d.$ to the decimal of £2.

9. Distinguish between interest and discount.

Find the difference between the amount of £247 „ 10*s.* for 2 years and the present worth of the same sum due after 2 years, at 5 per cent.

10. Draw the shapes and name (as descriptive of magnitude) the following products:

(1) 1 yard × 1 yard. (2) 1 foot × 1 inch. (3) 1 yard × $\frac{1}{3}$ foot.

How many bricks of which the length, breadth, and thickness are 9, 6, 3 inches respectively, will be required to build a wall, whereof the length, height, and thickness are 72, 8, and $1\frac{1}{2}$ feet?

11. A person sells out of the $3\frac{1}{2}$ per cents. at $92\frac{3}{4}$ and, realizes £18550: if he invest two-fifths of the produce in the 4 per cents. at 96 and the remainder in the 3 per cents. at 90; find the alteration in his income.

12. Find the square root of 603729, and of 12·56636 to three places of decimals.

13. If 5 pumps, each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine; what must be the length of stroke of each of 15 pumps which, working 10 hours a day for 12 days, would empty the same mine, the strokes of the former set of pumps being performed four times as fast as those of the latter?

Second Division A, 1862.

1. The product of two numbers is 1270374 and half of one of them is 3129; what is the other number?

What will remain after subtracting 213 as often as possible from 83216?

2. Distinguish between prime and composite numbers. Shew how to resolve a composite number into its prime factors, and by so doing for the numbers 1071, 1092, 2310, find their greatest common measure.

3. The total stock of gold coin and bullion in the Bank of England on a certain day being of the value of £16,548,126, and the weight of it 354160 lbs., determine the value of an ounce of gold.

4. From the rule for the multiplication of vulgar fractions deduce the rule for division.

Multiply the sum of $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{3}{4}$, by the difference between $\frac{4}{5}$, and $\frac{2}{3}$.

Reduce to its simplest form $2\frac{1}{2}+\dfrac{1}{3\frac{1}{3}+\dfrac{1}{4\frac{1}{4}}}$.

5. Express as the fraction of £10 the difference between £$8\frac{3}{5}$ and £$8\times\frac{3}{5}$; and find the value of $\frac{2}{3}$ of a ton of sugar when $\frac{3}{16}$ of a ton is worth £6 „ 5*s.*

6. Give rules for the division of decimals.

Divide ·01 by ·01001 and ·01001 by ·01.

Find the value of ·3375 of a ton; and express 18*s.* „ 11½*d.* as a decimal of a guinea.

7. Define the terms interest, discount, present worth.

Find the difference between the simple and compound interest of £649 „ 15*s.* for 2 years at 5 per cent.

8. Find the present worth of £132 „ 3*s.* due 2¼ years hence at 4½ per cent. simple interest.

9. Find the value of 14764 articles at £1 „ 17*s.* „ $8\frac{1}{4}$*d.* each; and of 191 acres „ 3 roods „ 37 poles at £42 „ 3*s.* „ 4*d.* per acre.

10. An analysis of the Board of Trade returns for 1861, respecting shipwrecked lives, gives the following results:

Saved by life-boats, $13\frac{1}{2}$ per cent.; by rocket and mortar apparatus, 8 per cent.; by ships' boats, &c. 62 per cent.; by individual exertion, $\frac{1}{2}$ per cent.; lost, 16 per cent. Determine the number of lives saved by the several means enumerated corresponding to the loss of 864 lives.

11. A monolith of red granite in the Isle of Mull is said to be about 108 feet in length and to have an average transverse section of 113 square feet. If shaped for an obelisk it would probably lose one-third of its bulk, and then weigh about 600 tons. Determine the number of cubic yards in such an obelisk and the weight in pounds of a cubic foot of granite.

12. A person invests £5187 „ 10*s.* in the 3 per cents. at 83, and when the funds have risen to 84, he transfers three-fifths of his capital to the 4 per cents. at 96: find the alteration in his income.

13. Find the square root of 767376; and the length of the side of a square whose area is equal to that of a rectangle, the sides of which are 47·14 yards and 210 yards.

October, 1862, (*A*).

1. Subtract thirty million twenty-six thousand and three, from forty-five million seven thousand and twenty-one.

2. Define a vulgar fraction, and shew that a fraction remains unaltered if the numerator and denominator be multiplied by the same number.

Add together $\frac{1}{24}$, $\frac{1}{56}$, $\frac{1}{21}$ and $\frac{1}{12}$; and find what fraction their sum is of $2\frac{2}{7}$ of $1\frac{1}{6}$ of $\frac{2\frac{2}{3}}{3}$.

3. Find the value of $\frac{5}{6}$ of $\frac{1}{7\frac{1}{2}}$ of 3 square yards 6 feet, at $\frac{9}{25}$ of $\frac{1}{5\frac{1}{2}}$ of 4*s.* „ 2*d.* per foot.

4. Prove the rule for division of decimals.

From a rod 2·078 inches long, portions are cut off each equal to ·0037 of an inch long; find how many such portions can be cut off and what will be the length of the remainder.

5. The price of oats being 30*s.* per quarter, it costs 17*s.* „ 6*d.* per week to keep a horse; if oats cost only 26*s.* per quarter the expense would be 16*s.* „ $2\frac{1}{4}$*d.*; what quantity of oats does a horse eat per year?

6. Extract the square root of 120409, and the cube root of $3\frac{1}{8}$ to two places of decimals. The breadth of a room is twice its height and half its length, the contents are 4096 cubic feet; find the dimensions of the room.

7. If 10 scudi be worth 52·5 francs, 16 shillings worth 20 francs, and 12 carlini worth 4*s.* „ 2*d.*; how many carlini are equivalent to 500 scudi?

8. Point out the difference between interest and discount.

The interest on a sum at simple interest is £28, and the discount £21 „ 17*s.* „ 6*d.* for the same time; what is the sum?

9. A spirit merchant buys two sorts of spirits in equal quantities, one at 1 shilling per gallon more than the other; he mixes them and sells the mixture for 16*s.* „ 6*d.* per gallon, gaining 10 per cent. on his outlay. What was the price paid per gallon by the merchant?

10. A person buys a farm of 150 acres for £4624, and after repairing the buildings, lets it at 30*s.* per acre, thereby getting a return of $4\frac{1}{2}$ per cent. for his money: how much did he expend on repairs?

11. A person having his property in the 3 per cents. which are at $96\frac{1}{8}$, sells out and invests in the Great Eastern railway £100 stock which is at $55\frac{7}{8}$ and pays a dividend of $1\frac{3}{4}$ per cent.: the brokerage for buying or selling is $\frac{1}{8}$ per cent.: will this increase or diminish his income?

12. If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days; how long will it take 2 men and 2 boys to reap 10 acres?

First Division A, 1863.

1. ***Multiply*** 30040769503 ***by*** 172814412 ***in three lines; and*** £571 „ 13*s*. „ 4*d*. ***by*** 147.

Since $172800000 + 14400 + 12 = 172814412$,

if we multiply the given quantity by 12, then multiply that result by 1200, and then that result by 12000, we shall obtain three lines, which, if added together, will give the product required. Thus

$$\begin{array}{r}
30040769503 \\
12 \\
\hline
360489234036 \\
1200 \\
\hline
432587080843200 \\
12000 \\
\hline
5191044970118400000
\end{array}$$

Hence

$$\begin{array}{r}
360489234036 \\
432587080843200 \\
5191044970118400000 \\
\hline
5191477917688477236
\end{array}$$

Again $7 \times 7 \times 3 = 147$;

therefore

$$\begin{array}{rcrcr}
\pounds & & s. & & d. \\
571 & „ & 13 & „ & 8 \\
 & & & & 7 \\
\hline
4001 & „ & 15 & „ & 8 \\
 & & & & 7 \\
\hline
28012 & „ & 9 & „ & 8 \\
 & & & & 3 \\
\hline
84037 & „ & 9 & „ & 0
\end{array}$$

2. ***Divide*** £43009 „ 9*s*. „ 4*d*. ***by*** 64; ***and*** £2726 „ 6*s*. „ $8\frac{1}{4}$*d*. ***by*** 43.

$$\begin{array}{r|rcrcr}
 & \pounds & & s. & & d. \\
8 & 43009 & „ & 9 & „ & 4 \\
\hline
8 & 5376 & „ & 3 & „ & 8 \\
\hline
 & 672 & „ & 0 & „ & 5\frac{1}{2}
\end{array}$$

```
         £    s.  d.
   43 ) 2726 „ 6 „ 8¼ ( 63
         146
          17
          20
         ----
         346 ( 8
           2
          12
         ---
          32
           4
         ----
         129 ( 3
         000
```

therefore £63 „ 8*s.* „ 0¾*d.* *Ans.*

3. 7031 *at* 14*s.* „ 6½*d.*, *and* 6754¾ *at* £2 „ 1*s.* „ 5*d.*

Using contracted multiplication, *i.e.* writing 14*s.* „ 6½*d.* as the decimal of £1, and multiplying by 7031 *inverted*, we have

```
   ·7270833
       1307
  ---------
   50895833
     218125
       7270
  ---------
  5112·1228,
```

whence, by inspection, £5112 „ 2*s.* „ 5½*d.* is the *Ans.*

Again, finding in the same way the cost of 6754·75 articles at £2·0708$\dot{3}$ we have,

```
  ·020708333
      574576
  ----------
   124250000
    14495833
     1035417
       82833
       14496
        1035
  ----------
  13987·9614,
```

hence the cost is £13987 „ 19*s.* „ 2¾*d.*

4. *Interest on* £8712 „ 10*s.* *at* 4 *per cent. for* 15 *months.*

Discount on £13735 *at* 3¾ *per cent. for* 8 *months.*

To multiply by 4 and divide by 100, and then to take $\frac{15}{12}$ of that result is to multiply by $\frac{4}{100} \times \frac{15}{12}$, or by $\frac{1}{20}$.

$$\begin{array}{r} \pounds \quad s. \quad d. \\ 20\,)\,\underline{8712 \text{ „ } 10 \text{ „ } 0} \\ 435 \text{ „ } 12 \text{ „ } 6 \; \textit{Ans.} \end{array}$$

Also the interest on £100 for 8 months is $\frac{2}{3}$ of $\frac{15}{4}$, or $2\frac{1}{2}$.

$$102\tfrac{1}{2} : 13735 :: 2\tfrac{1}{2} : x,$$

$$\frac{205}{2} \times x = 13735 \times \frac{5}{2},$$

$$x = \frac{13735 \times 5}{205} = \frac{13735}{41} = 335.$$

5. *What cost* 1150 *three per cents. at* $92\frac{1}{2}$? *and what three per cents. at* $93\frac{3}{4}$ *will* £6000 *buy?*

$$92\tfrac{1}{2} : x :: \overset{\text{st.}}{100} : \overset{\text{st.}}{1150},$$

$$2x = \frac{185}{2} \times 23,$$

$$x = \frac{4255}{4}$$

$$= \pounds 1063 \text{ „ } 15s.$$

Also,

$$93\tfrac{3}{4} : 6000 :: 100 : \textit{Ans.},$$

$$\textit{Ans.} \times \frac{375}{4} = 6000 \times 100,$$

$$\textit{Ans.} = \frac{6000 \times 100 \times 4}{375}$$

$$= 400 \times 4 \times 4$$

$$= 6400 \text{ stock.}$$

6. *Add* $\frac{2}{3}$ *of* $\frac{5}{7}$ *of* $44\frac{1}{10}$, $\frac{3}{8}$ *of* $\frac{3}{11}$ *of* $9\frac{7}{9}$, *and* $\frac{8}{23}$ *of* 1863.

Take $\frac{2}{31}$ *of* £4 „ 0*s.* „ 1*d.* from $\frac{5}{43}$ *of* £7 „ 14*s.* „ 1*d.*

$$\frac{2}{3} \times \frac{5}{7} \times \frac{441}{10} = \frac{63}{3} = 21,$$

$$\frac{3}{8} \times \frac{3}{11} \times \frac{88}{9} = 1,$$

$$\frac{8}{23} \times 1863 = 8 \times 81 = 648;$$

therefore $\quad 21 + 1 + 648 = 670.$

Again,
$$\frac{5}{43} \text{ of } £7\tfrac{169}{240}$$
$$= \frac{5}{43} \times \frac{1849}{240}$$
$$= \frac{5 \times 43}{240}$$
$$= 215 \text{ pence},$$
$$\frac{2}{31} \text{ of } 4\tfrac{1}{240}$$
$$= \frac{2}{31} \times \frac{961}{240}$$
$$= \frac{2 \times 31}{240}$$
$$= 62 \text{ pence};$$

therefore $\quad 215 - 62 = 153d. = 12s. ,, 9d.$

7. *Express* $\frac{5}{64}$ *as a decimal; and a day as a decimal of a leap year.*

```
6·4) ·500 (·078125        3·66) ·01000000 (·002732, &c.
      520                        2680
        80                        1180
        160                          820
         320                          88
         000
```

8. *Value* £·925; *and* ·0833 *of* £41 ,, 13*s.* ,, 4*d.* *Divide* 999 *by* ·37; *and* ·1599 *by* 4100.

By inspection, ·925 = 18*s.* ,, 6*d.*

Also, using contracted multiplication, $41\cdot\dot{6}$ may be multiplied by ·0833 in inverted order,

```
·004166
    338
-------
  33333
   1250
    125
-------
 3·4708
```

therefore $\quad$ £3 ,, 9*s.* ,, 5*d.* *Ans.*

3·7) 9990·0 (2700 *Ans.*
259
000

4·1) ·0001599 (·000039
369
000

9. *Find the square root of* 16777216, *and of* $44\frac{4}{9}$, *and of* $\frac{2}{5}$ *to* 4 *places.*

$$
\begin{array}{r|l}
 & 16777216\ (4096 \\
 & 16 \\
809) & 7772 \\
 & 7281 \\
8186) & 49116 \\
 & 49116 \\
 & \cdots\cdots
\end{array}
$$

Next, $$44\frac{4}{9}=\frac{400}{9};$$

therefore the required square root is $\frac{20}{3}$, or $6\frac{2}{3}$, or $6\cdot\dot{6}$.

Again, $$\sqrt{\frac{2}{5}}=\sqrt{\frac{10}{25}}=\frac{\sqrt{10}}{5}=\frac{3\cdot16227,\ \&c.}{5}$$
$$=\cdot63245,\ \&c.$$

10. *In which way had one better buy sugar, at* 3 *guineas per cwt., or at £*2 „ 16*s.* „ 4*d. per quintal of* 100 *lbs.? and how much is one buying when the gain by the more advantageous way is a guinea?*

112 lbs. for 63*s.* is at rate of $\frac{63}{112}$ *s.* per lb.

The quintal for $56\frac{1}{3}$*s.* is at rate of $\frac{169}{300}$ *s.* per lb.

Excess in price per lb. when bought by quintal is

$$\frac{169}{300}-\frac{63}{112}=\frac{4732-4725}{8400}\text{ of a shilling}$$
$$=\frac{7}{8400}=\frac{1}{1200}\text{ of a shilling}$$
$$=\frac{1}{100}\text{ of a penny.}$$

For this excess to amount to a guinea the quantity bought must be

$12\times21\times100$, or 25200 lbs., or 225 cwt.

11. *Three trees have their distances as* 3 : 4 : 5, *and a rope of* 492 *feet long just goes round them. Find their respective distances.*

The perimeter of the triangle being 492 feet, and the sides, in the ratio of 3 : 4 : 5, the respective distances are

$$\frac{3}{12} \text{ of } 492 = 3 \times 41 = 123,$$

$$\frac{4}{12} \text{ of } 492 = 4 \times 41 = 164,$$

$$\frac{5}{12} \text{ of } 492 = 5 \times 41 = 205.$$

12. *On what sum is the daily interest at* 4 *per cent. one penny?*

$$100 \times 365 : Ans. \times 1 :: 4 \times 240d. : 1d.,$$

$$Ans. \times 4 \times 240 = 100 \times 365,$$

$$Ans. = \frac{25 \times 73}{48}$$

$$= £38 \text{ ,, } 0s. \text{ ,, } 5d.$$

13. *If a grain of gold is worth* $2\frac{1}{2}d.$, *what should a sovereign weigh? Supposing the alloy in a sovereign to be* $\frac{1}{11}$ *of the whole, what would it be worth if it were all gold?*

$$240 \div 2\tfrac{1}{2} = 240 \times \frac{2}{5}$$

$$= 96 \text{ grains}$$

$$= 4 \text{ dwts.}$$

Also, the alloy having no value, if $\frac{10}{11}$ of a sovereign be worth 20*s.*, $\frac{1}{11}$ must be worth 2*s.*

Therefore a sovereign all gold would be worth 22*s.*

14. *If* 16 *darics make* 17 *guineas,* 19 *guineas make* 24 *pistoles,* 31 *pistoles make* 38 *sequins, then how many sequins are there in* 1581 *darics?*

$$\overset{\text{darics.}}{1581} = \frac{\overset{\text{guineas.}}{1581 \times 17}}{16}$$

$$= \frac{\overset{\text{pistoles.}}{1581 \times 17 \times 24}}{16 \times 19}$$

$$= \frac{\overset{\text{sequins.}}{1581 \times 17 \times 24 \times 38}}{16 \times 19 \times 31}$$

$$= 51 \times 17 \times 3 \text{ sequins}$$

$$= 2601 \text{ sequins.}$$

First Division B, 1863.

1. Multiply 43002073252 by 133112191 in 3 lines; and £607 „ 13*s*. „ 8*d*. by 135.

2. Divide £3388 „ 7*s*. „ $6\frac{3}{4}$*d*. by 33; and £21919 „ 18*s*. „ $1\frac{1}{2}$*d*. by $13\frac{3}{4}$.

3. 6864 at 13*s*. „ $9\frac{1}{4}$*d*.; and $8864\frac{3}{4}$ at £2 „ 5*s*. „ 10*d*.

4. Interest on £6787 „ 10*s*. at 3 per cent. for 16 months. Discount on £237655 at $3\frac{1}{4}$ per cent. for $\frac{3}{4}$ year.

5. What cost 2250 three per cents. at $91\frac{1}{4}$? and what three per cents. at $87\frac{1}{2}$ will £3500 buy?

6. Add $\frac{3}{4}$ of $\frac{3}{7}$ of $3\frac{1}{9}$, and $\frac{5}{8}$ of $\frac{5}{9}$ of $203\frac{9}{25}$, and $\frac{10}{207}$ of 1863.

Take $\frac{2}{29}$ of £3 „ 10*s*. „ 1*d*. from $\frac{8}{31}$ of £4 „ 0*s*. „ 1*d*.

7. Express $\frac{13}{32}$ as a decimal, and a pound as decimal of a hundred weight.

8. Value £·3025; and ·1433 of £83 „ 6*s*. „ 8*d*. Divide 299 by 13, and ·3621 by 7100.

9. Find the square root of 930372004, and of $60\frac{21}{25}$, and of $\frac{3}{5}$ to 4 places.

10. Which way had one better buy coffee, at 6 guineas a cwt. or at £5 „ 12*s*. „ 4*d*. per quintal of 100 lbs? And how

much is one buying when the loss on the less advantageous way is £1?

11. A rope 495 feet long just goes round three trees whose distances from each other are as 4 : 5 : 6. Find the distances.

12. On what sum is the daily interest at 5 per cent. one groat?

13. If six grains of silver are worth five farthings, what should a crown weigh?

If the alloy in silver coin is $\frac{1}{21}$ of the mass, what would a crown be worth if it were all silver?

14. If 2 guineas make 3 Napoleons, and 15 rix-dollars make 4 Napoleons, and 6 ducats make 7 rix-dollars, how many ducats are there in £490?

Second Division A, 1863.

1. Multiply 13 tons „ 5 cwt. „ 3 qrs. „ 11 lbs. by 24.

2. Divide £13043 „ 3*s.* „ $3\frac{3}{4}$*d.* by 679; and £65931 „ 12*s.* „ 9*d.* by $6\frac{3}{4}$.

3. 17392 at $6\frac{3}{4}$*d.*; $8044\frac{1}{4}$ at £2 „ 14*s.* „ 8*d.*

4. If 19 men finish a work in 437 days, how long would it take 23 men?

5. If 45 cwt. carried 65 miles cost 9*s.* „ 9*d.*, what will 60 cwt. carried 90 miles cost?

6. Interest on £3712 „ 10*s.* at $4\frac{1}{2}$ per cent. for $3\frac{1}{2}$ years.

Discount on £55447 at $4\frac{1}{2}$ per cent. due after $2\frac{1}{2}$ years.

7. Find the greatest common measure of 68635 and 19721, and the least common multiple of 8, 9, 10, 12.

8. Add $\frac{2}{7}, \frac{5}{8}, \frac{3}{11}$. Add $\frac{2}{3}, \frac{5}{9}, \frac{8}{27}, \frac{11}{81}$. Take $\frac{3}{4}$ of $\frac{3}{7}$ of 84 from $\frac{3}{5}$ of $\frac{2}{11}$ of $504\frac{1}{6}$.

9. What decimal is a day of a year? and 3*s.* „ $7\frac{3}{4}$*d.* of 18*s.* „ $2\frac{3}{4}$*d.*? Divide 7821 by ·079, and ·10304 by 9200.

10. Find the square root of 67108864; and of ·1 to two places of decimals.

11. *A* offers for an estate £83000, *B* offers £96000 after 3 years. Which is the better offer, and by how much, allowing five per cent. compound interest?

12. If the 3 per cents. are at $92\frac{5}{8}$, and the four per cents. at $123\frac{1}{2}$, in which should one invest? and how much is one investing when the difference in income is a shilling?

13. What must be the gross produce of an estate that after paying a ten per cent. income-tax, and a rate of 2*s.* „ $1\frac{1}{2}$*d.* on £1 on the residue, there may remain £2574 per annum?

14. If a population is now ten millions, and the births being 1 in 20, the deaths are 1 in 30, what will the population become in 5 years?

15. Can $\sqrt{2}$, $\sqrt{3}$, $\sqrt{10}$ be sides of a triangle?

October, 1863, (*B*).

1. Multiply 40837 by 99989; and £10796 „ 8*s.* „ $3\frac{1}{2}$*d.* by 96.

Divide £3609 „ 5*s.* „ 5*d.* by 25; and £7817 „ 12*s.* „ $10\frac{1}{2}$*d.* by 127.

2. How many ducats of 4*s.* „ $11\frac{3}{4}$*d.* each are worth 55926 rix-dollars of 4*s.* „ $10\frac{1}{2}$*d.* each?

3. What is the dividend on a bankrupt's estate, when his debts are £4800 and his property £3680?

4. If 36 men finish a work in 44 days, how long will it take 66 men?

5. If $4\frac{1}{4}$ tons are carried 40 miles for 14*s.* „ 2*d.*, how far will $5\frac{1}{2}$ tons be carried for £1 „ 7*s.* „ 6*d.*?

6. 30848 at $6\frac{3}{4}$*d.*; 9836 at 7*s.* „ $2\frac{1}{2}$*d.*; $3044\frac{3}{4}$ at £2 „ 16*s.* „ 4*d.*

7. Find by practice the value of 37 lbs. „ 3 oz. „ 9 dwt. „ 15 gr. troy, at £2 „ 13*s.* „ 4*d.* per ounce.

8. Find the interest on £328500 at 5 per cent. for 200 days; and the discount on £13051 due after 18 months at $3\frac{1}{2}$ per cent.

9. What is the amount of £14025 at 4 per cent. compound interest for 4 years?

10. Find the greatest common measure of 30012 and 13237, and the least common multiple of 16, 24, 30, 36.

11. Add $\frac{3}{5}$ of $\frac{3}{7}$ of $15\frac{5}{8}$, $\frac{3}{7}$ of $\frac{2}{11}$ of $12\frac{5}{6}$, $\frac{4}{9}$ of $\frac{4}{23}$ of 1863. Take $\frac{10}{33}$ of £4 „ 10*s.* „ 9*d.* from $\frac{20}{39}$ of £6 „ 6*s.* „ 9*d.*

12. Simplify $\frac{3\frac{3}{4}}{5}$, and $\frac{22\frac{1}{2}}{2\frac{1}{4}}$; and divide the sum of $2\frac{2}{3}$ and $2\frac{1}{2}$ by their difference.

13. Express $5\frac{1}{4}$*d.* as decimal of a shilling; and 8*s.* „ $4\frac{3}{4}$*d.* as decimal of £3 „ 7*s.* „ 2*d.* Value £·3125; and ·613 of £12 „ 10*s.* Divide ·1089 by 33; and 911·6 by ·0086; and ·005829 by ·00067.

14. Find the square root of 88804, of $10\frac{38}{49}$, and of ·0036.

15. If tea is bought for £28 per cwt., and sold at 5*s.* „ $7\frac{1}{2}$*d.* per lb., what is gained per cent.?

16. If a national debt of £512000000 has an eighth part of its then existing amount paid off every year, how soon will it be reduced to less than half its original amount?

Previous Examination. March, 1864.

1. Multiply £53 „ 19*s.* „ $7\frac{1}{2}$*d.* by $2\frac{1}{2}$; and 23 years „ 35 days „ 13 hours by 16. Divide £7000 „ 7*s.* „ 11*d.* by 365; and 203 tons „ 17 cwt. „ 0 qrs. „ 8 lbs. „ 13 oz. by $3\frac{3}{4}$.

2. If 37 tons carried 57 miles cost £8 „ 15*s.* „ 9*d.*, what will 63 tons carried 83 miles cost?

3. Find by Practice the amount of 46384 at $8\frac{1}{4}$*d.*; of $6088\frac{3}{4}$ at £2 „ 9*s.* „ 2*d.*; of 1292 cwt. „ 3 qrs. „ 8 lbs. at 35*s.* per cwt.

4. Find the interest on £1875 at $3\frac{3}{4}$ per cent. for 16 months; and the discount on £6191 „ 10*s.* „ 1*d.* at $3\frac{3}{4}$ per cent. for 4 months.

5. Find the amount, at compound interest, of £1864 at $4\frac{1}{2}$ per cent. in 3 years.

6. Find the greatest common measure of 40033 and 129645; and the least common multiple of 144 and 180.

7. Add $\frac{2}{9}, \frac{9}{10}, \frac{10}{11}$; add $\frac{3}{4}$ of $\frac{3}{7}$ of $31\frac{1}{5}$, $\frac{2}{3}$ of $\frac{4}{5}$ of $24\frac{3}{8}$, $\frac{2}{7}$ of $\frac{3}{8}$ of 1864.

From $\frac{33}{1232}$ cwt. take $\frac{8}{13}$ of 2 lbs. „ 7 oz.

8. Express $\frac{17}{125}$ as a decimal; and ·111 as an ordinary fraction.

Find what decimal 4*s.* „ $8\frac{1}{4}$*d.* is of £1 „ 4*s.* „ $5\frac{1}{4}$*d.*

9. Divide 655·36 by 25·6; and 100 by ·25; and ·25 by 100.

10. Extract the square root of 40054818769; and $1\frac{11}{25}$; and $\frac{2}{3}$.

11. Find the solid content of a box 6 ft. „ 9 in. long, 3 ft. „ 8 in. broad, and 3 ft. „ 4 in. high.

12. *A*, *B*, *C*, contribute respectively to an undertaking £105, £165, £285, and they gain £195; how shall they divide it equitably?

13. How much will a man save out of an annual income of £706, who spends a guinea and a half a day?

14. If copper is bought for £50 a ton, and sold for 6*d.* a lb., what is gained per cent.?

Previous Examination. March, 1865.

1. Explain the method of representing numbers by figures in the ordinary or decimal system of notation.

Express in figures ninety-nine million and nine thousand.

2. Divide 3672965 by $2 \times 3 \times 6$ (Short Division). Explain every step of the process.

3. A person left a sum of money which was divided equally amongst 43 poor people, such that after a deduction of 6*d.* in the pound, each received £2 „ 18*s.* „ 3*d.* What sum did he leave?

4. Define a fraction, and shew from your definition that $\frac{1}{2}=\frac{3}{6}$.

5. Add together $253\frac{1}{2}$, $47\frac{1}{7}$, $5\frac{1}{3}$, and $\frac{2}{3}$ of $7\frac{1}{2}$; and divide the result by ·01.

6. Subtract ,03 from $,\dot{0}\dot{3}$; and divide the result by $,10\dot{2}$.

7. Find the decimal of a week which differs from a day by less than the millionth part of a week.

8. The decimal subdivisions of a pound sterling being florins, cents, and mils, amongst how many people must £1 „ 5 florins „ 6 cents be divided in order that the share of each may be 1 florin „ 4 mils?

9. What sum of money will amount to £138 „ 2*s.* „ 6*d.* in 15 months at 5 per cent. per annum simple interest?

10. Find the discount on £520 „ 17*s.* „ 6*d.* due $3\frac{1}{2}$ years hence, the rate of interest being $4\frac{1}{2}$ per cent. per annum.

11. A room whose height is 11 feet, and length twice its breadth, takes 143 yards of paper 2 feet wide for its four walls; how much carpet will it require?

12. A person on leaving England exchanged his money for French money at the rate of 25 francs for a sovereign, and on arriving at Munich received 135 Bavarian gulden for 15 Napoleons: what was his loss in English money, supposing a gulden to be worth 1*s.* „ $8\frac{1}{2}$*d.*?

13. The road between two towns, *A* and *B*, distant 15 miles, goes over a hill, whose summit is 3 miles from *A*. Two pedestrians set out at the same time from *A* and *B*, the former going 4 miles an hour uphill, and $5\frac{1}{4}$ down, the latter $3\frac{1}{2}$ uphill, and $4\frac{1}{2}$ down; where will they meet?

Previous Examination, 1866.

1. Describe the common system of numerical notation.

Write down in figures six hundred and fifteen million four hundred thousand and twenty-three; and express in words the number 1659800205.

2. Multiply 985437 by 5769, explaining the process.

Divide 2154597775 by 68585.

3. The sum of £275 „ 15*s.* „ $1\frac{1}{2}$*d.* is divided among 18 persons; what will each receive?

Find the cost of 356 articles at £7 „ 15*s.* „ $7\frac{3}{4}$*d.* each, by practice.

4. Prove that the numerator and denominator of a fraction may be multiplied by the same integer without altering its value.

Simplify $\frac{1003}{1829}$.

Find the value of $\frac{1}{3\frac{1}{5}} - \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{2} + \frac{\frac{4}{7}}{4\frac{4}{7}}$.

5. Enunciate the rule for the addition of fractions.

Divide $2\frac{2}{15}$ of $2\frac{5}{8}$ by $2\frac{2}{3} - 2\frac{1}{4}$.

Reduce $\frac{4245}{2264}$ of £1 to the fraction of a Prussian dollar. (£1 = $6\frac{2}{3}$ dollars.)

6. Reduce to decimals the fractions $\frac{17}{125}$ and $\frac{372}{1250}$.

Convert into vulgar fractions the decimals ·0006875 and $\cdot\dot{4}2857\dot{1}$.

7. Multiply ·007853 by ·00476.

Divide ·2219904 by ·3854.

8. Extract the square roots of 51825601; ·013689.

Find the value of $\sqrt{2-\sqrt{2}}$ to 6 places of decimals.

9. The sum of £328 „ 3*s.* „ $6\frac{1}{2}$*d.* is to be divided between four men, *A*, *B*, *C*, and *D*, in such proportion, that for every £3 given to *A*, *B* is to receive £5, *C* £8, and *D* £9. What sum did each receive?

10. What sum will amount to £425 „ 19*s.* „ $4\frac{4}{5}$*d.* in ten years at $3\frac{1}{2}$ per cent. simple interest; and in how many years more will it amount to £435 „ 11*s.* „ 7*d.*?

Find the true discount on £125 due two years hence, at 4 per cent. per annum.

11. Find the amount of £1000 in 6 years at 5 per cent. compound interest.

12. If 15 men take 17 days to mow 300 acres of grass, how long will 27 men take to mow 167 acres?

13. A rectangular cistern, whose length is $13\frac{3}{8}$ feet and breadth 6 feet, contains $294\frac{1}{4}$ cubic feet of water: what is the depth of the water; and what is its weight, when a cubic inch of water weighs 252·5 grains?

Previous Examination, 1867.

1. What do you understand by 25? Might 25 admit of more than one meaning?

2. A man died in 1867 aged 92, his son died in 1823 at the age of 17. How old was the father when the son was born?

3. Write down all the numbers of 4 digits you can form with the digits 3, 6, 0, 4 and add them together.

4. Divide 79875 by 63 by short division: give a reason for your method of determining the remainder.

5. Multiply ·0204 by 40·2, and divide 99·9666 by ·037.

6. Extract the square root of 20 to 4 places of decimals.

7. Prove that a fraction is not altered by multiplying its numerator and denominator by the same quantity.

Add together $\frac{7}{33}, \frac{5}{12}, \frac{8}{44}, \frac{13}{28}, \frac{15}{56}$.

8. Find the compound interest on £3945·3125 for 4 years at 4 per cent.

9. A bankrupt can pay 6*s*. „ 8*d*. in £: if his assets were £500 more, he could pay 7*s*. „ 4*d*. Find his debts and his assets.

10. The manufacturer will supply a certain article at $1\frac{1}{2}d.$; if a tradesman charges 2*d*. what profit per cent. does he make?

11. The price of diamonds per carat varies as the square

of their **weight**: if a diamond of 2 carats is worth £32, what is the value of a diamond of 3 carats ?

12. Wishing to pay 18 kreutzer, I give 1 thaler and receive in change 22 kreutzer 10 silber-groschen and half a gulden. I know that 1 thaler is 30 silber-groschen and 1 gulden is 60 kreutzer. Find for me how many gulden are worth 4 thaler.

13. A man walks a certain distance and rides back, in 3 hrs. 45 min.: he could ride both ways in $2\frac{1}{2}$ hours. How long would it take him to walk both ways ?

14. In a constituency in which each elector may vote for two candidates, half of the constituency vote for *A*, but divide their votes among *B*, *C*, *D*, *E* in the proportions of 4, 3, 2, 1 ; of the remainder, half vote for *B*, and divide their votes among *C*, *D*. *E*, in the proportions of 3, 1, 1 ; two-thirds of the remainder vote for *D* and *E*, and 540 do not vote at all ; find the order on the poll ; and the whole number of electors.

Previous Examination, 1868.

1. Find the product and quotient of 390625 and 244140625.

2. Shew how many yards of carpet (2 ft. wide) it will take to furnish a room whose floor measures 8 yards by $7\frac{1}{4}$ yards.

If the same room be $9\frac{1}{2}$ feet high, find how many cubic feet it contains.

3. An Austrian souverain and gold ducat are worth 13*s*. „ 11*d*., and 9*s*. „ 6*d*. respectively. How many ducats are equivalent to 4560 souverains ?

4. Distinguish between a vulgar fraction and a decimal fraction.

Multiply $999\frac{4945}{4946}$ by 999.

5. State the rule for the multiplication of decimals ; and verify it by means of the products $1{\cdot}23 \times {\cdot}0011$, and $29000 \times {\cdot}01$.

6. Divide ${\cdot}\dot{3}\dot{7}$ by ${\cdot}\dot{1}4\dot{8}$; and shew that $\dfrac{{\cdot}\dot{1}2\dot{3}}{{\cdot}\dot{4}\dot{1}} = \dfrac{123123}{414141}$.

7. What are the aliquot parts $\left(\text{greater than } \dfrac{1}{112}\right)$ of 1 cwt. ?

Find by practice the value of 3119 things at £4 „ 7*s.* „ 7*d.* each; and of 9 tons „ 19 cwt. „ 3 qrs. „ 14 lbs. at £28 per ton.

8. Find the square root of 121·2201; and the cube root of $423987\frac{457}{512}$.

The latter root is a mixed number, whose fractional part is $\frac{1}{8}$.

9. A debt of £18 „ 15*s.* „ 10*d.* is paid in crowns, shillings, and pennies, whose numbers are proportioned to 3, 2, 1. Find the number of each coin.

10. Define interest and discount. Find the amount of £2160 „ 12*s.* „ 6*d.* in 1 yr. „ 73 dys. at 5 per cent. interest.

11. Find the discount on £4120 „ 8*s.* „ 7*d.* due 9 months hence; interest being reckoned at 4 per cent. per annum.

Does the creditor or debtor gain, by computing interest instead of discount?

12. When wheat is 15*s.* per bushel 8 men can be fed for 12 days at a certain cost. For how many days can 6 men be fed for the same cost when wheat is 12*s.* per bushel?

13. A four-wheeled carriage travels round on a circular railway. The circumferences of the two wheels of the carriage, and of the two circles of rails, are proportional to 6, 7, 7000, 7014. Find the number of revolutions made by each of the four wheels in a complete circuit.

Previous Examination, 1869.

1. In subtraction how do you evade the difficulty of taking a greater digit from a less? Illustrate your answer by taking 791 from 943.

2. What is a measure? a common measure? the greatest common measure?

Find the greatest common measure of

$$13 \times 17 \times 19,\ 17 \times 19 \times 21,\ 19 \times 21 \times 13.$$

3. Divide 24763 by 56 by short division. Explain how you determine the remainder. Justify your method.

4. Add together $23\cdot07\dot{6}$, $19\cdot\dot{2}4\dot{5}$, $31\cdot2\dot{0}\dot{3}$; and multiply $3\cdot62015$ by $1\cdot00236$.

5. Add $\frac{2}{5}$ of $\frac{3}{7}$ to $\frac{3}{7}$ of $2\frac{1}{3}$; and multiply the result by

$$\left(\frac{2}{3} \text{ of } \frac{5}{6}\right) \div \left(\frac{5}{4} + \frac{4}{5}\right).$$

6. What fraction is 8 lbs. troy „ 1 oz. „ 19 dwts. „ 9 grs. of 13 lbs. „ 7 oz. „ 5 dwts. „ 15 grs.? If 8 lbs. „ 1 oz. „ 19 dwts. „ 6 grs. cost £10 „ 6*s.* „ 6*d.*, what will 13 lbs. „ 7 oz. „ 5 dwts. „ 15 grs. cost?

7. "15 ft. „ 4 in. × 14 ft. „ 6 in." Write the preceding in words, and explain what it may mean with reference to the next question.

8. Find the cost of varnishing the floor of a room 14 ft. „ 6 in. broad, and 15 ft. „ 4 in. long, at 6*d.* per square yard.

9. Find by practice the cost of (i) 1842 articles at £2 „ 6*s.* „ $9\frac{1}{4}$*d.* each; (ii) 2 tons „ 7 cwt. „ 22 lbs. „ 5 oz. at £32 per ton.

10. A scuttle of coals is charged 6*d.* when coals are 27*s.* a ton; how much ought the scuttle to hold?

11. Find the difference between the interest and the discount on £313 „ 19*s.* for 8 months at 6 per cent.

12. The interest on £300 from 2 June to 20 September is £2 „ 5*s.* „ $2\frac{1}{2}$*d.*; at what rate is it calculated?

13. A tradesman's prices are 20 per cent. above cost price: if he allow a customer 10 per cent. on his bill, what profit does he make?

14. On a stream, *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back in 5 hours „ 15 min.; and from *A* to *C* in 7 hours: how long would it take to go from *C* to *A*?

15. I have a certain sum of money wherewith to buy a certain number of nuts; and I find that if I buy at the rate of 40 a penny I shall spend 5*d.* too much; if 50 a penny, 10*d.* too little. How much have I to spend?

ANSWERS TO THE EXAMINATION PAPERS.

I. *Winchester.*

1. $(5\frac{1}{2})^2 = 30\frac{1}{4}$.
2. £357668 „ 3*s.* „ $5\frac{1}{4}$*d.*; and 11 cwt. „ 2 qrs. „ 18 lbs. „ $10\frac{1}{2}$ oz.
3. £19379 „ 15*s.* „ 5*d.*; and £2131 „ 14*s.* „ $11\frac{1}{2}$*d.*
4. 5406720; and £270336. 5. 4032. 6. 11.
7. $5\frac{71}{72}$ sum; $1\frac{55}{72}$ dif.; $8\frac{13}{72}$ prod.; $1\frac{127}{152}$ quot. 8. 32 days.
9. 80 yds. 10. $4\frac{1}{2}$ miles.

II. *Winchester.*

1. $(5\frac{1}{2})^2 = 30\frac{1}{4}$. 2. £359550 „ 12*s.* „ $8\frac{5}{8}$*d.*; and 11 cwt. „ 2 qrs. „ 18 lbs. „ 10 oz. „ 8 drs.
3. $55\frac{1}{8}$ sq. feet; and $38\frac{19}{24}$ cubic feet. 4. £987 „ 10*s.*
5. $1\frac{1}{2}$ miles per hour; and 30 minutes.
6. Quot. of sum by dif. is $7\frac{238}{355}$; and of dif. by sum $\frac{355}{2723}$.
7. $\frac{7}{9}$; and ·175. 8. 566·4 geographical = 652·40$\dot{8}$ ordinary miles. 9. 18. 10. $4\frac{2}{25}$.
11. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$, whence his mistake: the sums in proper proportion are 3*s.*, 2*s.*, 1*s.* 6*d.*

III. *Winchester.*

1. Seven billion, three hundred and ninety-two thousand five hundred and eighty-six million, forty-four thousand, and one. 5000021000030.
2. 104803155405621; and 604071. 3. $\frac{115}{162}$; and 1.
4. ·04976767; and 120·712. 5. ·778125; and ·30$\dot{5}$.
6. £1 „ 2*s.* „ 6*d.* 7. 89·4132 &c.; and ·000365.

8. £13 „ 2*s*. „ 6*d*.; and £3 „ 15*s*. 9. £3 „ 13*s*. „ $8\frac{1}{2}d$.
10. G.C.M. 128; L.C.M. 2304. 11. £1. 12. £35 „ 15*s*.
13. Rate of stream to that of boat as 1 : 23.

IV. *Eton.*

1. £94 „ 0*s*. „ 11*d*. 2. $9\frac{85}{128}$. 3. 132.
4. 5 per cent. 5. $2\cdot067\dot{3}0769\dot{2}$. 6. $80\frac{3}{25}$ acres.
7. $8\frac{1080}{5291}$ days. 8. $3\frac{9}{17}$ per cent.
9. £11 „ 0*s*. „ $1\frac{21}{101}d$. 10. 85.

V. *Eton.*

1. £584 „ 6*s*. „ 8*d*. 2. £75. 3. £2 „ 6*s*. „ 8*d*.
4. 32. 5. £3 „ 10*s*. „ 8*d*. 6. Cf. § 79, page 132.
7. 128, 96, 72, 54. 8. $31\frac{1}{2}$ miles per hour. 9. 9 taps.

VI. *Eton.*

1. 1 rood „ 19 poles. 2. 190 yds. „ 1 ft.
3. £5 „ 0*s*. „ $0\frac{1}{4}d$. 4. £173 „ 6*s*. „ $4\frac{1}{8}d$. 5. £2000.
6. Brown £5000, Jones £3600, Robinson £1400.
7. $34\frac{5}{8}$ miles per hour.

VII. *Harrow.*

1. 1263. 2. $13\frac{7}{8}$; and 6.
3. $\frac{6109}{10000} = \cdot6109$. 4. $\cdot\dot{0}1298\dot{7}$; and $3\frac{9}{220}$.
5. $\frac{80}{101}$. 6. £1 „ 13*s*. „ 9*d*.; and $2\cdot109375$.
7. 5 days. 8. 7·8217 pence.
9. Increase his income by 13*s*. „ 2·0247*d*.

VIII. *Harrow.*

1. $2\frac{2}{35}$ inches; and $83\frac{2}{7}$ yards. 2. 45 gallons.
3. (1) $1\frac{1769}{3600}$; (2) $\frac{1200}{5369}$; product $\frac{1}{3}$.
4. £1 „ 7*s*. „ $3\frac{3}{8}d$.; and $\cdot\dot{2}5396\dot{8}$. 5. 16*s*. „ $4\frac{17}{25}d$.
6. $8\frac{16}{25}$ miles per hour.
7. £47 „ 13*s*. „ $0\frac{3}{4}d$.; and £405 „ 3*s*. „ $4\frac{1}{2}d$.

IX. *Harrow.*

1. Cf. § 37. G. C. M. 12; L. C. M. 30240.
2. $\frac{6}{17}$ is greatest, and $\frac{4}{15}$ least; for reasons cf. § 51, Ex. 3. p. 79. Quot. required is $1\frac{17}{225}$.
3. $10\frac{42}{47}$ inches; also 6·39 &c. inches. 4. 57 years, nearly.
5. $\frac{125}{348744}$; and ·00035842 &c.; also ·00860229 of an inch.
6. Interest £859 „ 9*s.* „ $0\frac{1}{2}$*d.*; discount £737 „ 14*s.* „ $6\frac{1}{2}$*d.*
7. $3\frac{1}{8}$ per cent. (nearly). 8. He would lose £25 „ 2*s.* „ $9\frac{52}{77}$*d.*

X. *Harrow.*

1. Cf. § 12. Sixty-four. 2. 115093215. 3. 19840.
4. $24\cdot\dot{9}7502\dot{4}$; 500·5; also 4 *hundreds;* 1 *hundredth;* and *one.*
5. The quotient obtained is too small by ·000886.
6. 16·734 inches, nearly. Also 23·665 inches.
7. As 1 : 4; also 4860 cubic 3-inches.
8. 1 pole. 9. $\frac{2}{225}$. 10. 9087·791, &c.
11. *A*, by 4. 12. 16·5 men, 19·8 women, 29·7 children.
13. £129 „ 12*s.*; and £573 „ 13*s.* „ $7\frac{1}{4}$*d.*
14. On the shares he makes $112\frac{1}{2}$ per cent.; in the funds his annual income would be £184; and that would be 69 per cent. on the money originally invested.
15. 7925 miles.

XI. *Ladies' College, Cheltenham.*

1. Cf. § 10, 11. 1249387. Cf. § 19.
2. (1) 6. (2) 83. (3) 2223.
3. Cf. § 53. (1) $2\frac{13}{60}$. (2) $7\frac{11}{72}$. (3) $11\frac{31}{96}$.
4. (*a*) $\frac{4}{7}$. (*b*) $4\frac{31}{37}$. (*c*) $1\frac{68}{119}$. (*d*) $\frac{1}{11}$.
5. (1) $7\frac{1}{2}$. (2) $\frac{2}{9}$. (3) $\frac{7}{25}$. (4) $\frac{45}{841}$. (5) $\frac{255}{364}$.
6. $6\frac{6}{7}$ hours.

7. Cf. § 77, 78. (1) ·3125. (2) ·007080078125.
(3) ·2375. (4) $\cdot 5\dot{6}$. (5) $\cdot 1\dot{9}753086\dot{4}$. (6) $\cdot 000\dot{1}\dot{7}$.

8. (1) 10500. (2) ·002. (3) 24·3. (4) 2040000.

9. 34·65925 cub. in.; 240·689 ozs.

10. (1) £837 „ 3*s.* „ $11\frac{1}{2}d.$ (2) £1737 „ 18*s.* „ $9\frac{3}{4}d.$
(3) £89 „ 5*s.* „ 3*d.* (4) £148 „ 16*s.* „ 9*d.* nearly.

11. (*a*) 3937 yards. (*b*) $166\frac{2}{3}$ hours.

12. $16\frac{109}{176}$ seconds.

XII. *Ladies' College, Cheltenham.*

1. (1) 210012. (2) Eleven thousand one hundred and ten. Cf. § 12. (3) *a.* would leave them all unaltered in value. *b.* would reduce the value of them all one hundred-thousand-fold. *c.* would make the value of the number *eleven units*, together with *eleven hundredths*.

3. (1) $14\frac{37}{60}$. (2) $\frac{2}{3}$. (3) $\frac{931}{1191}$.

4. Cf. § 48; 46; 47.

5. (1) $16\frac{59}{60}$. (2) $\frac{901}{3969}$, and $\frac{43}{63}$. (3) £15. (4) 250.

6. ·263. Cf. § 75. 7. ·001; ·01; 1.

8. (1) £24 „ 15*s.* (2) 12*s.* „ 6*d.* (3). ·074865; and 678·704. (4) 58 yds. more.

XIII. *College of Preceptors.*

1. £53 „ 10*s.* „ 10*d.* 2. $\frac{1}{7}$. 3. £162. 4. 24 days.

5. $\cdot\dot{2}$; also ·00203, and 7000. 6. 2; also 8 poles „ 2 yds.

7. 11*d.* 8. 9 months. 9. £800. 10. £80.

11. $1\cdot 33\dot{6}$.

XIV. *Diocesan Training Colleges.*

Section I. (1) 21488. (2) £2 „ 8*s.* „ $7\frac{3}{4}d.$ (3) £23 „ 15*s.*
(4) 6464280. (5) 10*d.* (6) $1\frac{1}{2}d.$

Section II. (1) $52479\frac{171}{583}$. (2) 4*s.* „ $5\frac{1}{3}d.$ (3) 240.

Section III. (1) $18\frac{579}{1562}$. (2) $\frac{143360}{237861}$. (3) 1·21485*d.*

Section IV. (1) ·67540029, &c. (2) $8\frac{283}{424}$ yrs. (3) $\cdot 06\dot{4}8\dot{1}$.

Section V. (1) $9\frac{60}{77}$ min. (2) $5d.$ (3) 25 per cent.

Section VI. (1) 102026 with remainder 29. (2) ·0108.
(3) 4.

Section VII. (1) £88 „ 16*s.* „ $11\frac{43}{300}d.$ (2) 5 per cent.
(3) The four per cents. £5 „ 2*s.* „ $0\frac{24}{25}d.$

XV. *Diocesan Training Colleges.*

General.

(1) $\cdot 0000\dot{7}1428\dot{5}$; $771\cdot\dot{4}2857\dot{1}$; $\cdot 00\dot{7}1428\dot{5}$.
(2) £1407 „ 14*s.* „ 2*d.* (3) $79\frac{37}{42}$ yds. ; and £18 „ 9*s.* „ $5\frac{3}{14}d.$ (4) £2 „ 18*s.* „ $3\frac{45}{73}d.$
(5) 8th Dec., 1784. (6) 5 min. „ $40\frac{316}{345}$ sec. (7) 2325.
(8) 12 days after *A* stops.

XVI. *Diocesan Training Colleges.*

Section I. (1) 45942521. (2) 320 times. (3) £90 „ 12*s.* „ 8*d.*
(4) £3105 „ 3*s.* (5) 238 ac. „ 0 r. „ 20 p. (6) 215.

Section II. (1) 200. (2) £1 „ 8*s.* „ 6*d.* (3) $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{63}{90}$.

Section III. (1) $\frac{7}{13}$. (2) $27\frac{3}{8}$. (3) £1 „ 17*s.* „ $4\frac{16}{125}d.$

Section IV. (1) ·2625. (2) $\cdot 0224185\dot{6}\dot{0}$; $\cdot 1793\dot{4}\dot{8}$; $39\cdot 45\dot{6}$.
(3) £97 „ 19*s.* „ $5\frac{3}{8}d.$

Section V. (1) $8\frac{428}{785}d.$ (2) 25 for 1*s.* „ 6*d.* (3) £640.

Section VI. (1) £7 „ 0*s.* „ $10\frac{13}{32}d.$ (2) Cf. § 89.
(3) 1·8, &c. feet.

Section VII. (1) £649 „ 1*s.* „ $8\frac{20}{21}d.$ (2) £1 „ 19*s.* „ 4*d.* ; $21\frac{88}{277}$.
(3) £9000.

XVII. *Diocesan Training Colleges.*

General.

(1) 1. (2) $\frac{15}{24165152}$. (3) £140 „ 12*s.*
(4) 4·90099501 gallons. (5) £20 „ 17*s.* „ 11·56224*d.*
(6) $9\frac{1}{4}$. (7) 193·649, &c. (8) £3078.
(9) $9\frac{3}{11}$ miles. (10) $9\frac{1}{16}$ miles. (11) 150 ; and 250.

XVIII. *Civil Service Commission.*

1. 19s. „ 2d. 2. 512. 3. ·100554. 4. 2 ft. „ 3 in.

5. $\frac{1}{2}$ hour. 6. 4 per cent. 7. $9\frac{3}{5}$ hours.

XIX. *Civil Service Commission.*

1. 8 : 15. 2. $\frac{9\cdot3}{19}$, or $50\frac{3}{17}$. 3. 1·794634, &c.

4. 12 „ 9' „ 0'' „ 0''' „ 8'''' „ 11'''''.

5. 12 cubic feet „ $1296\frac{127}{144}$ cubic inches.

6. 2·565789, &c. 7. 26·5435416, &c.

8. He gains £29 „ 16s. „ $7\frac{53}{263}d$. 9. 50 per cent.

10. £2 „ 15s. „ 3d. 11. 150 men. 12. $17\frac{1}{2}$ days.

13. 3 minutes „ $46\frac{2}{3}$ sec. 14. 5 minutes before 11.

15. One-sixth. See question 76, p. 159.

XX. *Civil Service Commission.*

1. 26·1509125. 2. 18s. „ 9d. 3. $22\frac{1}{2}$ years.

4. Gains $5\frac{55}{123}$ per cent. 5. 18495000.

6. He gains £52 „ 10s. 7. $88\frac{6}{13}$.

8. Out of every 100 of the population 75 would be Roman Catholics, 10 Dissenters; or for every 100 Roman Catholics, $13\frac{1}{3}$ Dissenters.

XXI. *Civil Service Commission.*

1. $128\frac{12}{13}$. 2. Reduced by $1\frac{53}{250}$ per cent.

3. $7092\frac{75}{141}$ stock, and $8\frac{2}{47}$ per cent. 4. $82\frac{1}{2}$.

5. Cf. § 108, p. 245. 6. 10 gulden.

7. £1 „ 19s. $4\frac{1}{2}d$. nearly. 8. He gains £2 „ 3s. „ $2\frac{3}{4}d$.

XXII. *Direct Commissions.*

1. 104 times. 2. 14080 steps. 3. 20 days.

4. 7 tons „ 4 cwt. 5. £1527 „ 3s. „ 9d.

6. $6\frac{84}{121}$; and 9. 7. 4840, and $\frac{5}{11}$.

8. ·07546, and ·008. 9. £11 „ 2s., and £1 „ 15s.

10. ·000175; and 8·426, &c.

XXIII. *Direct Commissions.*

1. 37 times. 2. £22 „ 19*s.* „ 8*d.* 3. 88.

4. £13 „ 3*s.* „ $6\frac{1}{2}$*d.* 5. £2039 „ 1*s.* „ 3*d.* 6. $\frac{7}{16}$, and $\frac{3}{5}$.

7. 1·8019, and ·0074. 8. $\frac{4}{27}$; also 4*s.* „ $10\frac{1}{2}$*d.*

9. ·000027, and 22·004.

XXIV. *Direct Commissions.*

1. Seven million two hundred thousand inches.

2. 3*s.* „ 11*d.* 3. £37 „ 2*s.*

4. 2 lbs. „ 10 oz. „ 11 dwt. „ 12 grs. 5. 24 cwt.

6. $\cdot\dot{0}0099\dot{9}$. 7. £1 „ 9*s.* 8. ·002988, &c.

9. £33 „ 14*s.* „ $4\frac{1}{2}$*d.* 10. 3007.

XXV. *Direct Commissions.*

1. £12704 „ 13*s.* „ 4*d.*; and £1 „ 5*s.* „ 6*d.*

2. 2 miles „ 150 yds. „ 2 feet. 3. 1320 steps; 1100 yds.

4. 3 tons „ 18 cwt. „ 3 qrs. 5. $3\frac{1}{3}$ per cent.

6. $\frac{1}{999}$; also 1. 7. 14·443; also 135·45. 8. 5·4 pence.

9. ·2 10. ·0000015625; also 27·0011.

XXVI. *Direct Commissions.*

1. 57. 2. 198252 inches.

3. 17 cwt. „ 2 qrs. „ 19 lbs. „ 4 oz. 4. 1*s.* „ $10\frac{1}{2}$*d.*

5. 6 hours. 6. $5\frac{3}{32}$. 7. ·2375.

8. 129107 sq. yds. 9. 2040. 10. 5090.

XXVII. *Staff College.*

1. £13810 „ 17*s.* 2. £127 „ 3*s.* „ 9*d.*

3. 52 feet „ 10 inches.

4. 12 cwt. saltpetre, $1\frac{3}{5}$ cwt. sulphur, $2\frac{2}{5}$ cwt. charcoal.

5. $5\frac{1}{2}$ per cent. 6. An increase of £142 „ 16*s.*

7. Cf. § 48. $\frac{30}{157}$ 8. 1, and $\frac{1}{64}$.

9. ·984375; Cf. § 76, p. 128, $\frac{1}{7}$; product $\cdot\dot{0}\dot{9}$.

10. ·05, 507·001, 2·3452, and ·00003696 remainder.

XXVIII. *Oxford Local.*

1. £95. 2. £37 „ 13*s.* „ $2\frac{1}{2}d.$

3. £977 „ 14*s.* „ $0\frac{3}{4}d.$; and £12 „ 2*s.* „ $1\frac{1}{2}d.$

4. $\frac{9}{107}$; and $4\frac{1}{8}$. 5. £1. 6. ·0004; 4000; 40.

7. $\frac{3}{8}$; $\frac{3}{80}$; $\frac{3}{275}$. 8. $\frac{1}{880}$; $\cdot 0011\dot{3}\dot{6}$. 9. 7014; and ·04.

10. As £14 „ 5*s.* „ 7·2*d.* : £14 „ 17*s.* „ 2·11584*d.*; or as 3·57 : 3·714704. 11. 3738.

12 864 planks; and £233 „ 8*s.* „ 7*d.* 13. 21 days.

XXIX. *Oxford Local.*

1. 408 lbs.; and 2*d.* each.

2. £49 „ 19*s.* „ $11\frac{3}{4}d.$; 1 ton „ 1 cwt. „ 0 qr. „ 26 lbs. „ 15 oz.

3. £901 „ 5*s.*; £10 „ 11*s.* „ 6*d.* 4. $\frac{29}{31}$; and 2.

5. 10 shillings. 6. 1·2; 240; 9.

7. $\frac{1}{30}$; and $\cdot 00347\dot{2}$. 8. 1·124864; and 29496.

9. As £1 „ 4*s.* „ 11·52*d.* : £1 „ 5*s.* „ 11·660544*d.*; or as 39 : 40·5808.

10. The rate of interest is the same in each case.

11. 96. 12. 18 men.

XXX. *Cambridge Local.*

1. The one is larger than the other by forty-nine thousand nine hundred and fifty, *i.e.* by 49950.

2. 60768396; of 129847 and 40068. 3. 4763, 763, and 63.

4. $22\frac{2}{3}$. 5. As 2464, 2268, 2625, 2700.

6. $\frac{9}{56}$. 7. 74·9265, and ·00749265.

8. ·163; quotient, divisor, dividend.

9. ·975, $\frac{975}{1000}$, Cf. § 75. 10. ·096.

11. ·000$\dot{5}3\dot{5}$, $\frac{107}{199800}$. 12. 908 yards.

13. 18*s.* „ $2\frac{3}{11}$*d.* 14. £4643 „ 15*s.*

XXXI. *Cambridge Local.*

1. 37217. 2. 457327. 3. 17*s.* „ 4*d.*

4. $\frac{49}{72}$. 5. $\frac{26}{45}$; $\frac{11}{125}$; and 1*s.* „ $7\frac{1}{4}$*d.* 6. ·0129.

7. 31 qrs. „ 2 bus. 8. £3496 „ 0*s.* „ 5*d.*

9. 1 ton „ 3 cwt. „ 1 qr. „ $0\frac{1}{8}$ lb. 10. 48·5 per cent.

11. 1 yd. „ 0 ft. „ 3·3708 inches.

12. 13·80 &c. pence, assuming that the copper alloy in a common shilling has *no* value. The question fails to supply either the relative values or the relative weights of silver and of copper.

XXXII. *Oxford Responsions, Michaelmas*, 1862.

1. 1. 2. $\frac{29}{311}$, and 1008. 3. ·458$\dot{3}$, and $\frac{3}{16}$.

4. ·001221, 12, 12000, 11. 5. £1 „ 4*s.* „ $4\frac{1}{2}$*d.*

6. ·02, 8·$\dot{3}$, $1\frac{3}{4}$, $\frac{61}{200}$. 7. 140 minæ. 8. 156 yards.

9. 21 men. 10. £88 „ 8*s.*, and £30 „ 9*s.*

11. £37 „ 10*s.* 12. £1010. 13. £60, £40, £30, £24.

XXXIII. *Oxford Responsions, Trinity Term*, 1865.

1. 2571, 315. 2. $\frac{18}{143}$, $\frac{49}{288}$. 3. $\frac{144}{175}$, 36205.

4. ·02265625, and ·003625, ·00003625, ·3625.

5. $\frac{1}{640}$, $\frac{1}{18}$, $\frac{7}{220}$. 6. £3 „ 3*s.* „ $1\frac{1}{2}$*d.*, ·03125 cwt.

7. 327, 2·03. 8. 1·0164, &c. 9. 15 days.

10. £25 „ 10*s.*; £16 „ 4*s.* „ $9\frac{3}{5}$*d.* 11. 1600.

XXXIV. *Oxford Responsions, Michaelmas Term*, 1865.

1. 1, and $\frac{11}{15}$. 2. £450. 3. $\frac{1}{96}$, $\frac{25}{252}$.

4. $\frac{11}{8000}$, $\frac{1}{33}$, $7\frac{17}{88}$, ·7, ·07, 70. 5. 9*s.* „ 9*d.*, and ·2.

6. 13·25, 2020.　　7. *B* wins by $7\frac{1}{2}$ minutes.

8. $37\frac{1}{2}$ per cent.　　9. £76 „ 13*s.*; £2 „ 10*s.* „ $9\frac{7}{8}$*d.*

10. £1 „ 13*s.* „ 4*d.*　　11. $46\frac{29}{40}$ sovereigns, and £4 „ 4*s.* „ $11\frac{5}{11}$*d.*

Cambridge Previous Examination.
First Division B, 1856.

1. Cf. § 11, 12. 694 with remainder 2, Cf. § 27.

2. £15 „ 6*s.* „ 3*d.*, £33 „ 9*s.* „ $6\frac{1}{4}$*d.*

3. 1, and 1*s.* „ 6*d.*　　4. $\frac{216}{4715}$ of a penny.

5. £69218, and 17 cwt. „ $3\frac{1}{25}$ lbs.; $\frac{9}{91}$; Cf. § 56.

6. Cf. § 88, p. 119.　　7. ·3, and ·0003; 13*s.* „ $6\frac{3}{4}$*d.*

8. 1250, 125, ·00000125. The sum is 105·6555. $\frac{33}{56}$ and $\frac{2}{7}$.

9. Cf. § 48, and § 66.　　10. £1388 „ 17*s.* „ $9\frac{1}{3}$*d.*

11. £3 „ 13*s.* „ 4*d.*　　12. 4 years.　　13. 4 per cent.

14. 84.　　15. 7*s.*; and 5*s.* „ 6*d.*　　16. 3·162, &c.

Second Division A, 1856.

1. Cf. § 25, p. 21, and § 45, p. 73.

2. £609 „ 9*s.* „ $5\frac{69}{160}$*d.*, £2 „ 15*s.* „ $11\frac{1}{4}$*d.*　　3. April 16.

4. 128 square feet „ 68 square inches.
If room be not rectangular, the area would consist of 128 parallelograms whose sides are feet, and 68 parallelograms whose sides are inches, but whose angles are not right angles, but are angles equal to those contained between the sides of the room.

5. 2; and 5.　　6. $\frac{37}{58}$; $\frac{5}{4}$; also $\frac{1}{4}$.　　7. $1\frac{1}{3}$ hours.

8. ·002021, 20210, ·1902, 1902000000, 1902000. Also ·0002938.　　9. 7*s.* „ $10\frac{1}{2}$*d.*, and 23·125 for both.

10. Cf. § 88 and § 5, p. 2. No, see § 26.　　11. 64 : 63.

12. 135 days.　　13. £$91\frac{3}{7}$.　　14. 750 stock.

15. 23515302409; 10192; and ·214.

Second Division B, 1856.

1. Cf. § 25 and § 45.
2. £1191 „ 10*s.* „ $1\frac{1}{5}$*d.*, £195 „ 11*s.* „ $6\frac{3}{4}$*d.*
3. April 8. 4. 101 square feet „ 56 square inches.
5. £1 ; $\frac{9}{25}$. 6. $\frac{68}{23}$ and $\frac{3}{4}$; $\frac{10}{9}$.
7. Rate of stream is $1\frac{1}{2}$ miles; against stream 1 hour.
8. ·00001, 10100, 11900000. Arithmetically impossible, since the subtrahend is the larger quantity. The result is $-$·1925.
9. ·91875, likewise ·00091875. £7 „ 17*s.*
10. They *cannot* be arranged as a proportion. No, see § 26.
11. 25 : 8. 12. 100 men.
13. £80. 14. $843\frac{3}{4}$ stock.
15. (*a*) 10260271849. (β) The number 10573009 is not a perfect square; if the figures be altered to 10582009, the root would be 3253. (γ) ·789.

October, 1856.

1. Cf. § 44, 45. 2. $\frac{4}{5}$; and 7 : 13.
3. (*a*) $\frac{25}{225}$, $\frac{90}{225}$, $\frac{18}{225}$, $\frac{75}{225}$. (β) $\frac{2399}{3080}$, and $\frac{1}{16}$.
4. 2·1, 210. 5. 432, $\cdot 00\dot{8}5714\dot{2}$.
6. £104 „ 7*s.* „ 0*d.* „ $1\frac{5}{7}$ *far.*, £9 „ 9*s.* „ 8*d.* „ $3\frac{5}{77}$ *far.*
7. 3 days „ 10 hours „ 13 min. „ 57 sec. 8. £223 „ 15*s.* „ $11\frac{5}{8}$*d.*
9. £25 „ 5 flo. „ 3 cents „ 1·25 mils, or 255·31225 florins.
10. £82970 „ 8 flo. „ 9 cents. „ 1·242 times.
11. £1002 „ 9 flo. „ 6 cents „ 5·88 mils. 12. $14\frac{1}{8}$ marks.
13. 4*s.* „ 2*d.* „ $2\frac{1}{2}$ *far.* 14. $4\frac{18265}{21918}$. 15. £21 „ 5*s.*
16. $3\frac{27}{91}$ per cent.

First Division B, 1857.

1. £70 „ 17*s.* „ $0\frac{3}{4}$*d.* 2. £17. 3. 4. 1.
5. 90 additional men. 6. 750, and $133333\cdot\dot{3}$.
7. ·004, and ·00375. 8. ·025, and ·0001.
9. £146 „ 9 flo. „ 6 cents „ 5 mils, or £146 „ 19*s.* „ 3*d.* „ $2\frac{3}{8}$ *far.*

10. £94 „ 11*s.* „ 6*d.*, and £6000. 11. £420.

12. £1320. 13. £316 „ 17*s.* „ 6*d.* 14. £560.

Second Division A, 1857.

1. 60 lbs. 2. $10\frac{1}{2}$ days. 3. $\frac{77}{60}$, and $\frac{1}{20}$.

4. $\frac{2}{7}$. 5. £520. 6. ·0108575, and ·02525.

7. ·001385, &c. 8. 3·8 feet. 9. £5000.

10. £410, and 30 years. 11. 8*s.* „ 4*d.*

12. 25802 ounces, or 14 cwt. „ 1 qr. „ 16 lbs. „ 10 oz.

13. £462. 14. £960000 capital, and £95238 „ 1*s.* „ $10\frac{6}{7}$*d.* receipts.

Second Division B, 1857.

1. 180 lbs. 2. 16 days. 3. $\frac{19}{20}$, and $1\frac{43}{60}$.

4. $\frac{4}{21}$. 5. £1040. 6. ·0325725, and ·1515.

7. $\cdot 008\dot{3}$. 8. 7·6 feet. 9. £6666 „ 13*s.* „ 4*d.*

10. £545, and 24 years. 11. £2 „ 1*s.* „ 8*d.*

12. 116109 ounces, or 3 tons „ 4 cwt. „ 3 qrs. „ 4 lbs. „ 13 oz.

13. £372. 14. £1500000 capital, and £150000 receipts.

October, 1857 (*A*).

1. The wheat. 2. 4*s.* „ 2*d.* 3. $\frac{2}{7}$. 4. $\frac{1}{15}$.

5. 89 of the alloyed are equivalent to 87 of the standard gold.

6. 12000 and ·012. 7. $00994\dot{3}1\dot{8}$. 8. 2667·76, &c

9. $4\frac{1}{6}$ per cent. 10. £82; £1 „ 1*s.* „ 0·075*d.* difference.

11. 250. 12. 256·001, &c., and ·809, &c.

13. 141 acres „ 3 roods. 14. 14 payments.

October, 1857 (*B*).

1. The wheat. 2. 5. 3. $\frac{2}{9}$. 4. $\frac{1}{30}$*d.*

5. 14 of alloyed gold are equivalent to 13 of standard gold.

6. 11000, and ·011. 7. ·6745. 8. 933·7, &c.

9. $4\frac{1}{9}$. 10. £41; and 10*s.* „ 6·0243*d.* difference.

11. £250. 12. 255·998, &c., and ·809, &c.

13. 204 acres „ 0 roods „ 30 poles. 14. 14 payments.

First Division B, 1858.

1. One hundred and twenty-seven million eight hundred thousand and twenty-one.

2. $\frac{1}{72}$, and 1*d*. 3. £6 „ 7 flo. „ 1 cent „ 0·41$\dot{6}$ mils.

4. 1·236, and ·04944. 5. $\frac{1232}{5525}$. 6. 15 hours.

7. 41000, and 6·3099. 8. He loses £446 „ 3*s*. „ $0\frac{3}{8}d$.

9. £100. 10. £3783. 11. £5 „ 17*s*. „ $0\frac{1}{4}d$.

12. Cf. § 43.

Second Division A, 1858.

1. 232003014. 2. $\frac{7}{20}$, and 1*s*. „ 6*d*.

3. 12·47, and 623·5. 4. £18 „ 6 flo. „ 2 cents „ 7·08$\dot{3}$ mils.

5. $\frac{4}{25}$, and $\frac{16}{625}$. 6. 35·0112, &c. ounces.

7. ·25, and 59·52. 8. £3 „ 17*s*. „ 9*d*. 9. 20807·97, &c.

10. 13. 11. 19*s*. „ 8*d*. 12. Cf. § 43.

October, 1858 (*A*).

1. 381274954. 2. 7970 miles. 3. $\frac{19}{120}$, and 1*s*.

4. $\frac{87}{68}$, and $\frac{1679}{4624}$. 5. 2·0535, and 102·675.

6. ·0875, and 4·5 dollars. 7. ·42804, and 369.

8. 1000 men. 9. 8 feet „ $10\frac{2}{3}$ inches. 10. £1 „ 12*s*. „ 6*d*.

11. *A*'s loss £20, *B*'s loss £26 „ 13*s*. „ 4*d*., *C*'s loss £33 „ 6*s*. „ 8*d*.

12. $1\frac{23}{33}$ miles.

First Division B, 1859.

1. 847021; 36865365. 2. 6075.

3. £59 „ 10*s*., and £3 „ 14*s*. 4. 2*s*. „ 3*d*.

5. 16 men. 6. 500 ; $\frac{2}{3}$; 1.

7. 1082·69869, 74·84, 22600. 8. $\frac{11}{40}$, ·275.

9. 3163, 490·07. 10. $1\frac{3}{4}$ hours. 11. Loses $\frac{1}{4}$ per cent.

12. 30. 13. £382 „ 10*s.* 14. $2\frac{1}{2}$ years.

15. £25 loss in cash, £31 gain in income.

Second Division A, 1859.

1. 41160090 sum, 16468734 difference, 355733452311336 product. 2. 75. 3. Cf. § 40, p. 48 ; $\frac{8}{9}$.

4. Cf. § 31, (7), p. 37 ; 213·4. 5. 1, and 1.

6. 3 cwt. „ 1 qr. „ 6 lbs. 7. £16000. 8. (1) £1153 „ 19*s.* (2) £444 „ 16*s.* „ 8*d.* (3) £719 „ 11*s.* „ 8*d.*

9. 3005, ·12, $\cdot\dot{3}$. 10. £50 „ 15*s.* „ $1\frac{1}{2}d.$

11. The latter by 7·3118, &c., pence. 12. £1237 „ 5*s.*

13. 5 per cent. 14. $2\frac{2}{9}d.$ 15. £6000.

October, 1859 (*A*).

1. 59691 sum, 8821 difference, 871301360 product.

2. 7 and 3. 3. 28080000 lbs. 4. £711 „ 18*s.*

5. £79 „ 17*s.* „ 6*d.* 6. £4 „ 15*s.* 7. ·2. 8. $\frac{8}{9}$.

9. 4*s.* „ $7\frac{1}{2}d.$ 10. £30. 11. 81 days. 12. £2940.

13. ·0000101. 14. £13, £25, £50, £100, £150.

15. 63 yds. 16. 2·14, &c., 3.22, &c., 8·50, &c., 29·42, &c.

First Division B, 1860.

1. Cf. § 11, 12 ; 564, cf. § 19. 2. Cf. § 44 and 48 ; $\frac{1}{17}$.

3. Cf. § 64 and 65. ·00000403 sum, ·000000000003888 product.

4. £·0703125. 5. 2530·6. 6. 26*s.* „ 6*d.* per acre.

7. £20 *A*'s share, £40 *B*'s share. 8. £1176.

9. $173\frac{67}{121}$ acres. 10. 968 years, and 4 miles „ $30\frac{70}{99}$ yards.

11. 150 per cent.

12. He gained 106 cash, and increased his income by $£118\frac{41}{97}$.

Second Division A, 1860.

1. 9843750 sum, 9687500 difference, 762939453125 product, 125 quotient. 2. $486\frac{598}{905}$.

3. 10·0191 sum, 10·0009 difference, ·0091091 product, 110 first quotient, $\cdot 000\dot{9}\dot{0}$ second quotient.

4. $(100s. \div 8s.) \times 75$ gallons $= 937\frac{1}{2}$ gallons,
$(8s. \div 100s.) \times 75$ gallons $= 6$ gallons.

5. Cf. p. 165, £95 „ 17*s*. 6. £25 „ 6*s*.; £34 „ 10*s*.

7. £310 „ 14*s*.; £331 „ 17*s*. „ 6*d*.; £500 „ 10*s*. 8. £3 „ 5*s*. „ 1*d*.

9. Since 5 per cent. is found by taking $\frac{1}{20}$th part, see p. 202, and since pounds when considered as shillings have to be divided by 20, the proper interest would be thus obtained from them: while the shillings and fractional parts of a shilling, when brought to the fraction of a twelfth of a pound, are shillings divided by 20 (which is taking 5 per cent. of them), and multiplied by 12, which brings them into pence.

It is more simple to divide at once by 20, as suggested at p. 202.

£31 „ 1*s*. „ $8\frac{1}{3}d$. is 5 per cent., and £29 „ 10*s*. „ $7\frac{19}{100}d$. is $4\frac{3}{4}$ per cent.

10. Bank stock is best in ratio of 320 : 319.

11. £134 „ 6*s*. „ $9\frac{6}{25}d$. 12. 352 persons. 13. 2·97 pence.

October, 1860 (*A*).

1. 1666350 sum, 1639900 difference, 21862578125 product, 125 quotient.

2. The square ·000057289761, the square root ·087.

2. 32 furlongs $\div$ 4 furlongs $= 8$, £2 $\times 8 =$ £16,

$$4 \text{ furlongs} \div 32 \text{ furlongs} = \frac{1}{8},\ £2 \times \frac{1}{8} = 5s.$$

4. $32399\frac{25}{108}$, and 3 days „ 11 hours „ 13 min.

5. 10*s*. „ $4\frac{1}{2}d$., and 12 lbs. 6. Cf. § 40, p. 48.

7. £26393 „ 15*s*. 8. £24 „ 7*s*. „ 8*d*., and £23 „ 2*s*.

9. £4 „ 5*s*. 10. £457 „ 8*s*. „ 9*d*., and £273 „ 15*s*.

11. £392, £784, £1176, £1568. 12. Loses 25 per cent.

13. 403 yards.

First Division B, 1861.

1. 438, cf. § 19. 2. 24 with remainder 5397; and 24 furlongs „ 149 yds. „ 2 ft. „ 9 in. 3. $7\frac{1861}{3885}$.

4. $\frac{23}{100}$, and £89 „ 6*s*. „ $1\frac{1}{8}$*d*.

5. ·$42106\dot{4}8\dot{1}$, and £1 „ 5*s*. „ 3*d*. 6. 145, 14·503, and 48 ft.

7. £9 „ 9*s*. 8. 15 days. 9. £1085 „ 12*s*. „ $0\frac{4}{5}$*d*.

10. £22 „ 2*s*. „ $2\frac{96}{169}$*d*.

11. £60 „ 6*s*. „ 6*d*., and £2 „ 19*s*. „ $2\frac{59}{77}$*d*. per cent.

12. $640\frac{1}{2}$ quarters.

Second Division B, 1861.

1. Three hundred and twenty-four million, nine hundred and thirty-seven thousand, five-hundred and ninety-four.

2. Cf. § 44, 48. Order of magnitude is (1), (3), (2); $\frac{161}{33}$.

3. ·8499745, and ·30685. 4. 8*s*. „ 2*d*., and ·$081\dot{6}$.

5. In a year and 6 days there are 32054400 seconds.

6. Total gain £16, gain per cent. $12\frac{1}{2}$. 7. 1*s*. „ 6*d*.

8. ·000045, &c. 9. £17 „ 11*s*. 10. £8 „ 9*s*. „ $11\frac{3}{4}$*d*.

11. 8 per cent. 12. $27\frac{1}{13}$ feet, and $1\frac{721}{779}$ seconds.

13. £253 „ 0*s*. „ $9\frac{603}{733}$*d*.

October, 1861 (*A*).

1. 7833958 quotient. 2. £13 „ 18*s*. 3. 3 and 4.

4. $\frac{1}{32}$, $\frac{27}{37}$, $\frac{361}{495}$; £2 „ 7*s*. „ 1*d*. 5. 15 days. 6. £156.

7. At 5 minutes *before* nine, the engine of the second train would have run into the last carriage of the first.

8. £22 „ 0*s*. „ $4\frac{4}{5}$*d*. 9. £800. 10. £2812 „ 3*s*. „ $2\frac{2}{5}$*d*.

11. £33 „ 15*s*. „ $8\frac{476}{743}$ income, £33 „ 2*s*. „ $2\frac{1122}{1435}$*d*. gain.

First Division B, 1862.

1. 10001001. Sum is 2259, cf. § 16. Also 12484 required number.
2. 5995, with remainder 19. 3. £17 „ 9*s.* „ 6*d.*
4. 17 lbs. Excess in weight is 266 lbs. „ 7 oz. „ 6 dwts. „ 16 grs.; excess in value £12457 „ 8*s.* „ 1*d.*
5. Cf. § 56. $3\frac{11}{374}$; $\frac{12531}{22120}$. 6. Cf. § 75. (1) 12·66806; (2) 4962.
7. (1) £5109 „ 7*s.* „ $10\frac{1}{2}$*d.*; £93 „ 14*s.* „ $1\frac{1}{2}$*d.*
8. 6 cwt. „ 3 qrs., and £·3703125. 9. Cf. § 99, 100. £47 „ 5*s.*
10. Cf. question 10 in preceding paper. 9216 bricks.
11. £19 „ 14*s.* „ 5*d.* 12. 777, and 3·544, &c. 13. $2\frac{1}{2}$ ft.

Second Division A, 1862.

1. 203; 146 remainder. 2. Cf. § 32 and § 40. G.C.M. is 21.
3. £3 „ 17*s.* „ $10\frac{1}{2}$*d.* 4. $\frac{29}{150}$; $2\frac{71}{91}$.
5. $\frac{19}{50}$; £22 „ 4*s.* „ $5\frac{1}{3}$*d.*
6. $\cdot\dot{9}9900\dot{0}$: 1·001; 6 cwt. „ 3 qrs.; and $\cdot 902\dot{7}$.
7. Cf. § 94 and § 99; £1 „ 12*s.* „ 5·85*d.* 8. £120.
9. £27820 „ 18*s.* „ 3*d.*, £8095 „ 4*s.* „ $2\frac{1}{4}$*d.*
10. By lifeboats 729, rockets 432, ships' boats 3348, individuals 27. 11. $301\frac{1}{3}$ cubic yards; $165\frac{65}{389}$ lbs.
12. His income at first was £187 „ 10*s.*; afterwards it was £196 „ 5*s.*; hence £8 „ 15*s.* was the increase.
13. 876; 99·49, &c.

October, 1862 (*A*).

1. 14981018. 2. Cf. § 44 and 48; $\frac{4}{21}$ and $\frac{9}{112}$.
3. $1\frac{1}{3}$*d.*
4. 561 portions, with a remainder ·0023 of an inch long.
5. 17 qrs. „ $0\frac{1}{2}$ bushel.
6. 347, and 1·49, &c. Breadth 16 feet, length 32, height 8.
7. 6048 carlini. 8. Cf. § 99. £100.
9. 14*s.* „ 6*d.*, and 15*s.* „ 6*d.*
10. £376. 11. Neither increase nor diminish. 12. 4 days.

First Division B, 1863.

1. 57240967479503354972, £82037 „ 5*s.*
2. £102 „ 13*s.* „ $6\frac{3}{4}$*d.*, £1594 „ 3*s.* „ 6*d.*
3. £4726 „ 3*s.*, £20315 „ 1*s.* „ $0\frac{1}{2}$*d.* 4. £271 „ 10*s.*, £5655.
5. £2053 „ 2*s.* „ 6*d.*; £4000. 6. $161\frac{11}{18}$; 15*s.* „ 10*d.*
7. ·40625; $\cdot 0089\dot{2}8571\dot{4}$.
8. 6*s.* „ $0\frac{3}{5}$*d.*; £11 „ 18*s.* „ 10*d.*; 2300; ·000051.
9. 30502; 7·8; ·7745, &c.
10. The gain is $\frac{1}{50}$ of a penny per lb. by buying per quintal; also by buying 12000 the loss is £1.
11. 132, 165, 198 feet. 12. £121 „ 13*s.* „ 4*d.*
13. 288 grains, or 12 dwt.; 5*s.* „ 3*d.* 14. 2250 ducats.

Second Division A, 1863.

1. 319 tons „ 0 cwt. „ 1 qr. „ 12 lbs.
2. £19 „ 4*s.* „ $2\frac{1}{4}$*d.*, £9767 „ 13*s.*
3. £489 „ 3*s.*, £21987 „ 12*s.* „ 4*d.*
4. 361 days. 5. 18*s.* 6. £584 „ 14*s.* „ $4\frac{1}{2}$*d.*, £5607.
7. 37 and 360. 8. $1\frac{113}{616}$. $1\frac{53}{81}$, and 28.
9. ·002739, &c.; ·2; 99000; ·0000112.
10. 8192, and ·31, &c. 11. *A*'s offer, by £82 „ 17*s.* „ 6*d.*
12. £$\frac{2000}{182903}$ is the gain by investing £100 in the 4 per cents. Also £457 „ 5*s.* „ $1\frac{1}{8}$*d.* 13. £3200. 14. 10791224.
15. As any two of them are greater than the third, they can.

October, 1863 (*B*).

1. 4083250793, £1036455 „ 16*s.* £144 „ 7*s.* „ 5*d.*, £61 „ 11*s.* „ $1\frac{1}{2}$*d.*
2. 54756. 3. 15*s.* „ 4*d.* 4. 24 days. 5. 60 miles.
6. £867 „ 12*s.*, £3545 „ 1*s.* „ 2*d.*, £8576 „ 0*s.* „ 10*d.*
7. £1193 „ 5*s.* „ 8*d.* 8. £9000, £651.
9. £16407 „ 5*s.* „ 3·912*d.* 10. 61, and 720.
11. 149, £1 „ 17*s.* „ 6*d.* 12. $\frac{3}{4}$, 10, 31.

13. ·4375, ·125, 6*s.* „ 3*d.* £7 „ 13*s.* „ 3*d.* ·0033, 106000, 8·7.

14. 298, 3·2857, &c., ·06. 15. $12\frac{1}{2}$ per cent.

16. In six years.

Previous Examination, March, 1864 (*A*).

1. (1) £134 „ 19*s.* „ $0\frac{3}{4}$*d.* (2) 369 yrs. „ 203 days „ 16 hrs.
   (3) £19 „ 3*s.* „ 7*d.*
   (4) 54 tons „ 7 cwt. „ 0 qrs. „ 24 lbs. „ 12 oz.

2. £21 „ 15*s.* „ 9*d.*

3. (1) £1594 „ 9*s.* (2) £14968 „ 3*s.* „ $6\frac{1}{2}$*d.*
   (3) £2262 „ 8*s.* „ 9*d.*

4. (1) £93 „ 15*s.* (2) £76 „ 8*s.* „ $9\frac{16}{81}$*d.*

5. £2127 „ 2*s.* „ 8·077*d.*, &c. 6. 43; and 720.

7. $2\frac{31}{990}$; and 1 lb. „ 8 oz. 8. ·136; $\frac{111}{1000}$; ·1918158, &c.

9. 25·6; 400; ·0025. 10. 200137; $1\frac{1}{3}$, and ·81649, &c.

11. $82\frac{1}{2}$ cubic feet.

12. £36 „ 17*s.* „ $10\frac{2}{37}$*d.*; £57 „ 19*s.* „ $9\frac{19}{37}$*d.*; £100 „ 2*s.* „ $8\frac{16}{37}$*d.*

13. £131 „ 2*s.* „ 6*d.* 14. 12 per cent.

Previous Examination, March, 1865.

1. 99009000. 2. 102026, with 29 remainder.

3. £125 „ 4*s.* „ 9*d.* 4. Cf. § 44, 48.

5. $31097\frac{13}{21}$. 6. ·00296, &c. 7. ·142857.

8. 15. 9. £130. 10. £70 „ 17*s.* „ 6*d.*

11. $37\frac{5}{9}$ sq. yds. 12. 9*s.* „ $4\frac{1}{2}$*d.* 13. $8\frac{4}{9}$ miles from *A*.

Previous Examination, December, 1866.

1. 615400023. One thousand six hundred and fifty-nine million, eight hundred thousand, two hundred and five.

2. 5684986053; 31415. 3. £15 „ 6*s.* „ $4\frac{3}{4}$*d.*; £2770 „ 9*s.* „ 11*d.*

4. $\frac{17}{31}$; 2. 5. 16, and $12\frac{1}{2}$.

6. ·136, and ·2976; $\frac{11}{16000}$, and $\frac{3}{7}$.

7. ·00003738028, and ·576.

8. 7199; ·117; also ·765366, &c.

9. *A*, £39 „ 7*s.* „ $7\frac{1}{2}$*d.*; *B*, £65 „ 12*s.* „ $8\frac{1}{2}$*d.*; *C*, £105 „ 0*s.* „ 4*d.*; *D*, £118 „ 2*s.* „ $10\frac{1}{2}$*d.*

10. £315 „ 10*s.* „ 8*d.*, and $317\frac{39021}{66262}$ days, or $\frac{57655}{66262}$ yrs. Also £9 „ 5*s.* „ $2\frac{2}{3}$*d.*

11. £1340 „ 1*s.* „ $10\frac{783}{800}$*d.* 12. $5\frac{129}{540}$ days.

13. 3 feet „ 8 inches. Also 8 tons „ 3 cwt. „ 3 qrs. „ $1\frac{4}{15}$ lbs.

Previous Examination, December, 1867.

1. Cf. § 11, 12. 2. 31 years. 3. 83772.

4. 1267, with remainder 54. 5. 82008; 2701·8.

6. 4·4721, &c. 7. $1\frac{1003}{1848}$. 8. 670 1455.

9. £15000; £5000. 10. $33\frac{1}{3}$. 11. 72.

12. 7 gulden. 13. 5 hours.

14. *A* had 3240 votes, *B* 2916, *D* 2052, *C* 1944, *E* 1728. Number of electors 6480.

Previous Examination, December, 1868.

1. 95367431640625, and 625.

2. 87 yards; also 4959 cubic feet. 3. 6680 ducats.

4. $998999\frac{3847}{4946}$. 5. Cf. § 71. 6. $3{\cdot}52\dot{2}\dot{7}$.

7. Cf. table on page 152. £13658 „ 12*s.* „ 5*d.*, and £279 „ 16*s.* „ 6*d.* 8. 11·01; and $75\frac{1}{8}$.

9. 66 crowns, 44 shillings, and 22 pennies.

10. £2290 „ 5*s.* „ 3*d.* 11. £120 „ 0*s.* „ 3*d.* the debtor.

12. 20 days.

13. The larger wheels make 1002 and 1000 revolutions, the smaller 1169 and $1166\frac{2}{3}$ revolutions.

Previous Examination, March, 1869.

1. 152. Cf. § 19. 2. 1·9. 3. 442, with remainder 11.

4. $73{\cdot}524\dot{9}42221\dot{4}$, and 3·628693554. 5. $\frac{20}{63}$.

6. £17 „ 4*s.* „ 2*d.* 7. Cf. § 116. 8. 12*s.* „ $4\frac{2}{9}$*d.*

9. £4307 „ 11*s.* „ $10\frac{1}{2}$*d.*, and £75 „ 10*s.* „ $4\frac{1}{2}$*d.*

10. $41\frac{13}{27}$ lbs. 11. 9*s.* „ $7\frac{23}{25}$*d.*

12. $2\frac{1}{2}$ (nearly) per cent. 13. £8. 14. $3\frac{1}{2}$ hours.

15. 5*s.* „ 10*d.*

CAMBRIDGE: PRINTED BY C. J. CLAY, M.A. AT THE UNIVERSITY PRESS.

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